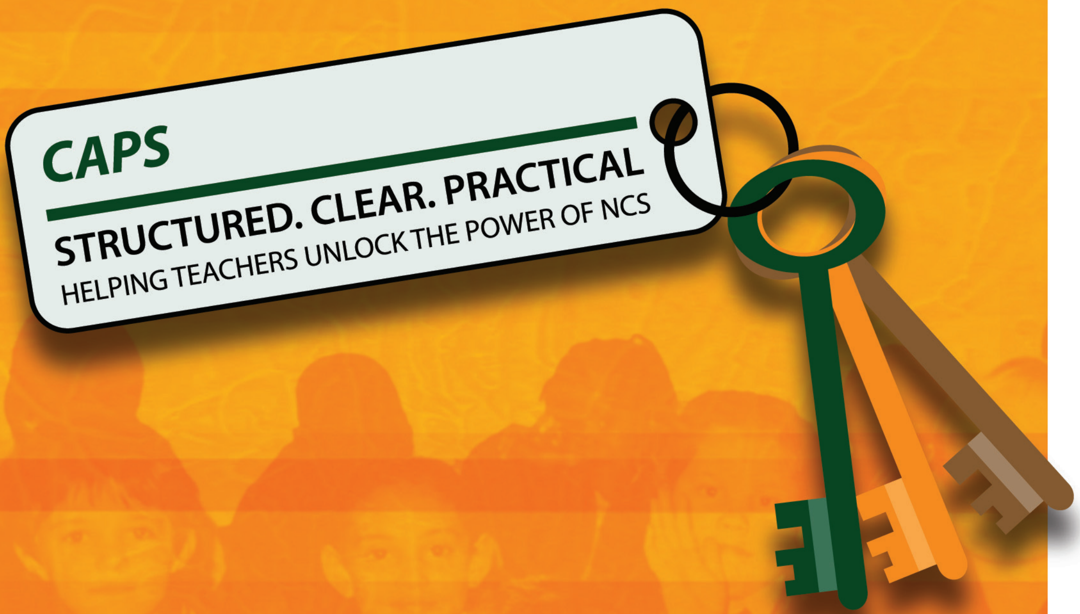
MATHEMATICS

National Curriculum Statement (NCS)

Curriculum and Assessment Policy Statement



Further Education and Training Phase
Grades 10-12

basic education
Department:
Basic Education
REPUBLIC OF SOUTH AFRICA

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CURRICULUM AND ASSESSMENT POLICY STATEMENT
GRADES 10-12

MATHEMATICS

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FOREWORD BY THE MINISTER

Our national curriculum is the culmination of our efforts over a period of seventeen years to transform the curriculum bequeathed to us by apartheid. From the start of democracy we have built our curriculum on the values that inspired our Constitution (Act 108 of 1996). The Preamble to the Constitution states that the aims of the Constitution are to:

- heal the divisions of the past and establish a society based on democratic values, social justice and fundamental human rights;
- improve the quality of life of all citizens and free the potential of each person;
- lay the foundations for a democratic and open society in which government is based on the will of the people and every citizen is equally protected by law; and
- build a united and democratic South Africa able to take its rightful place as a sovereign state in the family of nations.

Education and the curriculum have an important role to play in realising these aims.

In 1997 we introduced outcomes-based education to overcome the curricular divisions of the past, but the experience of implementation prompted a review in 2000. This led to the first curriculum revision: the *Revised National Curriculum Statement Grades R-9* and the *National Curriculum Statement Grades 10-12* (2002).

Ongoing implementation challenges resulted in another review in 2009 and we revised the *Revised National Curriculum Statement* (2002) to produce this document.

From 2012 the two 2002 curricula, for *Grades R-9* and *Grades 10-12* respectively, are combined in a single document and will simply be known as the *National Curriculum Statement Grades R-12.* The *National Curriculum Statement for Grades R-12* builds on the previous curriculum but also updates it and aims to provide clearer specification of what is to be taught and learnt on a term-by-term basis.

The *National Curriculum Statement Grades R-12* accordingly replaces the Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines with the

(a) Curriculum and Assessment Policy Statements (CAPS) for all approved subjects listed in this document;

(b) *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12*; and

(c) *National Protocol for Assessment Grades R-12.*

MRS ANGIE MOTSHEKGA, MP
MINISTER OF BASIC EDUCATION

CONTENTS

SECTION 1

INTRODUCTION TO THE CURRICULUM AND ASSESSMENT POLICY STATEMENTS FOR MATHEMATICS GRADES 10-12

1.1 Background

The *National Curriculum Statement Grades R-12 (NCS)* stipulates policy on curriculum and assessment in the schooling sector.

To improve implementation, the National Curriculum Statement was amended, with the amendments coming into effect in January 2012. A single comprehensive Curriculum and Assessment Policy document was developed for each subject to replace Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines in Grades R-12.

1.2 Overview

(a) The *National Curriculum Statement Grades R-12 (January 2012)* represents a policy statement for learning and teaching in South African schools and comprises the following:

- *(i) Curriculum and Assessment Policy Statements for each approved school subject;*
- *(ii) The policy document, National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12; and*
- *(iii) The policy document, National Protocol for Assessment Grades R-12 (January 2012).*

(b) The *National Curriculum Statement Grades R-12 (January 2012)* replaces the two current national curricula statements, namely the

- *(i) Revised National Curriculum Statement Grades R-9, Government Gazette No. 23406 of 31 May 2002, and*
- *(ii) National Curriculum Statement Grades 10-12 Government Gazettes, No. 25545 of 6 October 2003 and No. 27594 of 17 May 2005.*

(c) The national curriculum statements contemplated in subparagraphs b(i) and (ii) comprise the following policy documents which will be incrementally repealed by the *National Curriculum Statement Grades R-12 (January 2012)* during the period 2012-2014:

- *(i) The Learning Area/Subject Statements, Learning Programme Guidelines and Subject Assessment Guidelines for Grades R-9 and Grades 10-12;*
- *(ii) The policy document, National Policy on assessment and qualifications for schools in the General Education and Training Band, promulgated in Government Notice No. 124 in Government Gazette No. 29626 of 12 February 2007;*
- *(iii) The policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), promulgated in Government Gazette No.27819 of 20 July 2005;*

(iv) *The policy document, An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding learners with special needs, published in Government Gazette, No.29466 of 11 December 2006, is incorporated in the policy document, National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12; and*

(v) *The policy document, An addendum to the policy document, the National Senior Certificate: A qualification at Level 4 on the National Qualifications Framework (NQF), regarding the National Protocol for Assessment (Grades R-12), promulgated in Government Notice No.1267 in Government Gazette No. 29467 of 11 December 2006.*

(d) The policy document, *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12*, and the sections on the Curriculum and Assessment Policy as contemplated in Chapters 2, 3 and 4 of this document constitute the norms and standards of the *National Curriculum Statement Grades R-12.* It will therefore, in terms of *section 6A* of the *South African Schools Act, 1996 (Act No. 84 of 1996,)* form the basis for the Minister of Basic Education to determine minimum outcomes and standards, as well as the processes and procedures for the assessment of learner achievement to be applicable to public and independent schools.

1.3 General aims of the South African Curriculum

(a) The *National Curriculum Statement Grades R-12* gives expression to the knowledge, skills and values worth learning in South African schools. This curriculum aims to ensure that children acquire and apply knowledge and skills in ways that are meaningful to their own lives. In this regard, the curriculum promotes knowledge in local contexts, while being sensitive to global imperatives.

(b) The National Curriculum Statement Grades R-12 serves the purposes of:

- equipping learners, irrespective of their socio-economic background, race, gender, physical ability or intellectual ability, with the knowledge, skills and values necessary for self-fulfilment, and meaningful participation in society as citizens of a free country;
- providing access to higher education;
- facilitating the transition of learners from education institutions to the workplace; and
- providing employers with a sufficient profile of a learner’s competences.

(c) The National Curriculum Statement Grades R-12 is based on the following principles:

- Social transformation: ensuring that the educational imbalances of the past are redressed, and that equal educational opportunities are provided for all sections of the population;
- Active and critical learning: encouraging an active and critical approach to learning, rather than rote and uncritical learning of given truths;
- High knowledge and high skills: the minimum standards of knowledge and skills to be achieved at each grade are specified and set high, achievable standards in all subjects;
- Progression: content and context of each grade shows progression from simple to complex;

- Human rights, inclusivity, environmental and social justice: infusing the principles and practices of social and environmental justice and human rights as defined in the Constitution of the Republic of South Africa. The National Curriculum Statement Grades R-12 is sensitive to issues of diversity such as poverty, inequality, race, gender, language, age, disability and other factors;

- Valuing indigenous knowledge systems: acknowledging the rich history and heritage of this country as important contributors to nurturing the values contained in the Constitution; and

- Credibility, quality and efficiency: providing an education that is comparable in quality, breadth and depth to those of other countries.

(d) The National Curriculum Statement Grades R-12 aims to produce learners that are able to:

- identify and solve problems and make decisions using critical and creative thinking;

- work effectively as individuals and with others as members of a team;

- organise and manage themselves and their activities responsibly and effectively;

- collect, analyse, organise and critically evaluate information;

- communicate effectively using visual, symbolic and/or language skills in various modes;

- use science and technology effectively and critically showing responsibility towards the environment and the health of others; and

- demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.

(e) Inclusivity should become a central part of the organisation, planning and teaching at each school. This can only happen if all teachers have a sound understanding of how to recognise and address barriers to learning, and how to plan for diversity.

The key to managing inclusivity is ensuring that barriers are identified and addressed by all the relevant support structures within the school community, including teachers, District-Based Support Teams, Institutional-Level Support Teams, parents and Special Schools as Resource Centres. To address barriers in the classroom, teachers should use various curriculum differentiation strategies such as those included in the Department of Basic Education's *Guidelines for Inclusive Teaching and Learning* (2010).

1.4 Time Allocation

1.4.1 Foundation Phase

(a) The instructional time in the Foundation Phase is as follows:

SUBJECT	GRADE R (HOURS)	GRADES 1-2 (HOURS)	GRADE 3 (HOURS)
Home Language	10	8/7	8/7
First Additional Language		2/3	3/4
Mathematics	7	7	7
Life Skills	**6**	**6**	**7**
• Beginning Knowledge	(1)	(1)	(2)
• Creative Arts	(2)	(2)	(2)
• Physical Education	(2)	(2)	(2)
• Personal and Social Well-being	(1)	(1)	(1)
TOTAL	**23**	**23**	**25**

(b) Instructional time for Grades R, 1 and 2 is 23 hours and for Grade 3 is 25 hours.

(c) Ten hours are allocated for languages in Grades R-2 and 11 hours in Grade 3. A maximum of 8 hours and a minimum of 7 hours are allocated for Home Language and a minimum of 2 hours and a maximum of 3 hours for Additional Language in Grades 1-2. In Grade 3 a maximum of 8 hours and a minimum of 7 hours are allocated for Home Language and a minimum of 3 hours and a maximum of 4 hours for First Additional Language.

(d) In Life Skills Beginning Knowledge is allocated 1 hour in Grades R-2 and 2 hours as indicated by the hours in brackets for Grade 3.

1.4.2 Intermediate Phase

(a) The instructional time in the Intermediate Phase is as follows:

SUBJECT	HOURS
Home Language	6
First Additional Language	5
Mathematics	6
Natural Sciences and Technology	3,5
Social Sciences	3
Life Skills	**4**
• Creative Arts	(1,5)
• Physical Education	(1)
• Personal and Social Well-being	(1,5)
TOTAL	**27,5**

1.4.3 Senior Phase

(a) The instructional time in the Senior Phase is as follows:

SUBJECT	HOURS
Home Language	5
First Additional Language	4
Mathematics	4,5
Natural Sciences	3
Social Sciences	3
Technology	2
Economic Management Sciences	2
Life Orientation	2
Creative Arts	2
TOTAL	**27,5**

1.4.4 Grades 10-12

(a) The instructional time in Grades 10-12 is as follows:

SUBJECT	TIME ALLOCATION PER WEEK (HOURS)
Home Language	**4.5**
First Additional Language	**4.5**
Mathematics	**4.5**
Life Orientation	**2**
A minimum of any three subjects selected from **Group B** Annexure B, Tables B1-B8 of the policy document, *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R-12,* subject to the provisos stipulated in paragraph 28 of the said policy document.	**12 (3x4h)**
TOTAL	**27,5**

The allocated time per week may be utilised only for the minimum required NCS subjects as specified above, and may not be used for any additional subjects added to the list of minimum subjects. Should a learner wish to offer additional subjects, additional time must be allocated for the offering of these subjects

SECTION 2

Introduction

In Chapter 2, the Further Education and Training (FET) Phase Mathematics CAPS provides teachers with a definition of mathematics, specific aims, specific skills, focus of content areas and the weighting of content areas.

2.1 What is Mathematics?

Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making.Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.

2.2 Specific Aims

1. To develop fluency in computation skills without relying on the usage of calculators.

2. Mathematical modeling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible.

3. To provide the opportunity to develop in learners the ability to be methodical, to generalize, make conjectures and try to justify or prove them.

4. To be able to understand and work with number system.

5. To show Mathematics as a human creation by including the history of Mathematics.

6. To promote accessibility of Mathematical content to all learners. It could be achieved by catering for learners with different needs.

7. To develop problem-solving and cognitive skills. Teaching should not be limited to **"how"**but should **rather** feature the **"when"** and **"why"** of problem types. Learning procedures and proofs without a good understanding of why they are important will leave learners ill-equipped to use their knowledge in later life.

8. To prepare the learners for further education and training as well as the world of work.

2.3 Specific Skills

To develop essential mathematical skills the learner should:

- develop the correct use of the language of Mathematics;

- collect, analyse and organise quantitative data to evaluate and critique conclusions;

- use mathematical process skills to identify, investigate and solve problems creatively and critically;
- use spatial skills and properties of shapes and objects to identify, pose and solve problems creatively and critically;
- participate as responsible citizens in the life of local, national and global communities; and
- communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams.

2.4 Focus of Content Areas

Mathematics in the FET Phase covers ten main content areas. Each content area contributes towards the acquisition of the specific skills. The table below shows the main topics in the FET Phase.

The Main Topics in the FET Mathematics Curriculum

1.	**Functions**
2.	**Number Patterns, Sequences, Series**
3.	**Finance, growth and decay**
4.	**Algebra**
5.	**Differential Calculus**
6.	**Probability**
7.	**Euclidean Geometry and Measurement**
8.	**Analytical Geometry**
9.	**Trigonometry**
10.	**Statistics**

2.5 Weighting of Content Areas

The weighting of mathematics content areas serves two primary purposes: *firstly* the weighting gives guidance on the amount of time needed to address adequately the content within each content area; *secondly* the weighting gives guidance on the spread of content in the examination (especially end of the year summative assessment).

Weighting of Content Areas			
Description	**Grade 10**	**Grade 11**	**Grade. 12**
PAPER 1 (Grades 12:bookwork: maximum 6 marks)			
Algebra and Equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and Sequences	15 ± 3	25 ± 3	25 ± 3
Finance and Growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and Graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	**100**	**150**	**150**
PAPER 2: Grades 11 and 12: theorems and/or trigonometric proofs: maximum 12 marks			
Description	**Grade 10**	**Grade 11**	**Grade 12**
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	40 ± 3
Euclidean Geometry and Measurement	30 ± 3	50 ± 3	50 ± 3
TOTAL	**100**	**150**	**150**

2.6 Mathematics in the FET

The subject Mathematics in the Further Education and Training Phase forges the link between the Senior Phase and the Higher/Tertiary Education band. All learners passing through this phase acquire a functioning knowledge of the Mathematics that empowers them to make sense of society. It ensures access to an extended study of the mathematical sciences and a variety of career paths.

In the FET Phase, learners should be exposed to mathematical experiences that give them many opportunities to develop their mathematical reasoning and creative skills in preparation for more abstract mathematics in Higher/ Tertiary Education institutions.

SECTION 3

Introduction

Chapter 3 provides teachers with:

- specification of content to show progression;
- clarification of content with teaching guidelines; and
- allocation of time.

3.1 Specification of Content to show Progression

The specification of content shows progression in terms of concepts and skills from Grade 10 to 12 for each content area. However, in certain topics the concepts and skills are similar in two or three successive grades. The clarification of content gives guidelines on how progression should be addressed in these cases. The specification of content should therefore be read in conjunction with the clarification of content.

3.1.1 Overview of topics

Grade 10	Grade 11	Grade 12
1. FUNCTIONS		
Work with relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear and some quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions.	Extend Grade 10 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear and quadratic polynomial functions, exponential functions, some rational functions and trigonometric functions.	Introduce a more formal definition of a function and extend Grade 11 work on the relationships between variables in terms of numerical, graphical, verbal and symbolic representations of functions and convert flexibly between these representations (tables, graphs, words and formulae). Include linear, quadratic and some cubic polynomial functions, exponential and logarithmic functions, and some rational functions.
Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effect of the parameter which results in a vertical shift and that which results in a vertical stretch and /or a reflection about the *x* axis.	Generate as many graphs as necessary, initially by means of point-by-point plotting, supported by available technology, to make and test conjectures and hence generalise the effects of the parameter which results in a horizontal shift and that which results in a horizontal stretch and/or reflection about the *y* axis.	The inverses of prescribed functions and be aware of the fact that, in the case of many-to-one functions, the domain has to be restricted if the inverse is to be a function.
Problem solving and graph work involving the prescribed functions.	Problem solving and graph work involving the prescribed functions. Average gradient between two points.	Problem solving and graph work involving the prescribed functions (including the logarithmic function).
2. NUMBER PATTERNS, SEQUENCES AND SERIES		
Investigate number patterns leading to those where there is constant difference between consecutive terms, and the general term is therefore linear.	Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	Identify and solve problems involving number patterns that lead to arithmetic and geometric sequences and series, including infinite geometric series.
3. FINANCE, GROWTH AND DECAY		
Use simple and compound growth formulae $A = P(1 + in)$ and $A = P(1 + i)^n$ to solve problems (including interest, hire purchase, inflation, population growth and other real life problems).	Use simple and compound decay formulae $A = P(1 + in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). Link to work on functions.	(a) Calculate the value of n in the formulae $A = P(1 + i)^n$ and $A = P(1 - i)^n$ (b) Apply knowledge of geometric series to solve annuity and bond repayment problems.
The implications of fluctuating foreign exchange rates.	The effect of different periods of compounding growth and decay (including effective and nominal interest rates).	Critically analyse different loan options.

4. ALGEBRA					
	(a) Understand that real numbers can be irrational or rational.		Take note that there exist numbers other than those on the real number line, the so-called non-real numbers. It is possible to square certain non-real numbers and obtain negative real numbers as answers. Nature of roots.		
	(a) Simplify expressions using the laws of exponents for rational exponents. (b) Establish between which two integers a given simple surd lies. (c) Round real numbers to an appropriate degree of accuracy (to a given number of decimal digits).		(a) Apply the laws of exponents to expressions involving rational exponents. (b) Add, subtract, multiply and divide simple surds.		Demonstrate an understanding of the definition of a logarithm and any laws needed to solve real life problems.
	Manipulate algebraic expressions by: • multiplying a binomial by a trinomial; • factorising trinomials; • factorising the difference and sums of two cubes; • factorising by grouping in pairs; and • simplifying, adding and subtracting algebraic fractions with denominators of cubes (limited to sum and difference of cubes).		Revise factorisation.		• Take note and understand, the Remainder and Factor Theorems for polynomials up to the third degree. • Factorise third-degree polynomials (including examples which require the Factor Theorem).
	Solve: • linear equations; • quadratic equations; • literal equations (changing the subject of a formula); • exponential equations; • linear inequalities; • system of linear equations; and • word problems.		Solve: • quadratic equations; • quadratic inequalities in one variable and interpret the solution graphically; and • equations in two unknowns, one of which is linear the other quadratic, algebraically or graphically.		

5. DIFFERENTIAL CALCULUS					
					(a) An intuitive understanding of the concept of a limit. (b) Differentiation of specified functions from first principles. (c) Use of the specified rules of differentiation. (d) The equations of tangents to graphs. (e) The ability to sketch graphs of cubic functions. (f) Practical problems involving optimization and rates of change (including the calculus of motion).
6. PROBABILITY					
	(a) Compare the relative frequency of an experimental outcome with the theoretical probability of the outcome. (b) Venn diagrams as an aid to solving probability problems. (c) Mutually exclusive events and complementary events. (d) The identity for any two events A and B: *P(A or B) = P(A) + (B) - P(A and B)*		(a) Dependent and independent events. (b) Venn diagrams or contingency tables and tree diagrams as aids to solving probability problems (where events are not necessarily independent).		(a) Generalisation of the fundamental counting principle. (b) Probability problems using the fundamental counting principle.
7. EUCLIDEAN GEOMETRY AND MEASUREMENT					
	(a) Revise basic results established in earlier grades. (b) Investigate line segments joining the mid-points of two sides of a triangle. (c) Properties of special quadrilaterals.		(a) Investigate and prove theorems of the geometry of circles assuming results from earlier grades, together with one other result concerning tangents and radii of circles. (b) Solve circle geometry problems, providing reasons for statements when required. (c) Prove riders.		(a) Revise earlier (Grade 9) work on the necessary and sufficient conditions for polygons to be similar. (b) Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem); • that equiangular triangles are similar; • that triangles with sides in proportion are similar; • the Pythagorean Theorem by similar triangles; and • riders.

	Solve problems involving volume and surface area of solids studied in earlier grades as well as spheres, pyramids and cones and combinations of those objects.		Revise Grade 10 work.		
8. TRIGONOMETRY					
	(a) Definitions of the trigonometric ratios sin θ, cos θ and tan θ in a right-angled triangles. (b) Extend the definitions of sin θ, cos θ and tan θ to $0^\circ \le \theta \le 360^\circ$. (c) Derive and use values of the trigonometric ratios (without using a calculator for the special angles $\theta \in \{0^\circ;30^\circ;45^\circ;60^\circ;90^\circ\}$ (d) Define the reciprocals of trigonometric ratios.		(a) Derive and use the identities: $\tan\theta = \frac{\sin\theta}{\cos\theta}$ and $\sin^2\theta + \sin^2\theta = 1$. (b) Derive the reduction formulae. (c) Determine the general solution and / or specific solutions of trigonometric equations. (d) Establish the sine, cosine and area rules.		Proof and use of the compound angle and double angle identities
	So Solve problems in two dimensions.		Solve problems in 2-dimensions.		Solve problems in two and three dimensions.
9. ANALYTICAL GEOMETRY					
	Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1 ; y_1)$ and $(x_2 ; y_2)$, a formula for calculating: • the distance between the two points; • the gradient of the line segment joining the points; • conditions for parallel and perpendicular lines; and • the co-ordinates of the mid-point of the line segment joining the points.		Use a Cartesian co-ordinate system to derive and apply : • the equation of a line through two given points; • the equation of a line through one point and parallel or perpendicular to a given line; and • the inclination of a line.		Use a two-dimensional Cartesian co-ordinate system to derive and apply: • the equation of a circle (any centre); and • the equation of a tangent to a circle at a given point on the circle.
10. STATISTICS					
	(a) Collect, organise and interpret univariate numerical data in order to determine: • measures of central tendency; • five number summary; • box and whisker diagrams; and • measures of dispersion.		(a) Represent measures of central tendency and dispersion in univariate numerical data by: • using ogives; and • calculating the variance and standard deviation of sets of data manually (for small sets of data) and using calculators (for larger sets of data) and representing results graphically. (b) Represent Skewed data in box and whisker diagrams, and frequency polygons. Identify outliers.		(a) Represent bivariate numerical data as a scatter plot and suggest intuitively and by simple investigation whether a linear, quadratic or exponential function would best fit the data. (b) Use a calculator to calculate the linear regression line which best fits a given set of bivariate numerical data. (c) Use a calculator to calculate the correlation co-efficient of a set of bivariate numerical data and make relevant deductions.

3.2 Content Clarification with teaching guidelines

In Chapter 3, content clarification includes:

- teaching guidelines;
- sequencing of topics per term; and
- pacing of topics over the year.
- Each content area has been broken down into topics. The sequencing of topics within terms gives an idea of how content areas can be spread and re-visited throughout the year.
- The examples discussed in the Clarification Column in the annual teaching plan which follows are by no means a complete representation of all the material to be covered in the curriculum. They only serve as an indication of some questions on the topic at different cognitive levels. Text books and other resources should be consulted for a complete treatment of all the material.
- The order of topics is not prescriptive, but ensure that part of trigonometry is taught in the first term and more than six topics are covered / taught in the first two terms so that assessment is balanced between paper 1 and 2.

3.2.1 Allocation of Teaching Time

Time allocation for Mathematics: 4 hours and 30 minutes, e.g. six forty five-minutes periods, per week in grades 10, 11 and 12.

Terms	Grade 10		Grade 11		Grade 12	
		No. of weeks		No. of weeks		No. of weeks
Term 1	Algebraic expressions Exponents Number patterns Equations and inequalities Trigonometry	3 2 1 2 3	Exponents and surds Equations and inequalities Number patterns Analytical Geometry	3 3 2 3	Patterns, sequences and series Functions and inverse functions Exponential and logarithmic functions Finance, growth and decay Trigonometry - compound angles	3 3 1 2 2
Term 2	Functions Trigonometric functions Euclidean Geometry MID-YEAR EXAMS	4 1 3 3	Functions Trigonometry (reduction formulae, graphs, equations) MID-YEAR EXAMS	4 4 3	Trigonometry 2D and 3D Polynomial functions Differential calculus Analytical Geometry MID-YEAR EXAMS	2 1 3 2 3
Term 3	Analytical Geometry Finance and growth Statistics Trigonometry Euclidean Geometry Measurement	2 2 2 2 1 1	Measurement Euclidean Geometry Trigonometry (sine, area, cosine rules) Probability Finance, growth and decay	1 3 2 2 2	Geometry Statistics (regression and correlation) Counting and Probability Revision TRIAL EXAMS	2 2 2 1 3
Term 4	Probability Revision EXAMS	2 4 3	Statistics Revision EXAMS	3 3 3	Revision EXAMS	3 6

The detail which follows includes examples and numerical references to the Overview.

3.2.2 Sequencing and Pacing of Topics

MATHEMATICS: GRADE 10 PACE SETTER

1	TERM 1										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Algebraic expressions			Exponents		Number patterns	Equations and inequalities		Trigonometry		
Assessment	Investigation or project						Test				
Date completed											

2	TERM 2										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Functions				Trigonometric functions	Euclidean Geometry					
Assessment	Assignment / Test								MID-YEAR EXAMINATION		
Date completed											

3	TERM 3									
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**
Topics	Analytical Geometry		Finance and growth		Statistics		Trigonometry		Euclidean Geometry	Measurement
Assessment	Test						Test			
Date completed										

4	TERM 4									
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**
Topics	Probability		Revision							Admin
Assessment	Test						Examinations			
Date completed										

Paper 1: 2 hours	
Algebraic expressions and equations (and inequalities) exponents	30
Number Patterns	10
Functions and graphs	30
Finance and growth	15
Probability	15
Total marks	100

Paper 2: 2 hours	
Euclidean geometry and measurement	20
Analytical geometry	15
Trigonometry	50
Statistics	15
Total marks	100

MATHEMATICS: GRADE 11 PACE SETTER											
1	**TERM 1**										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Exponents and surds			Equations and inequalities			Number patterns		Analytical Geometry		
Assessment	Investigation or project						Test				
Date completed											
2	**TERM 2**										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Functions				Trigonometry (reduction formulae, graphs, equations)						
Assessment	Assignment / Test								MID-YEAR EXAMINATION		
Date completed											
3	**TERM 3**										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	
Topics	Measurement	Euclidean Geometry			Trigonometry (sine, cosine and area rules)		Finance, growth and decay		Probability		
Assessment	Test					Test					
Date completed											

4	**TERM 4**										Paper 1: 3 hours		Paper 2: 3 hours	
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	Algebraic expressions and equations (and inequalities) Number patterns Functions and graphs Finance, growth and decay Probability	45 25 45 15 20	Euclidean Geometry and measurement Analytical geometry Trigonometry Statistics	40 30 60 20
Topics	Statistics			Revision			FINAL EXAMINATION			Admin				
Assessment	Test													
Date completed											Total marks	150	Total marks	150

MATHEMATICS: GRADE 12 PACE SETTER

1	TERM 1										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Number patterns, sequences and series			Functions: Formal definition; inverses			Functions: exponential and logarithmic	Finance, growth and decay		Trigonometry	
Assessment	Test					Investigation or project		Assignment			
Date completed											

2	TERM 2										
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	**WEEK 11**
Topics	Trigonometry		Functions: polynomials	Differential Calculus			Analytical Geometry				
Assessment	Test								MID-YEAR EXAMINATION		
Date completed											

3	TERM 3									
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**
Topics	Geometry		Statistics		Counting and Probability		Revision	**TRIAL EXAMINATION**		
Assessment	Test									
Date completed										

4	TERM 4										Paper 1: 3 hours		Paper 2: 3 hours	
Weeks	**WEEK 1**	**WEEK 2**	**WEEK 3**	**WEEK 4**	**WEEK 5**	**WEEK 6**	**WEEK 7**	**WEEK 8**	**WEEK 9**	**WEEK 10**	Algebraic expressions and equations (and inequalities)	25	Euclidean Geometry and measurement	50
Topics	Revision			FINAL EXAMINATION						Admin	Number patterns	25	Analytical Geometry	40
Assessment											Functions and graphs	35	Trigonometry	40
											Finance, growth and decay	15	Statistics	20
											Differential Calculus	35		
											Counting and probability	15		
Date completed											Total marks	150	Total marks	150

3.2.3 Topic allocation per term

GRADE 10: TERM 1			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem-solving (P)
3	**Algebraic expressions**	1. Understand that real numbers can be rational or irrational. 2. Establish between which two integers a given simple surd lies. 3. Round real numbers to an appropriate degree of accuracy. 4. Multiplication of a binomial by a trinomial. 5. Factorisation to include types taught in grade 9 and: • trinomials • grouping in pairs • sum and difference of two cubes 6. Simplification of algebraic fractions using factorization with denominators of cubes (limited to sum and difference of cubes).	**Examples to illustrate the different cognitive levels involved in factorisation:** 1. Factorise fully: 1.1. $m^2 - 2m + 1$ (revision) Learners must be able to recognise the simplest perfect squares. (R) 1.2. $2x^2 - x - 3$ This type is routine and appears in all texts. (R) 1.3. $\frac{y^2}{2} - \frac{13y}{2} + 18$ Learners are required to work with fractions and identify when an expression has been "fully factorised". (R) 2. Simplify $\frac{1-2x}{4x^2-1} - \frac{x+4}{2x^2-3x+1} + \frac{1}{1-x}$ (C)
2	**Exponents**	1. Revise laws of exponents learnt in Grade 9 where $x, y > 0$ and $m, n \in Z$: • $x^m \times x^n = x^{m+n}$ • $x^m \div x^n = x^{m-n}$ • $(x^m)^n = x^{mn}$ • $x^m \times y^m = (xy)^m$ Also by definition: • $x^{-n} = \frac{1}{x^n}$, $x \neq 0$, and • $x^0 = 1$, $x \neq 0$ 2. Use the laws of exponents to simplify expressions and solve equations, accepting that the rules also hold for $m, n \in Q$.	**Examples:** 1. Simplify: $(3 \times 5^2)^3 - 75$ A simple two-step procedure is involved. (R) 2. Simplify $\frac{9^x - 1}{3^x + 1}$ Assuming this type of question has not been taught, spotting that the numerator can be factorised as a difference of squares requires insight. (P) 3. Solve for x: 3.1 $2^x = 0{,}125$ (R) 3.2 $2x^{\frac{3}{2}} = 54$ (R) 3.3 $3^{x+1} + 3^{x-1} = \frac{10}{9}$ (C) 3.4 $x^{\frac{1}{2}} + 3x^{\frac{1}{4}} - 18 = 0$ (C)

GRADE 10: TERM 1			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Numbers and patterns**	Patterns: Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term (without using a formula-see content overview) is therefore linear.	**Comment:** • Arithmetic sequence is done in Grade 12, hence $T_n = a + (n-1)d$ is not used in Grade 10. **Examples:** 1. Determine the 5th and the *n*th terms of the number pattern 10; 7; 4; 1; …. There is an algorithmic approach to answering such questions. (R) 2. If the pattern MATHSMATHSMATHS… is continued in this way, what will the 267th letter be? It is not immediately obvious how one should proceed, unless similar questions have been tackled. (P)
2	**Equations and Inequalities**	1. Revise the solution of linear equations. 2. Solve quadratic equations (by factorisation). 3. Solve simultaneous linear equations in two unknowns. 4. Solve word problems involving linear, quadratic or simultaneous linear equations. 5. Solve literal equations (changing the subject of a formula). 6. Solve linear inequalities (and show solution graphically). Interval notation must be known.	**Examples**: 1. Solve for *x*: $\frac{2x-3}{3} - 3x = \frac{2x}{6}$ (R) 2. Solve for *m*: $2m^2 - m = 1$ (R) 3. Solve for *x* and *y*: $x + 2y = 1$; $\frac{x}{3} + \frac{y}{2} = 1$ (C) 4. Solve for *r* in terms of *V*, Π and *h*: $V = \Pi r^2 h$ (R) 5. Solve for *x*: $-1 \le 2 - 3x \le 8$ (C)

GRADE 10: TERM 1

No of Weeks	Topic	Curriculum statement	Clarification
3	**Trigonometry**	1. Define the trigonometric ratios sin θ, cos θ and tan θ, using right-angled triangles. 2. Extend the definitions of sin θ, cos θ and tan θ for $0^0 \leq \theta \leq 360^0$. 3. Define the reciprocals of the trigonometric ratios cosec θ, sec θ and cot θ, using right-angled triangles (these three reciprocals should be examined in grade 10 only). 4. Derive values of the trigonometric ratios for the special cases (without using a calculator) $\theta \in \{0^0; 30^0; 45^0; 60^0; 90^0\}$. 5. Solve two-dimensional problems involving right-angled triangles. 6. Solve simple trigonometric equations for angles between 0^0 and 90^0. 7. Use diagrams to determine the numerical values of ratios for angles from 0^0 to 360^0.	**Comment**: It is important to stress that: • similarity of triangles is fundamental to the trigonometric ratios sin θ, cos θ and tan θ; • trigonometric ratios are independent of the lengths of the sides of a similar right-angled triangle and depend (uniquely) only on the angles, hence we consider them as functions of the angles; • doubling a ratio has a different effect from doubling an angle, for example, generally $2\sin\theta \neq \sin 2\theta$; and • Solve equation of the form $\sin x = c$, or $2\cos x = c$, or $\tan 2x = c$, where c is a constant. **Examples**: 1. If $5\sin\theta + 4 = 0$ and $0^0 \leq \theta \leq 270^0$, calculate the value of $\sin^2\theta + \cos^2\theta$ without using a calculator. (R) 2. Let *ABCD* be a rectangle, with AB = 2*cm*. Let *E* be on *AD* such that $A\hat{B}E = 45^0$ and $B\hat{E}C = 75^0$. Determine the area of the rectangle. (P) 3. Determine the length of the hypotenuse of a right-angled triangle *ABC*, where $\hat{B} = 90^0$, $\hat{A} = 30^0$ and AB = 10cm (K) 4. Solve for *x*: $4\sin x - 1 = 3$ for $x \in [0°;90°]$ (C)

Assessment Term 1:

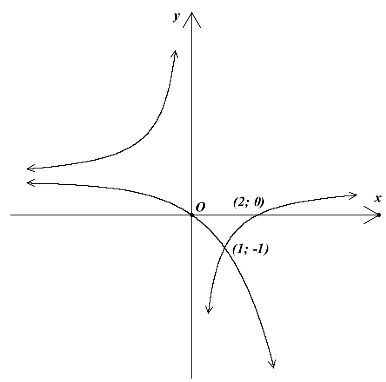
1. Investigation or project (only one project per year) (at least 50 marks)

   Example of an investigation:

   Imagine a cube of white wood which is dipped into red paint so that the surface is red, but the inside still white. If one cut is made, parallel to each face of the cube (and through the centre of the cube), then there will be 8 smaller cubes. Each of the smaller cubes will have 3 red faces and 3 white faces. Investigate the number of smaller cubes which will have 3, 2, 1 and 0 red faces if 2/3/4/…/*n* equally spaced cuts are made parallel to each face. This task provides the opportunity to investigate, tabulate results, make conjectures and justify or prove them.

2. Test (at least 50 marks and 1 hour). Make sure all topics are tested.

   Care needs to be taken to set questions on all four cognitive levels: approximately 20% knowledge, approximately 35% routine procedures, 30% complex procedures and 15% problem-solving.

GRADE 10: TERM 2			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
(4 + 1) **5**	**Functions**	1. The concept of a function, where a certain quantity (output value) uniquely depends on another quantity (input value). Work with relationships between variables using tables, graphs, words and formulae. Convert flexibly between these representations. Note: that the graph defined by $y = x$ should be known from Grade 9. 2. Point by point plotting of basic graphs defined by $y = x^2$, $y = \frac{1}{x}$ and $y = b^x$; $b > 0$ and $b \neq 1$ to discover shape, domain (input values), range (output values), asymptotes, axes of symmetry, turning points and intercepts on the axes (where applicable). 3. Investigate the effect of a and q on the graphs defined by $y = a.f(x) + q$, where $f(x) = x$, $f(x) = x^2$, $f(x) = \frac{1}{x}$ and $f(x) = b^x$, $b > 0, b \neq 1$. 4. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \sin\theta$ for $\theta \in [0°;360°]$ 5. Study the effect of a and q on the graphs defined by: $y = a\sin\theta + q$; $y = a\cos\theta + q$; and $y = a\tan\theta + q$ where $a, q \in$ Q for $\theta \in [0°;360°]$. 6. Sketch graphs, find the equations of given graphs and interpret graphs. **Note**: Sketching of the graphs must be based on the observation of number 3 and 5.	**Comments**: • A more formal definition of a function follows in Grade 12. At this level it is enough to investigate the way (unique) output values depend on how input values vary. The terms independent (input) and dependent (output) variables might be useful. • After summaries have been compiled about basic features of prescribed graphs and the effects of parameters a and q have been investigated: a: a vertical stretch (and/or a reflection about the x-axis) and q a vertical shift. The following examples might be appropriate: • Remember that graphs in some practical applications may be either discrete or continuous. **Examples:** 1. Sketched below are graphs of $f(x) = \frac{a}{x} + q$ and $g(x) = nb^x + t$. The horizontal asymptote of both graphs is the line y = 1. Determine the values of a , b , n , q and t . (C)  2. Sketch the graph defined by $y = -\sin x + \frac{1}{2}$ for $x \in [0°;360°]$ (R)

GRADE 10: TERM 2

Weeks	Topic	Curriculum statement	Clarification
3	**Euclidean Geometry**	1. Revise basic results established in earlier grades regarding lines, angles and triangles, especially the similarity and congruence of triangles. 2. Investigate line segments joining the mid-points of two sides of a triangle. 3. Define the following special quadrilaterals: the kite, parallelogram, rectangle, rhombus, square and trapezium. Investigate and make conjectures about the properties of the sides, angles, diagonals and areas of these quadrilaterals. Prove these conjectures.	**Comments:** • Triangles are similar if their corresponding angles are equal, or if the ratios of their sides are equal: Triangles ABC and DEF are similar if $\hat{A} = \hat{D}$, $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$. They are also similar if $\frac{AB}{DE} = \frac{BC}{EF} = \frac{CA}{FD}$. • We could define a parallelogram as a quadrilateral with two pairs of opposite sides parallel. Then we investigate and prove that the opposite sides of the parallelogram are equal, opposite angles of a parallelogram are equal, and diagonals of a parallelogram bisect each other. • It must be explained that a single counter example can disprove a Conjecture, but numerous specific examples supporting a conjecture do not constitute a general proof. **Example**: In quadrilateral KITE, KI = KE and IT = ET. The diagonals intersect at M. Prove that: 1. IM = ME and (R) 2. KT is perpendicular to IE. (P) As it is not obvious, first prove that $\Delta KIT \equiv \Delta KET$.
3	**Mid-year examinations**		

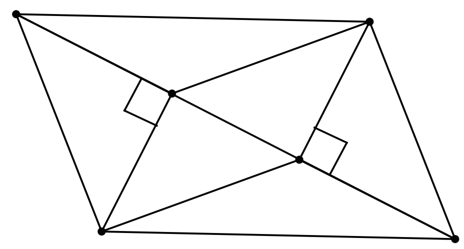
Assessment term 2:

1. Assignment / test (at least 50 marks)
2. Mid-year examination (at least 100 marks)

   One paper of 2 hours (100 marks) **or** Two papers - one, 1 hour (50 marks) and the other, 1 hour (50 marks)

GRADE 10: TERM 3			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	Represent geometric figures on a Cartesian co-ordinate system. Derive and apply for any two points $(x_1;y_1)$ and $(x_2;y_2)$ the formulae for calculating the: 1. distance between the two points; 2. gradient of the line segment connecting the two points (and from that identify parallel and perpendicular lines); and 3. coordinates of the mid-point of the line segment joining the two points.	**Example**: Consider the points $P(2;5)$ and $Q(-3;1)$ in the Cartesian plane. 1.1. Calculate the distance PQ. (K) 1.2 Find the coordinates of R if M $(-1;0)$ is the mid-point of PR. (R) 1.3 Determine the coordinates of S if PQRS is a parallelogram. (C) 1.4 Is PQRS a rectangle? Why or why not? (R)
2	**Finance and growth**	Use the simple and compound growth formulae $A = P(1+in)$ and $A = P(1+i)^n$] to solve problems, including annual interest, hire purchase, inflation, population growth and other real-life problems. Understand the implication of fluctuating foreign exchange rates (e.g. on the petrol price, imports, exports, overseas travel).	

<table>
<tr><th colspan="4">GRADE 10: TERM 3</th></tr>
<tr><th>Weeks</th><th>Topic</th><th>Curriculum statement</th><th>Clarification</th></tr>
<tr><td>2</td><td>Statistics</td><td>1. Revise measures of central tendency in ungrouped data.
2. Measures of central tendency in grouped data:
calculation of mean estimate of grouped and ungrouped data and identification of modal interval and interval in which the median lies.
3. Revision of range as a measure of dispersion and extension to include percentiles, quartiles, interquartile and semi interquartile range.
4. Five number summary (maximum, minimum and quartiles) and box and whisker diagram.
5. Use the statistical summaries (measures of central tendency and dispersion), and graphs to analyse and make meaningful comments on the context associated with the given data.</td><td>Comment:
In grade 10, the intervals of grouped data should be given using inequalities, that is, in the form 0≤x<20 rather than in the form 0-19, 20-29, …
Example:
The mathematics marks of 200 grade 10 learners at a school can be summarised as follows:
<table><tr><th>Percentage obtained</th><th>Number of candidates</th></tr><tr><td>0≤x<20</td><td>4</td></tr><tr><td>20≤x<30</td><td>10</td></tr><tr><td>30≤x<40</td><td>37</td></tr><tr><td>40≤x<50</td><td>43</td></tr><tr><td>50≤x<60</td><td>36</td></tr><tr><td>60≤x<70</td><td>26</td></tr><tr><td>70≤x<80</td><td>24</td></tr><tr><td>80≤x<100</td><td>20</td></tr></table>1. Calculate the approximate mean mark for the examination. (R)
2. Identify the interval in which each of the following data items lies:
2.1. the median; (R)
2.2. the lower quartile; (R)
2.3. the upper quartile; and (R)
2.4. the thirtieth percentile. (R)</td></tr>
</table>

GRADE 10: TERM 3			
Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry**	Problems in two dimensions.	**Example:** Two flagpoles are 30 m apart. The one has height 10 m, while the other has height 15 m. Two tight ropes connect the top of each pole to the foot of the other. At what height above the ground do the two ropes intersect? What if the poles were at different distance apart? (P)
1	**Euclidean Geometry**	Solve problems and prove riders using the properties of parallel lines, triangles and quadrilaterals.	**Comment:** Use congruency and properties of quads, esp. parallelograms. **Example:** EFGH is a parallelogram. Prove that MFNH is a parallelogram.  (C)
2	**Measurement**	1. Revise the volume and surface areas of right-prisms and cylinders. 2. Study the effect on volume and surface area when multiplying any dimension by a constant factor *k*. 3. Calculate the volume and surface areas of spheres, right pyramids and right cones.	**Example:** The height of a cylinder is 10 cm, and the radius of the circular base is 2 cm. A hemisphere is attached to one end of the cylinder and a cone of height 2 cm to the other end. Calculate the volume and surface area of the solid, correct to the nearest cm^3 and cm^2, respectively. (R) **Comments:** • In case of pyramids, bases must either be an equilateral triangle or a square. • Problem types must include composite figure.

Assessment term 3: Two (2) <u>Tests</u> (at least 50 marks and 1 hour) covering all topics in approximately the ratio of the allocated teaching time.

GRADE 10: TERM 4

No of Weeks	Topic	Curriculum statement	Clarification
2	**Probability**	1. The use of probability models to compare the relative frequency of events with the theoretical probability. 2. The use of Venn diagrams to solve probability problems, deriving and applying the following for any two events A and B in a sample space S: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive ; and $P(A) + P(B) = 1$. Then $P(B) = P(\text{ not } (A)) = 1 - P(A)$.	**Comment:** • It generally takes a very large number of trials before the relative frequency of a coin falling heads up when tossed approaches 0,5. **Example**: In a survey 80 people were questioned to find out how many read newspaper S or D or both. The survey revealed that 45 read D, 30 read S and 10 read neither. Use a Venn diagram to find how many read 1. S only; (C) 2. D only; and (C) 3. both D and S. (C)
4	**Revision**		**Comment:** The value of working through past papers cannot be over emphasised.
3	**Examinations**		

Assessment term 4

1. Test (at least 50 marks)
2. Examination

Paper 1: 2 hours (100 marks made up as follows: 15±3 on number patterns, 30±3 on algebraic expressions, equations and inequalities , 30±3 on functions, 10±3 on finance and growth and 15±3 on probability.

Paper 2: 2 hours (100 marks made up as follows: 40±3 on trigonometry, 15±3 on Analytical Geometry, 30±3 on Euclidean Geometry and Measurement, and 15±3 on Statistics

GRADE 11: TERM 1			
No fWeeks	**Topic**	**Curriculum statement**	**Clarification**
			Where an example is given, the cognitive demand is suggested: knowledge (K), routine procedure (R), complex procedure (C) or problem solving (P)
3	**Exponents and surds**	1. Simplify expressions and solve equations using the laws of exponents for rational exponents where $x^{\frac{p}{q}} = \sqrt[q]{x^p};\ x > 0; q > 0$. 2. Add, subtract, multiply and divide simple surds. 3. Solve simple equations involving surds.	**Example**: 1. Determine the value of $9^{\frac{3}{2}}$. (R) 2. Simplify: $(3+\sqrt{2})(3-\sqrt{2})$. (R) 3. Solve for x: $\sqrt{x-2} = 3$. (P)
3	**Equations and Inequalities**	1. Complete the square. 2. Quadratic equations (by factorisation and by using the quadratic formula). 3. Quadratic inequalities in one unknown (Interpret solutions graphically). NB: It is recommended that the solving of equations in two unknowns is important to be used in other equations like hyperbola-straight line as this is normal in the case of graphs. 4. Equations in two unknowns, one of which is linear and the other quadratic. 5. Nature of roots.	**Example**: 1. I have 12 metres of fencing. What are the dimensions of the largest rectangular area I can enclose with this fencing by using an existing wall as one side?Hint: let the length of the equal sides of the rectangle be x metres and formulate an expression for the area of the rectangle.(C) (Without the hint this would probably be problem solving.) 2.1. Show that the roots of $x^2 - 2x - 7 = 0$ are irrational. (R) 2.2. Show that $x^2 + x + 1 = 0$ has no real roots. (R) 3. Solve for x: $x^2 \leq 4$. (R) 4. Solve for x: $(x+1)(2x-3) \leq 3$. (R) 5. Two machines, working together, take 2 hours 24 minutes to complete a job. Working on its own, one machine takes 2 hours longer than the other to complete the job. How long does the slower machine take on its own? (P)
2	**Number patterns**	Patterns: Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic.	**Example**: In the first stage of the World Cup Soccer Finals there are teams from four different countries in each group. Each country in a group plays every other country in the group once. How many matches are there for each group in the first stage of the finals? How many games would there be if there were five teams in each group? Six teams? *n* teams? (P)

GRADE 11: TERM 1			
No fWeeks	Topic	Curriculum statement	Clarification
3	**Analytical Geometry**	Derive and apply: 1. the equation of a line though two given points; 2. the equation of a line through one point and parallel or perpendicular to a given line; and 3. the inclination (θ) of a line, where $m = \tan\theta$ is the gradient of the line $(0° \le \theta < 180°)$.	**Example**: Given the points $A(2;5)$; $B(-3;-4)$ and $C(4;-2)$ determine: 1. the equation of the line AB; and (R) 2. the size of $B\hat{A}C$. (C)

Assessment Term 1:

1. <u>An Investigation or a project</u> (a maximum of one project in a year) (at least 50 marks)

Notice that an assignment is generally an extended piece of work undertaken at home. ***Linear programming questions can be used as projects.***

Example of an assignment: Ratios and equations in two variables.

(This assignment brings in an element of history which could be extended to include one or two ancient paintings and examples of architecture which are in the shape of a rectangle with the ratio of sides equal to the golden ratio.)

Task 1

If $2x^2 - 3xy + y^2 = 0$ then $(2x - y)(x-y) = 0$ so $x = \frac{y}{2}$ or $x = y$. Hence the ratio $\frac{x}{y} = \frac{1}{2}$ or $\frac{x}{y} = \frac{1}{1}$. In the same way find the possible values of

the ratio $\frac{x}{y}$ if it is given that $2x^2 - 5xy + y^2 = 0$

Task

Most paper is cut to internationally agreed sizes: A0, A1, A2, …, A7 with the property that the A1 sheet is half the size of the A0 sheet and similar to the A0

sheet, the A2 sheet is half the size of the A1 sheet and similar to the A1 sheet, etc. Find the ratio of the length x to the breadth y of A0, A1, A2, …, A7 paper (in simplest surd form).

Task 3

The golden rectangle has been recognised through the ages as being aesthetically pleasing. It can be seen in the architecture of the Greeks, in sculptures and in Renaissance paintings. Any golden rectangle with length x and breadth y has the property that when a square the length of the shorter side (y) is cut from it, another rectangle similar to it is left. The process can be continued indefinitely, producing smaller and smaller rectangles. Using this information, calculate the ratio $x: y$ in surd form.

Example of <u>project</u>: Collect the heights of at least 50 sixteen-year-old girls and at least 50 sixteen-year-old boys. Group your data appropriately and use these two sets of grouped data to draw frequency polygons of the relative heights of boys and of girls, in different colours, on the same sheet of graph paper. Identify the modal intervals, the intervals in which the medians lie and the approximate means as calculated from the frequencies of the grouped data. By how much does the approximate mean height of your sample of sixteen-year-old girls differ from the actual mean? Comment on the symmetry of the two frequency polygons and any other aspects of the data which are illustrated by the frequency polygons.

2. <u>Test</u> (at least 50 marks and 1 hour). Make sure all topics are tested.

Care needs to be taken to ask questions on all four cognitive levels: approximately 20% knowledge, approximately 35% routine procedures, 30% complex procedures and 15% problem-solving.

GRADE 11: TERM 2			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Functions**	1. Revise the effect of the parameters *a* and *q* and investigate the effect of *p* on the graphs of the functions defined by: 1.1. $y = f(x) = a(x+p)^2 + q$ 1.2. $y = f(x) = \frac{a}{x+p} + q$ 1.3. $y = f(x) = ab^{x+p} + q$ where *b>0, b≠1* 2. Investigate numerically the average gradient between two points on a curve and develop an intuitive understanding of the concept of the gradient of a curve at a point. 3. Point by point plotting of basic graphs defined by $y = \sin\theta$, $y = \cos\theta$ and $y = \sin\theta$ for $\theta \in [-360°;360°]$ 4. Investigate the effect of the parameter *k* on the graphs of the functions defined by $y = \sin(kx)$, $y = \cos(kx)$ and $y = \tan(kx)$. 5. Investigate the effect of the parameter *p* on the graphs of the functions defined by $y = \sin(x+p)$, $y = \cos(x+p)$ and $y = \tan(x+p)$. 6. Draw sketch graphs defined by: $y=a \sin k(x+p)$, $y=a \cos k(x+p)$ and $y=a \tan k(x+p)$ at most two parameters at a time.	**Comment:** • Once the effects of the parameters have been established, various problems need to be set: drawing sketch graphs, determining the defining equations of functions from sufficient data, making deductions from graphs. Real life applications of the prescribed functions should be studied. • Two parameters at a time can be varied in tests or examinations. **Example**: Sketch the graphs defined by $y = -\frac{1}{2}\sin(x+30°)$ and $f(x) = \cos(2x - 120°)$ on the same set of axes, where $-360° \le x \le 360°$. (C)

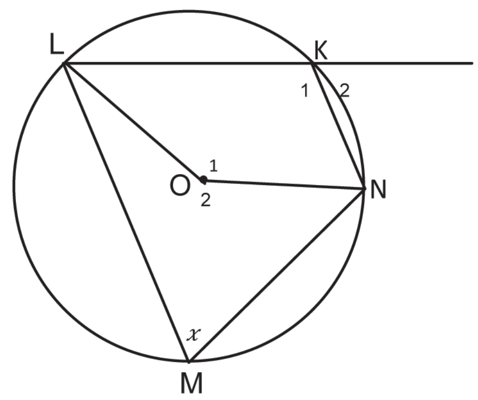
GRADE 11: TERM 2			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
4	**Trigonometry**	1. Derive and use the identities $\tan\theta = \frac{\sin\theta}{\cos\theta}$, $\theta \neq k.90°, k$ an odd integer; and $\sin^2\theta + \cos^2\theta = 1$. 2. Derive and use reduction formulae to simplify the following expressions: 2.1. $\sin(90° \pm \theta); \cos(90° \pm \theta)$; 2.2. $\sin(180° \pm \theta); \cos(180° \pm \theta)$; $\tan(180° \pm \theta)$; 2.3. $\sin(360° \pm \theta); \cos(360° \pm \theta)$; $\tan(360° \pm \theta)$; and 2.4. $\sin(-\theta); \cos(-\theta); \tan(-\theta)$. 3. Determine for which values of a variable an identity holds. 4. Determine the general solutions of trigonometric equations. Also, determine solutions in specific intervals.	**Comment:** • Teachers should explain where reduction formulae come from. **Examples**: 1. Prove that $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$. (R) 2. For which values of θ is $\frac{1}{\tan\theta} + \tan\theta = \frac{\tan\theta}{\sin^2\theta}$ undefined? (R) 3. Simplify $\frac{\cos(180° - x)\sin(x - 90°) - 1}{\tan^2(540° + x)\sin(90° + x)\cos(-x)}$ (R) 4. Determine the general solutions of $\cos^2\theta + 3\sin\theta = -3$. (C)
3	**Mid-year examinations**		

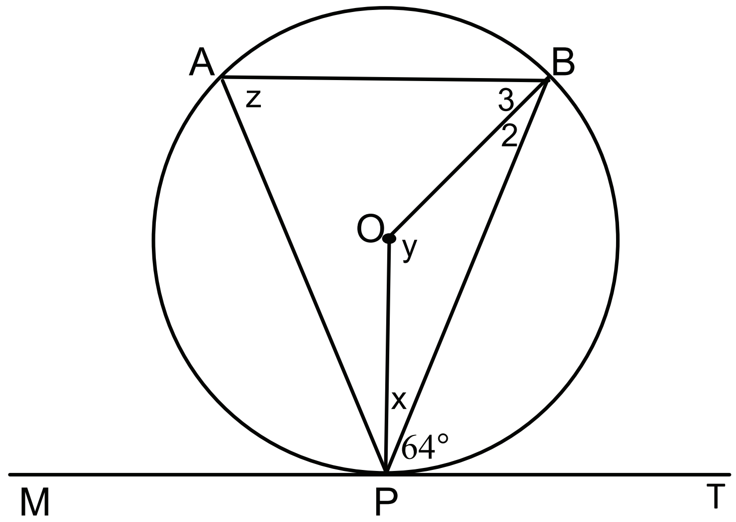
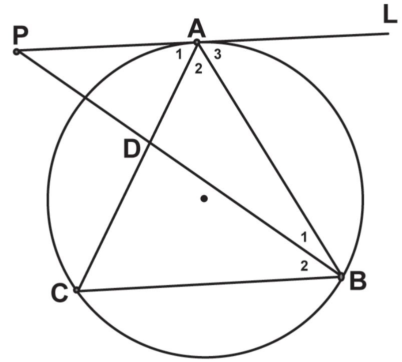
Assessment term 2:

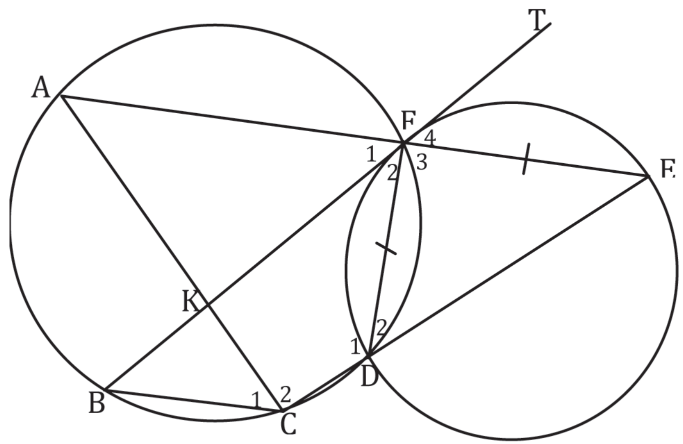
1. Assignment (at least 50 marks)
2. Mid-year examination:

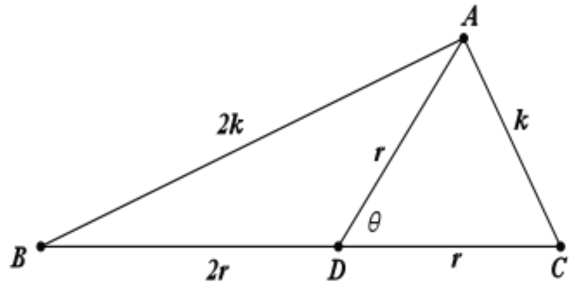
   Paper 1: 2 hours (100 marks made up as follows: general algebra (25±3) equations and inequalities (35±3); number patterns (15±3); functions (25±3)

   Paper 2: 2 hours (100 marks made up as follows: analytical geometry (30±3) and trigonometry (70±3)).

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
1	**Measurement**	1. Revise the Grade 10 work.	
3	**Euclidean Geometry**	Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact. Then investigate and prove the theorems of the geometry of circles: • The line drawn from the centre of a circle perpendicular to a chord bisects the chord; • The perpendicular bisector of a chord passes through the centre of the circle; • The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre); • Angles subtended by a chord of the circle, on the same side of the chord, are equal; • The opposite angles of a cyclic quadrilateral are supplementary; • Two tangents drawn to a circle from the same point outside the circle are equal in length; • The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment. Use the above theorems and their converses, where they exist, to solve riders.	**Comments:** Proofs of theorems can be asked in examinations, but their converses (wherever they hold) cannot be asked. **Example:** 1. *AB* and *CD* are two chords of a circle with centre O. *M* is on *AB* and *N* is on *CD* such that *OM* ⊥ *AB* and *ON* ⊥ *CD*. Also, *AB* = 50mm, *OM* = 40mm and *ON* = 20mm. Determine the radius of the circle and the length of *CD*. (C) 2. *O* is the centre of the circle below and $\hat{O}_1 = 2x$.  2.1. Determine $\hat{O}_2$ and $\hat{M}$ in terms of x. (R) 2.2. Determine $\hat{K}_1$ and $\hat{K}_2$ in terms of x. (R) 2.3. Determine $\hat{K}_1 + \hat{M}$. What do you notice? (R) 2.4. Write down your observation regarding the measures of $\hat{K}_2$ and $\hat{M}$. (R)

GRADE 11: TERM 3			
No. of weeks	Topic	Curriculum Statement	Clarification
			3. *O* is the centre of the circle above and MPT is a tangent. Also, $OP \perp MT$. Determine, with reasons, x, y and z. (C)  4. Given: $AB = AC$, $AP \parallel BC$ and $\hat{A}_2 = \hat{B}_2$. .  Prove that: 4.1 *PAL* is a tangent to circle *ABC*; (P) 4.2 *AB* is a tangent to circle *ADP*. (P)

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
			5. In the accompanying figure, two circles intersect at F and D.  *BFT* is a tangent to the smaller circle at *F*. Straight line *AFE* is drawn such that *FD* = *FE*. *CDE* is a straight line and chord *AC* and *BF* cut at K. Prove that: *5.1 BT // CE* (C) *5.2 BCEF* is a parallelogram (P) *5.3 AC* = *BF* (P)

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
2	**Trigonometry**	1. Prove and apply the sine, cosine and area rules. 2. Solve problems in two dimensions using the sine, cosine and area rules.	**Comment:** • The proofs of the sine, cosine and area rules are examinable. **Example:** In ΔABC, D is on BC, ADC = θ, $DA = DC = r$, $BD = 2r$, $AC = k$, and $BA = 2k$.  Show that $\cos\theta = \frac{1}{4}$. (P)
2	**Finance, growth and decay**	1. Use simple and compound decay formulae: $A = P(1 - in)$ and $A = P(1 - i)^n$ to solve problems (including straight line depreciation and depreciation on a reducing balance). 2. The effect of different periods of compound growth and decay, including nominal and effective interest rates.	**Examples**: 1. The value of a piece of equipment depreciates from R10 000 to R5 000 in four years. What is the rate of depreciation if calculated on the: 1.1 straight line method; and (R) 1.2 reducing balance? (C) 2. Which is the better investment over a year or longer: 10,5% p.a. compounded daily or 10,55% p.a. compounded monthly? (R) 3. R50 000 is invested in an account which offers 8% p.a. interest compounded quarterly for the first 18 months. The interest then changes to 6% p.a. compounded monthly. Two years after the money is invested, R10 000 is withdrawn. How much will be in the account after 4 years? (C) **Comment**: • The use of a timeline to solve problems is a useful technique. • Stress the importance of not working with rounded answers, but of using the maximum accuracy afforded by the calculator right to the final answer when rounding might be appropriate.

GRADE 11: TERM 3			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
2	**Probability**	1. Revise the addition rule for mutually exclusive events: $P(A\ or\ B)=PA+P(B)$, the complementary rule: $P(not\ A)=1-P(A)$ and the identity $\text{P(A or B)} = \text{P(A)} + \text{P(B)} - \text{P(A and B)}$ 2. Identify dependent and independent events and the product rule for independent events: $P(A\ and\ B)=P(A)\times P(B)$. 3. The use of Venn diagrams to solve probability problems, deriving and applying formulae for any three events A, B and C in a sample space S. 4. Use tree diagrams for the probability of consecutive or simultaneous events which are not necessarily independent.	**Comment**: • Venn Diagrams or Contingency tables can be used to study dependent and independent events. **Examples**: 1. P(A) = 0,45, P(B) = 0,3 and P(A or B) = 0,615. Are the events A and B mutually exclusive, independent or neither mutually exclusive nor independent? (R) 2. What is the probability of throwing at least one six in four rolls of a regular six sided die? (C) 3. In a group of 50 learners, 35 take Mathematics and 30 take History, while 12 take neither of the two. If a learner is chosen at random from this group, what is the probability that he/she takes both Mathematics and History? (C) 4. A study was done to test how effective three different drugs, A, B and C were in relieving headaches. Over the period covered by the study, 80 patients were given the opportunity to use all two drugs. The following results were obtained:from at least one of the drugs? (R) 40 reported relief from drug A 35 reported relief from drug B 40 reported relief from drug C 21 reported relief from both drugs A and C 18 reported relief from drugs B and C 68 reported relief from at least one of the drugs 7 reported relief from all three drugs. 4.1 Record this information in a Venn diagram. (C) 4.2 How many subjects got no relief from any of the drugs? (K) 4.3 How many subjects got relief from drugs A and B, but not C? (R) 4.4 What is the probability that a randomly chosen subject got relief from at least one of the drugs? (R)

GRADE 11: TERM 4			
No. of weeks	**Topic**	**Curriculum Statement**	**Clarification**
3	**Statistics**	1. Histograms 2. Frequency polygons 3. Ogives (cumulative frequency curves) 4. Variance and standard deviation of ungrouped data 5. Symmetric and skewed data 6. Identification of outliers	**Comments:** • Variance and standard deviation may be calculated using calculators. • Problems should cover topics related to health, social, economic, cultural, political and environmental issues. • Identification of outliers should be done in the context of a scatter plot as well as the box and whisker diagrams.
3	**Revision**		
3	**Examinations**		

Assessment term 4:

1. Test (at least 50 marks)
2. Examination (300 marks)

Paper 1: 3 hours (150 marks made up as follows: (25±3) on number patterns, on (45±3) exponents and surds, equations and inequalities, (45±3) on functions, (15±3) on finance growth and decay, (20±3) on probability).

Paper 2: 3 hours (150 marks made up as follows: (50±3) on trigonometry, (30±3) on Analytical Geometry, (50±3) on Euclidean Geometry and Measurement, (20±3) on Statistics.

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Patterns, sequences, series**	1. Number patterns, including arithmetic and geometric sequences and series 2. Sigma notation 3. Derivation and application of the formulae for the sum of arithmetic and geometric series: 3.1 $S_n = \frac{n}{2}[2a + (n-1)d]$; $S_n = \frac{n}{2}(a + l)$; 3.2 $S_n = \frac{a(r^n - 1)}{r - 1}; (r \neq 1)$; and 3.3 $S_\infty = \frac{a}{1-r}; (-1 < r < 1)$, $(r \neq 1)$	**Comment:** Derivation of the formulae is examinable. **Examples:** 1. Write down the first five terms of the sequence with general term $T_k = \frac{1}{3k-1}$. (K) 2. Calculate $\sum_{k=0}^{3} (3k-1)$. (R) 3. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. (C) 4. Determine the largest value of n such that $\sum_{i=1}^{n} (3i-2) < 2000$. (R) 5. Show that $0,999\dot{9} = 1$. (P)
3	**Functions**	1. Definition of a *function.* 2. General concept of the *inverse of a function* and how the domain of the function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function. 3. Determine and sketch graphs of the inverses of the functions defined by $y = ax + q$; $y = ax^2$ $y = b^x; (b > 0, b \neq 1)$ Focus on the following characteristics: domain and range, intercepts with the axes, turning points, minima, maxima, asymptotes (horizontal and vertical), shape and symmetry, average gradient (average rate of change), intervals on which the function increases /decreases.	**Examples:** 1. Consider the function f where $f(x) = 3x - 1$. 1.1 Write down the domain and range of f. (K) 1.2 Show that f is a one-to-one relation. (R) 1.3 Determine the inverse function f^{-1}. (R) 1.4 Sketch the graphs of the functions f, f^{-1} and $y = x$ line on the same set of axes. What do you notice? (R) 2. Repeat Question 1 for the function $f(x) = -x^2$ and $x \leq 0$. (C) **Caution:** 1. Do not confuse the inverse function f^{-1} with the reciprocal $\frac{1}{f(x)}$. For example, for the function where $f(x) = \sqrt{x}$, the reciprocal is $\frac{1}{\sqrt{x}}$, while $f^{-1}(x) = x^2$ for $x \geq 0$. 2. Note that the notation $f^{-1}(x) = ...$ is used only for one-to-one relation and must not be used for inverses of many-to-one relations, since in these cases the inverses are not functions.

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
1	**Functions: exponential and logarithmic**	1. Revision of the exponential function and the exponential laws and graph of the function defined by $y = b^x$ where $b > 0$ and $b \neq 1$ 2. Understand the definition of a logarithm: $y = \log_b x \Leftrightarrow x = b^y$, where $b > 0$ and $b \neq 1$. 3. The graph of the function define $y = \log_b x$ for both the cases $0 < b < 1$ and $b > 1$.	**Comment:** The four logarithmic laws that will be applied, only in the context of real-life problems related to finance, growth and decay, are: $\log_b(AB) = \log_b A + \log_b B$; $\log_b(\frac{A}{B}) = \log_b A - \log_b B$; $\log A^n = n\log A$; and $\log_B A = \frac{\log A}{\log B}$. They follow from the basic exponential laws (term 1 of grade 10). • Manipulation involving the logarithmic laws will not be examined. **Caution:** 1. Make sure learners know the difference between the two functions defined by $y = b^x$ and $y = x^b$ where b is a positive (constant) real number. 2. Manipulation involving the logarithmic laws will not be examined. **Examples:** 1. Solve for x : $75(1,025)^{x-1} = 300$ (R) 2. Let $f(x) = a^x$, $a > 0$. 2.1 Determine a if the graph of f goes through the point $(2;\frac{25}{16})$. (R) 2.2 Determine the function f^{-1}. (R) 2.3 For which values of x is $f^{-1}(x) > -1$? (C) 2.4 Determine the function h if the graph of h is the reflection of the graph of f through the y-axis. (C) 2.5 Determine the function k if the graph of k is the reflection of the graph of f through the x-axis. (C) 2.6 Determine the function p if the graph of p is obtained by shifting the graph of f two units to the left. (C) 2.7 Write down the domain and range for each of the functions f, f^{-1}, h, k and p. (R) 2.8 Represent all these functions graphically. (R)

GRADE 12: TERM 1			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Finance, growth and decay**	1. Solve problems involving present value and future value annuities. 2. Make use of logarithms to calculate the value of n, the time period, in the equations $A = P(1+i)^n$ or $A = P(1-i)^n$. 3. Critically analyse investment and loan options and make informed decisions as to best option(s) (including pyramid).	**Comment:** Derivation of the formulae for present and future values using the geometric series formula, $S_n = \frac{a(r^n - 1)}{r-1}; r \neq 1$, should be part of the teaching process to ensure that the learners understand where the formulae come from. The two annuity formulae: $F = \frac{x[(1+i)^n - 1]}{i}$ and $P = \frac{x[1-(1+i)^{-n}]}{i}$ hold only when payment commences one period from the present and ends after n periods. **Comment**: The use of a timeline to analyse problems is a useful technique. **Examples:** Given that a population increased from 120 000 to 214 000 in 10 years, at what annual (compound) rate did the population grow? (R) 1. In order to buy a car, John takes out a loan of R25 000 from the bank. The bank charges an annual interest rate of 11%, compounded monthly. The instalments start a month after he has received the money from the bank. 1.1 Calculate his monthly instalments if he has to pay back the loan over a period of 5 years. (R) 1.2 Calculate the outstanding balance of his loan after two years (immediately after the 24th instalment). (C)
2	**Trigonometry**	Compound angle identities: $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$; $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$; $\sin 2\alpha = 2\sin\alpha\cos\alpha$; $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$; $\cos 2\alpha = 2\cos^2\alpha - 1$; and $\cos 2\alpha = 1 - \sin^2\alpha$.	2. Accepting $\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$, derive the other compound angle identities. (C) 3. Determine the general solution of $\sin 2x + \cos x = 0$. (R) 4. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$. (C)

GRADE 12: TERM 1			
No. of Weeks	Topic	Curriculum statement	Clarification

Assessment Term 1:

1. Investigation or project. (at least 50 marks)

   Only one investigation or project per year is required.

- Example of an investigation which revises the sine, cosine and area rules:

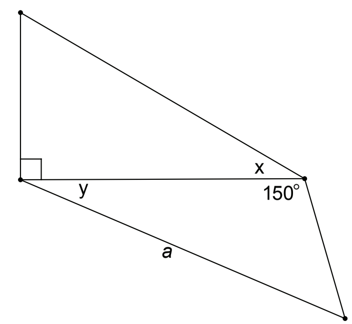
  Grade 12 Investigation: Polygons with 12 Matches
- How many different triangles can be made with a perimeter of 12 matches?
- Which of these triangles has the greatest area?
- What regular polygons can be made using all 12 matches?
- Investigate the areas of polygons with a perimeter of 12 matches in an effort to establish the maximum area that can be enclosed by the matches.

  Any extensions or generalisations that can be made, based on this task, will enhance your investigation. But you need to strive for quality, rather than simply producing a large number of trivial observations.

Assessment:

The focus of this task is on mathematical processes. Some of these processes are: specialising, classifying, comparing, inferring, estimating, generalising, making conjectures, validating, proving and communicating mathematical ideas.

2. Assignment or Test. (at least 50 marks)
3. Test. (at least 50 marks)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Trigonometry continued**	1. Solve problems in two and three dimensions.	**Examples:**  1. TP is a tower. Its foot, P, and the points Q and R are on the same horizontal plane. From Q the angle of elevation to the top of the building is x. Furthermore, $P\hat{Q}R = 150°$, $Q\hat{P}R$ = y and the distance between P and R is a metres. Prove that $TP = a\tan x(\cos y - \sqrt{3}\sin y)$ (C) 2. In ΔABC, $AD \perp BC$. Prove that: 2.1 $a = b\cos C + c\cos B$ where $a = BC$; $b = AC$ and $c = AB$. (R) 2.2 $\frac{\cos B}{\cos C} = \frac{c - b\cos A}{b - c\cos A}$ (on condition that $\hat{C} \neq 90°$). (P) 2.3 $\tan A = \frac{a\sin C}{b - a\cos C}$ (on condition that $\hat{A} \neq 90°$). (P) 2.4 $a + b + c = (b + c)\cos A + (c + a)\cos B + (a + b)\cos C$. (P)
1	**Functions: polynomials**	Factorise third-degree polynomials. Apply the Remainder and Factor Theorems to polynomials of degree at most 3 (no proofs required).	Any method may be used to factorise third degree polynomials but it should include examples which require the Factor Theorem. **Examples:** 1. Solve for x : $x^3 + 8x^2 + 17x + 10 = 0$ (R) 2. If $a(x) = x^5 - 2x^3 + px - 1$ is divided by $x - 1$, the remainder is $-\frac{1}{2}$. Determine the value of p. (P)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Differential Calculus**	1. An intuitive understanding of the limit concept, in the context of approximating the rate of change or gradient of a function at a point. 2. Use limits to define the derivative of a function f at any x : $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$. Generalise to find the derivative of f at any point x in the domain of f , i.e., define the derivative function $f'(x)$ of the function $f(x)$. Understand intuitively that $f'(a)$ is the gradient of the tangent to the graph of f at the point with x -coordinate a . 3. Using the definition (first principle), find the derivative, $f'(x)$ for *a*, *b* and c constants: 3.1 $f(x) = ax^2 + bx + c$; 3.2 $f(x) = ax^3$; 3.3 $f(x) = \frac{a}{x}$; and 3.3 $f(x) = c$.	**Comment:** Differentiation from first principles will be examined on any of the types described in 3.1, 3.2 and 3.3. **Examples:** 1. In each of the following cases, find the derivative of $f(x)$ at the point where $x = -1$, using the definition of the derivative: 1.1 $f(x) = x^2 + 2$ (R) 1.2 $f(x) = \frac{1}{2}x^2 + x - 2$ (R) 1.3 $f(x) = -x^3$ (R) 1.4 $f(x) = -\frac{x}{x}$ (C) **Caution**: Care should be taken not to apply the sum rule for differentiation 4.1 in a similar way to products. • Determine $\frac{d}{dx}((x+1)(x-1))$. • Determine $\frac{d}{dx}(x+1) \times \frac{d}{dx}(x-1)$. • Write down your observation. 2. Use differentiation rules to do the following: 2.1 Determine $f'(x)$ if $f(x) = (x+2)^2$ (R) 2.2 Determine $f'(x)$ if $f(x) = \frac{(x+2)^3}{\sqrt{x}}$ (C) (P)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
		4. Use the formula $\frac{d}{dx}(ax^n) = anx^{n-1}$, (for any real number *n*) together with the rules 4.1 $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$ and 4.2 $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)]$ (k a constant) 5. Find equations of tangents to graphs of functions. 6. Introduce the second derivative $f''(x) = \frac{d}{dx}(f'(x))$ of $f(x)$ and how it determines the concavity of a function. 7. Sketch graphs of cubic polynomial functions using differentiation to determine the co-ordinate of stationary points, and points of inflection (where concavity changes). Also, determine the x-intercepts of the graph using the factor theorem and other techniques. 8. Solve practical problems concerning optimisation and rate of change, including calculus of motion.	2.3 Determine $\frac{dy}{dt}$ if $y = \frac{t^2 - 1}{2t + 2}$ (R) 2.4 Determine $f'(\theta)$ if $f(\theta) = (\theta^{3/2} - 3\theta^{-1/2})^2$ (C) 3. Determine the equation of the tangent to the graph defined by $y = (2x+1)^2(x+2)$ where $x = \frac{3}{4}$. (C) 4. Sketch the graph defined by $y = -x^3 + 4x^2 - x$ by: 4.1 finding the intercepts with the axes; (C) 4.2 finding maxima, minima and the co-ordinate of the point of inflection; (R) 4.3 looking at the behaviour of y as $x \to \infty$ and as $x \to -\infty$. (P) (Remember: To understand points of inflection, an understanding of concavity is necessary. This is where the second derivative plays a role.) 5. The radius of the base of a circular cylindrical can is x cm, and its volume is 430 cm^3. 5.1 Determine the height of the can in terms of x. (R) 5.2 Determine the area of the material needed to manufacture the can (that is, determine the total surface area of the can) in terms of x. (C) 5.3 Determine the value of x for which the least amount of material is needed to manufacture such a can. (C) 5.4 If the cost of the material is R500 per m^2, what is the cost of the cheapest can (labour excluded)? (P)

GRADE 12: TERM 2			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Analytical Geometry**	1. The equation $(x-a)^2+(y-b)^2=r^2$ defines a circle with radius r and centre $(a;b)$. 2. Determination of the equation of a tangent to a given circle.	**Examples:** 1. Determine the equation of the circle with centre $(-1;2)$ and radius $\sqrt{6}$ (K) 2. Determine the equation of the circle which has the line segment with endpoints $(5;3)$ and $(-3;6)$ as diameter. (R) 3. Determine the equation of a circle with a radius of 6 units, which intersects the x-axis at $(-2;0)$ and the y-axis at $(0;3)$. How many such circles are there? (P) 4. Determine the equation of the tangent that touches the circle defined by $x^2-2x+y^2+4y=5$ at the point $(-2;-1)$. (C) 5. The line with the equation $y=x+2$ intersects the circle defined by $x^2+y^2=20$ at A and B. 5.1 Determine the co-ordinates of A and B. (R) 5.2 Determine the length of chord AB. (K) 5.3 Determine the co-ordinates of M, the midpoint of AB. (K) 5.4 Show that $OM \perp AB$, where O is the origin. (C) 5.5 Determine the equations of the tangents to the circle at the points A and B. (C) 5.6 Determine the co-ordinates of the point C where the two tangents in 5.5 intersect. (C) 5.7 Verify that $CA=CB$. (R) 5.8 Determine the equations of the two tangents to the circle, both parallel to the line with the equation $y=-2x+4$. (P)
3	**Mid-Year Examinations**		

Assessment term 2:

1. Assignment (at least 50 marks)
2. Examination (300 marks)

GRADE 12 TERM 3			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Euclidean Geometry**	1. Revise earlier work on the necessary and sufficient conditions for polygons to be similar. 2. Prove (accepting results established in earlier grades): • that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Mid-point Theorem as a special case of this theorem) ; • that equiangular triangles are similar; • that triangles with sides in proportion are similar; and • the Pythagorean Theorem by similar triangles.	**Example:** Consider a right triangle ABC with $\hat{B} = 90°$. Let $BC = a$ and $AB = c$. Let D be on AC such that $BD \perp AC$. Determine the length of BD in terms of a and c. (P)
1	**Statistics (regression and correlation)**	1. Revise symmetric and skewed data. 2. Use statistical summaries, scatterplots, regression (in particular the least squares regression line) and correlation to analyse and make meaningful comments on the context associated with given bivariate data, including interpolation, extrapolation and discussions on skewness.	**<u>Example:</u>** The following table summarises the number of revolutions x (per minute) and the corresponding power output y (horse power) of a Diesel engine: x: 400, 500, 600, 700, 750 y: 580, 1030, 1420, 1880, 2100 1. Find the least squares regression line $y = a + bx$ (K) 2. Use this line to estimate the power output when the engine runs at 800m. (R) 3. Roughly how fast is the engine running when it has an output of 1200 horse power? (R)

x	400	500	600	700	750
y	580	1030	1420	1880	2100

GRADE 12 TERM 3			
No. of Weeks	**Topic**	**Curriculum statement**	**Clarification**
2	**Counting and probability**	1. Revise: • dependent and independent events; • the product rule for independent events: P(A and B) = P(A) × P(B). • the sum rule for mutually exclusive events A and B: $P(A \text{ or } B) = P(A) + P(B)$ • the identity: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$ • the complementary rule: $P(\text{ not } A) = 1 - P(A)$ 2. probability problems using Venn diagrams, trees, two-way contingency tables and other techniques (like the fundamental counting principle) to solve probability problems (where events are not necessarily independent). 3. Apply the fundamental counting principle to solve probability problems.	**Examples:** 1. How many three-character codes can be formed if the first character must be a letter and the second two characters must be digits? (K) 2. What is the probability that a random arrangement of the letters BAFANA starts and ends with an 'A'? (R) 3. A drawer contains twenty envelopes. Eight of the envelopes each contain five blue and three red sheets of paper. The other twelve envelopes each contain six blue and two red sheets of paper. One envelope is chosen at random. A sheet of paper is chosen at random from it. What is the probability that this sheet of paper is red? (C) 4. Assuming that it is equally likely to be born in any of the 12 months of the year, what is the probability that in a group of six, at least two people have birthdays in the same month? (P)
3	**Examinations / Revision**		

Assessment Term 3:

1. Test (at least 50 marks)
2. Preliminary examinations (300 marks)

Important:

Take note that at least one of the examinations in terms 2 and 3 must consist of two three-hour papers with the same or very similar structure to the final NSC papers. The other examination can be replaced by tests on relevant sections.

GRADE 12: TERM 4			
No of Weeks	**Topic**	**Curriculum statement**	**Clarification**
3	**Revision**		
6	**Examinations**		

Assessment Term 4:

Final examination:

Paper 1: 150 marks: 3 hours		**Paper 2: 150 marks: 3 hours**	
Patterns and sequences	(25±3)	Euclidean Geometry and Measurement	(50±3)
Finance, growth and decay	(15±3)	Analytical Geometry	(40±3)
Functions and graphs	(35±3)	Statistics and regression	(20±3)
Algebra and equations	(25±3)	Trigonometry	(40±3)
Calculus	(35±3)		
Probability	(15±3)		

SECTION 4

4.1 Introduction

assessment is a continuous planned process of identifying, gathering and interpreting information about the performance of learners, using various forms of assessment. It involves four steps: generating and collecting evidence of achievement; evaluating this evidence; recording the findings and using this information to understand and assist in the learner's development to improve the process of learning and teaching.

Assessment should be both informal (Assessment for Learning) and formal (Assessment of Learning). In both cases regular feedback should be provided to learners to enhance the learning experience.

Although assessment guidelines are included in the Annual Teaching Plan at the end of each term, the following general principles apply:

1. Tests and examinations are assessed using a marking memorandum.

2. Assignments are generally extended pieces of work completed at home. They can be collections of past examination questions, but should focus on the more demanding aspects as any resource material can be used, which is not the case when a task is done in class under strict supervision.

3. At most one project or assignment should be set in a year. The assessment criteria need to be clearly indicated on the project specification. The focus should be on the mathematics involved and not on duplicated pictures and regurgitation of facts from reference material. The collection and display of real data, followed by deductions that can be substantiated from the data, constitute good projects.

4. Investigations are set to develop the skills of systematic investigation into special cases with a view to observing general trends, making conjecures and proving them. To avoid having to assess work which is copied without understanding, it is recommended that while the initial investigation can be done at home, the final write up should be done in class, under supervision, without access to any notes. Investigations are marked using rubrics which can be specific to the task, or generic, listing the number of marks awarded for each skill:

    - 40% for communicating individual ideas and discoveries, assuming the reader has not come across the text before. The appropriate use of diagrams and tables will enhance the investigation.
    - 35% for the effective consideration of special cases;
    - 20% for generalising, making conjectures and proving or disproving these conjectures; and
    - 5% for presentation: neatness and visual impact.

4.2 Informal or Daily Assessment

the aim of assessment for learning is to collect continually information on a learner's achievement that can be used to improve individual] learning.

Informal assessment involves daily monitoring of a learner's progress. This can be done through observations, discussions, practical demonstrations, learner-teacher conferences, informal classroom interactions, etc. Although informal assessment may be as simple as stopping during the lesson to observe learners or to discuss with learners how learning is progressing. Informal assessment should be used to provide feedback to the learners and to inform planning for teaching, it need not be recorded. This should not be seen as separate from learning activities taking place in the classroom. Learners or teachers can evaluate these tasks.

Self assessment and peer assessment actively involve learners in assessment. Both are important as these allow learners to learn from and reflect on their own performance. Results of the informal daily assessment activities are not formally recorded, unless the teacher wishes to do so. The results of daily assessment tasks are not taken into account for promotion and/or certification purposes.

4.3 Formal Assessment

All assessment tasks that make up a formal programme of assessment for the year are regarded as Formal Assessment. Formal assessment tasks are marked and formally recorded by the teacher for progress and certification purposes. All Formal Assessment tasks are subject to moderation for the purpose of quality assurance.

Formal assessments provide teachers with a systematic way of evaluating how well learners are progressing in a grade and/or in a particular subject. Examples of formal assessments include tests, examinations, practical tasks, projects, oral presentations, demonstrations, performances, etc. Formal assessment tasks form part of a year-long formal Programme of Assessment in each grade and subject.Formal assessments in Mathematics include tests, a June examination, a trial examination(for Grade 12), a project or an investigation.

The forms of assessment used should be age- and developmental- level appropriate. The design of these tasks should cover the content of the subject and include a variety of activities designed to achieve the objectives of the subject.

Formal assessments need to accommodate a range of cognitive levels and abilities of learners as shown below:

4.4 Programme of Assessment

The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999. Descriptors for each level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

Cognitive levels	Description of skills to be demonstrated	Examples
Knowledge **20%**	• Straight recall • Identification of correct formula on the information sheet (no changing of the subject) Use of mathematical facts • Appropriate use of mathematical vocabulary	1. Write down the domain of the function $y = f(x) = \frac{3}{x} + 2$ (Grade 10) 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle ....
Routine Procedures **35%**	• Estimation and appropriate rounding of numbers • Proofs of prescribed theorems and derivation of formulae • Identification and direct use of correct formula on the information sheet (no changing of the subject) • Perform well known procedures • Simple applications and calculations which might involve few steps • Derivation from given information may be involved • Identification and use (after changing the subject) of correct formula • Generally similar to those encountered in class	1. Solve for $x : x^2 - 5x = 14$ (Grade 10) 2. Determine the general solution of the equation $2\sin(x - 30°) + 1 = 0$ (Grade 11) 2. Prove that the angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is double the size of the angle $A\hat{C}B$ which the same arc subtends at the circle. (Grade 11)
Complex Procedures **30%**	• Problems involve complex calculations and/or higher order reasoning • There is often not an obvious route to the solution • Problems need not be based on a real world context • Could involve making significant connections between different representations • Require conceptual understanding	1. What is the average speed covered on a round trip to and from a destination if the average speed going to the destination is $100km/h$ and the average speed for the return journey is $80km/h$? (Grade 11) 2. Differentiate $\frac{(x+2)^2}{\sqrt{x}}$ with respect to x. (Grade 12)
Problem Solving **15%**	• Non-routine problems (which are not necessarily difficult) • Higher order reasoning and processes are involved • Might require the ability to break the problem down into its constituent parts	Suppose a piece of wire could be tied tightly around the earth at the equator. Imagine that this wire is then lengthened by exactly one metre and held so that it is still around the earth at the equator. Would a mouse be able to crawl between the wire and the earth? Why or why not? (Any grade)

The Programme of Assessment is designed to set formal assessment tasks in all subjects in a school throughout the year.

a) **Number of Assessment Tasks and Weighting:**

Learners are expected to have seven (7) formal assessment tasks for their school-based assessment. The number of tasks and their weighting are listed below:

		GRADE 10		GRADE 11		GRADE 12	
		TASKS	**WEIGHT (%)**	**TASKS**	**WEIGHT (%)**	**TASKS**	**WEIGHT (%)**
School-based Assessment	**Term 1**	**Project /Investigation** **Test**	**20** **10**	**Project /Investigation** **Test**	**20** **10**	**Test** **Project /Investigation** **Assignment**	**10** **20** **10**
	Term 2	**Assignment/Test** **Mid-Year Examination**	**10** **30**	**Assignment/Test** **Mid-Year Examination**	**10** **30**	**Test** **Mid-Year Examination**	**10** **15**
	Term 3	**Test** **Test**	**10** **10**	**Test** **Test**	**10** **10**	**Test** **Trial Examination**	**10** **25**
	Term 4	**Test**	**10**	**Test**	**10**		
School-based Assessment mark			**100**		**100**		**100**
School-based Assessment mark (as % of promotion mark)			**25%**		**25%**		**25%**
End-of-year examinations			**75%**		**75%**		
Promotion mark			**100%**		**100%**		

Note:

- Although the project/investigation is indicated in the first term, it could be scheduled in term 2. Only **ONE** project/investigation should be set per year.
- Tests should be at least ONE hour long and count at least 50 marks.
- Project or investigation must contribute 25% of term 1 marks while the test marks contribute 75% of the term 1 marks.
  The combination (25% and 75%) of the marks must appear in the learner's report.
- None graphic and none programmable calculators are allowed (for example, to factorise $a^2 - b^2 = (a-b)(a+b)$, or to find roots of equations) will be allowed. Calculators should only be used to perform standard numerical computations and to verify calculations by hand.
- **Formula sheet must not be provided for tests and for final examinations in Grades 10 and 11.**

b) Examinations:

In Grades 10, 11 and 12, 25% of the final promotion mark is a year mark and 75% is an examination mark.

All assessments in Grade 10 and 11 are internal while in Grade 12 the 25% year mark assessment is internally set and marked but externally moderated and the 75% examination is externally set, marked and moderated.

Mark distribution for Mathematics NCS end-of-year papers: Grades 10-12			
PAPER 1: Grades 12: bookwork: maximum 6 marks			
Description	**Grade 10**	**Grade 11**	**Grade. 12**
Algebra and equations (and inequalities)	30 ± 3	45 ± 3	25 ± 3
Patterns and sequences	15 ± 3	25 ± 3	25 ± 3
Finance and growth	10 ± 3		
Finance, growth and decay		15 ± 3	15 ± 3
Functions and graphs	30 ± 3	45 ± 3	35 ± 3
Differential Calculus			35 ± 3
Probability	15 ± 3	20 ± 3	15 ± 3
TOTAL	**100**	**150**	**150**
PAPER 2: Grades 11 and 12: theorems and/or trigonometric proofs: maximum 12 marks			
Description	**Grade 10**	**Grade 11**	**Grade 12**
Statistics	15 ± 3	20 ± 3	20 ± 3
Analytical Geometry	15 ± 3	30 ± 3	40 ± 3
Trigonometry	40 ± 3	50 ± 3	40 ± 3
Euclidean Geometry and Measurement	30 ± 3	50 ± 3	50 ± 3
TOTAL	**100**	**150**	**150**

Note:

- Modelling as a process should be included in all papers, thus contextual questions can be set on any topic.
- Questions will not necessarily be compartmentalised in sections, as this table indicates. Various topics can be integrated in the same question.
- **Formula sheet must not be provided for tests and for final examinations in Grades 10 and 11.**

4.5 Recording and reporting

- Recording is a process in which the teacher is able to document the level of a learner's performance in a specific assessment task.
  - It indicates learner progress towards the achievement of the knowledge as prescribed in the Curriculum and Assessment Policy Statements.
  - Records of learner performance should provide evidence of the learner's conceptual progression within a grade and her / his readiness to progress or to be promoted to the next grade.
  - Records of learner performance should also be used to monitor the progress made by teachers and learners in the teaching and learning process.
- Reporting is a process of communicating learner performance to learners, parents, schools and other stakeholders. Learner performance can be reported in a number of ways.

- These include report cards, parents' meetings, school visitation days, parent-teacher conferences, phone calls, letters, class or school newsletters, etc.

- Teachers in all grades report percentages for the subject. Seven levels of competence have been described for each subject listed for Grades R-12. The individual achievement levels and their corresponding percentage bands are shown in the Table below.

CODES AND PERCENTAGES FOR RECORDING AND REPORTING

RATING CODE	DESCRIPTION OF COMPETENCE	PERCENTAGE
7	Outstanding achievement	80 – 100
6	Meritorious achievement	70 – 79
5	Substantial achievement	60 – 69
4	Adequate achievement	50 – 59
3	Moderate achievement	40 – 49
2	Elementary achievement	30 – 39
1	Not achieved	0 - 29

Note: The seven-point scale should have clear descriptors that give detailed information for each level.

Teachers will record actual marks for the task on a record sheet; and indicate percentages for each subject on the learners' report cards.

4.6 Moderation of Assessment

Moderation refers to the process that ensures that the assessment tasks are fair, valid and reliable. Moderation should be implemented at school, district, provincial and national levels. Comprehensive and appropriate moderation practices must be in place to ensure quality assurance for all subject assessments.

4.7 General

This document should be read in conjunction with:

4.7.1 *National policy pertaining to the programme and promotion requirements of the National Curriculum Statement Grades R – 12*; and

4.7.2 The policy document, *National Protocol for Assessment Grades R – 12.*