CHAPTER 10

MATHEMATICS

The following report should be read in conjunction with the Mathematics question papers for the NSC November 2023 examinations.

10.1 PERFORMANCE TRENDS (2019–2023)

The number of candidates who wrote the Mathematics examination in 2023 decreased by 7 718 compared to that of 2022.

There was a significant improvement in the pass rate this year. Candidates who passed at the 30% level improved from 55% in 2022 to 63,5% in 2023. There was a corresponding improvement in the pass rate at the 40% level over the past two years from 36% to 43,6%.

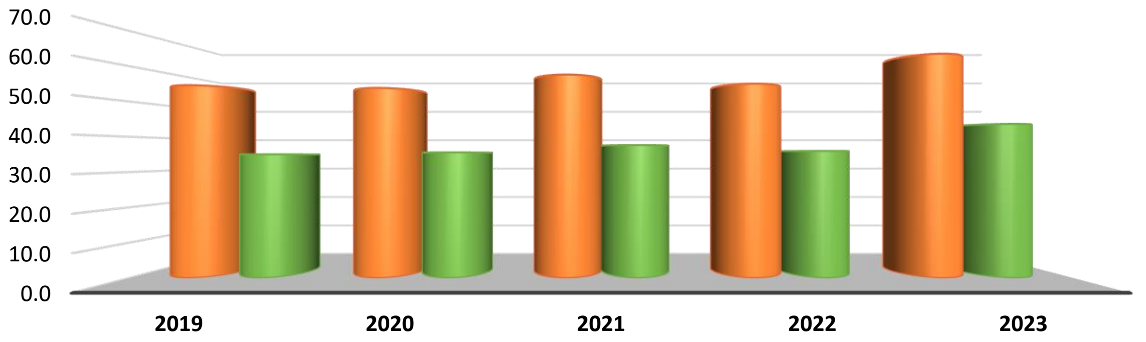
The percentage of distinctions over 80% improved from 2,7% in 2022 to 3,4% in 2023. Given the decrease in the size of the 2023 cohort, this converts into an increase in the total number of distinctions from 7 283 to 8 909.

The various commendable intervention strategies employed by teachers, subject advisors and provincial education departments were continued in 2023. The resourcefulness and diligence of the above-average candidates also contributed to the overall improvement in the subject.

Table 10.1.1 Overall achievement rates in Mathematics

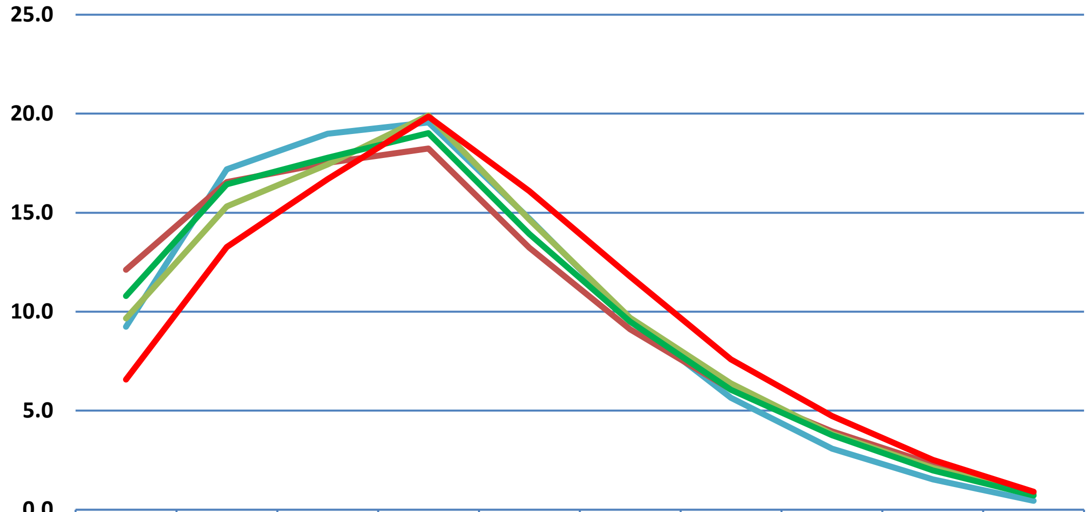
Year	No. wrote	No. achieved at 30% and above	% achieved at 30% and above	No. achieved at 40% and above	% achieved at 40% and above
2019	222 034	121 179	54,6	77 751	35,0
2020	233 315	125 526	53,8	82 964	35,6
2021	259 143	149 177	57,6	97 561	37,6
2022	269 734	148 346	55,0	97 041	36,0
2023	262 016	166 337	63,5	114 311	43,6

Graph 10.1.1 Overall achievement rates in Mathematics (percentage)



	2019	2020	2021	2022	2023
% achieved at 30% and above	54.6	53.8	57.6	55.0	63.5
% achieved at 40% and above	35.0	35.6	37.6	36.0	43.6

Graph 10.1.2 Performance distribution curves in Mathematics (percentage)



	0-9.9	10-19.9	20-29.9	30-39.9	40-49.9	50-59.9	60-69.9	70-79.9	80-89.9	90-100
2019	9.2	17.2	19.0	19.6	14.7	9.6	5.6	3.1	1.5	0.4
2020	12.1	16.5	17.5	18.2	13.2	9.1	6.1	3.9	2.3	0.9
2021	9.7	15.3	17.4	19.9	14.7	9.7	6.4	3.8	2.1	0.8
2022	10.8	16.4	17.8	19.0	13.9	9.5	6.1	3.8	2.0	0.7
2023	6.6	13.3	16.7	19.9	16.1	11.8	7.6	4.7	2.5	0.9

10.2 OVERVIEW OF CANDIDATES' PERFORMANCE IN PAPER 1

(a) Many candidates were able to answer the knowledge and routine questions correctly. This suggests that the candidates were well-prepared to deal with these questions in the paper. Unlike in the past, candidates scored some marks in most of the questions.

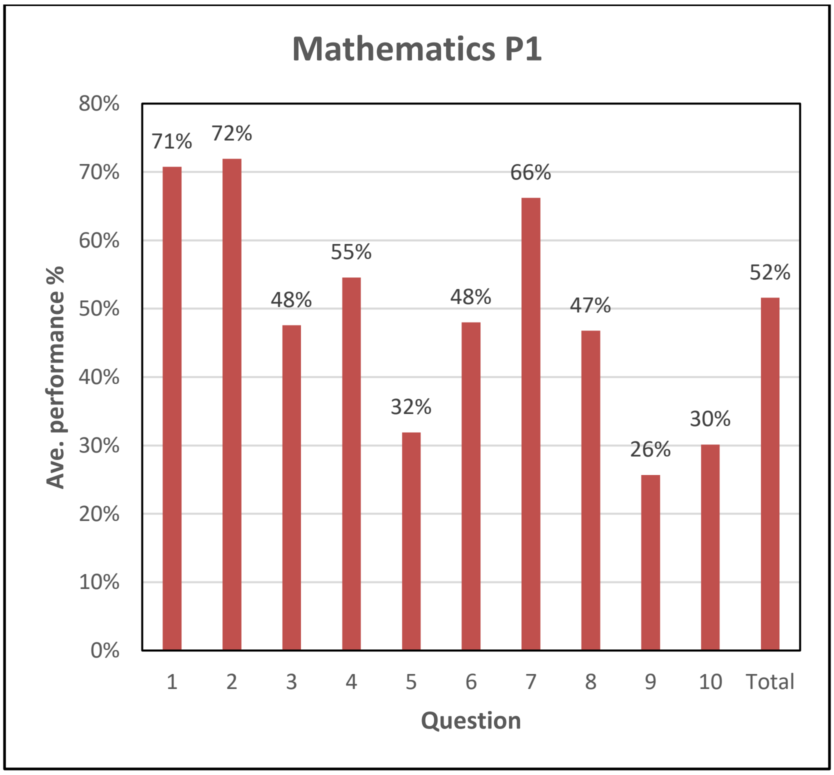
(b) The algebraic skills of the candidates are poor. Most candidates lacked fundamental and basic mathematical competencies which should have been acquired in the lower grades. This becomes an impediment to candidates when answering complex questions.

(c) While calculations and performing well-known routine procedures form the basis of answering questions in a Mathematics paper, a deeper understanding of definitions and concepts cannot be overlooked. Candidates did not fare well in answering questions that assessed an understanding of concepts.

10.3 DIAGNOSTIC QUESTION ANALYSIS OF PAPER 1

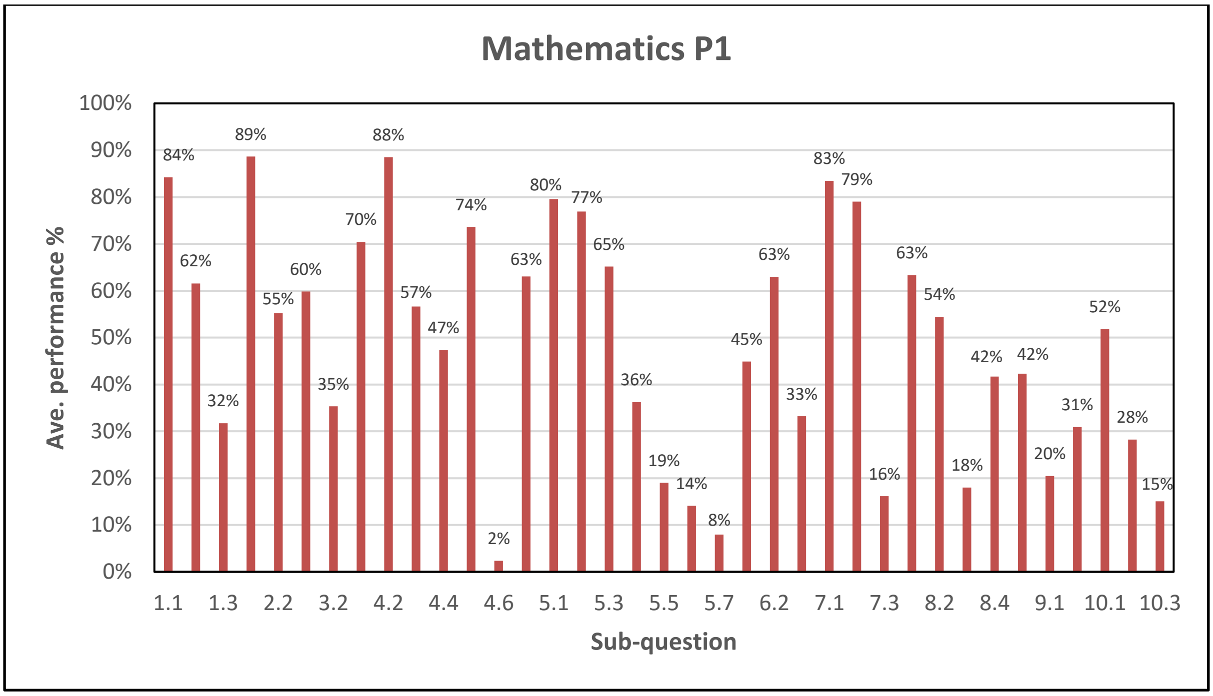
The following graph is based on data from a random sample of candidates' scripts. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 10.3.1 Average performance per question in Paper 1



Q	Topics
1	Equations, Inequalities & Algebraic and Exponential Manipulation
2	Number Patterns & Sequences
3	Number Patterns & Sequences
4	Functions, Inverse Functions & Graphs
5	Functions & Graphs
6	Finance
7	Calculus
8	Calculus
9	Calculus
10	Probability & Counting Principles

Graph 10.3.2 Average performance per sub-question in Paper 1



10.4 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 1

QUESTION 1: ALGEBRA

Common errors and misconceptions

(a) In Q1.1.2 some candidates did not write the equation correctly in standard form, so they incorrectly substituted '6' as the value of *c* instead of '–6'.

(b) In Q1.1.3 many candidates were able to square the equation correctly but then factorised $x^2 - 4x = 0$ incorrectly as $(x-2)(x+2) = 0$. Some divided $x^2 - 4x = 0$ through by *x* which meant that the candidate lost a solution to the equation.

(c) Many candidates struggled to solve the inequality in Q1.1.4 after correctly calculating the critical values. Some candidates drew a sketch but were unable to use it to write down the required answer. Another common error was the incorrect notation in the answer. Candidates wrote the answer as $x > -1$ or $x > 3$ instead of $x < -1$ or $x > 3$.

(d) The simultaneous equation given in fraction form in Q1.2 proved challenging for most candidates. The notion of isolating the one variable and substituting this into the second equation was well understood. However, candidates changed $\frac{1}{x} + \frac{1}{y} = 1$ to $x^{-1} + y^{-1} = 1$ and then after substitution incorrectly worked with the exponent of –1.

(e) In Q1.3 many candidates were able to write the exponents with separate bases, i.e. $2^m.2 + 2^m = 3^n.3^2 - 3^n$. However, some failed to factorise the resulting expression correctly, i.e. $2^m.(2+1) = 3^n.\left(3^2 - 1\right)$. The majority of the candidates just worked with the exponents and this resulted in an equation $m+1+m=n+2-n$ which did not lead them to solving the given problem.

Suggestions for improvement

(a) Learners must ensure that they understand what correct *standard form* is in a quadratic equation.

(b) Learners must be taught to check their solutions when using the squaring technique to solve an equation that is not originally quadratic.

(c) Emphasis must be given to working with lowest common denominators to solve fraction-based problems.

(d) When dealing with *surd* equations, learners should be reminded that they need to isolate the *radical* before they can square both sides of the equation. Teachers must emphasise that implicit restrictions are placed on *surd* equations and that learners should continue to test whether their answers satisfy the original equation.

(e) Learners need to be exposed to complex questions involving *surds* and *exponents*. Correct use of *exponential rules* must be revised, tested and re-explained from Grade 9 through to the final Grade 12 examination.

(f) Regular revision and emphasis on working with *prime bases* in *exponents* is important.

(g) Learners need to be exposed to problem-solving questions involving algebraic manipulation in all areas of algebra. This requires regular revision and testing of non-routine questions in the algebra section, focusing on a range of skills that candidates need to have mastered from Grade 10.

(h) As suggested in previous reports:
- Teachers should not take for granted that learners know how to round off a number to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12. Teachers should penalise learners in SBA tasks when they do not round off to the correct number of places.
- Teachers should take some time, preferably in Grade 10, to focus on teaching learners how to represent inequalities (e.g. $-3<x<5$; $x<-3$ or $x>5$) on a number line and also how to write an inequality from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasis on the correct notation is essential when writing down the solutions to inequalities.
- Linked to this, teachers should explain the difference between *and* and *or* in the context of inequalities. Learners cannot use these words interchangeably as they have different meanings.

QUESTION 2: PATTERNS

Common errors and misconceptions

(a) The most common errors in Q2.1 and its sub-questions were the use of incorrect formulae (i.e. $T_n=a(n-1)d$ rather than $T_n=a+(n-1)d$) or incorrect substitution of n as T_n or S_n.

(b) Many candidates substituted $n = 5$ into the given T_n formula in Q2.1.2 to answer Q2.1.1 rather than developing the *quadratic pattern*. The candidates knew how to develop the pattern as this was well done in Q2.1.2, when asked to show that the *quadratic pattern* had a general term of $T_n = 6n^2 - 9n + 6$, but the link was not made between determining the formula of a *quadratic pattern* and generation of the *quadratic pattern*.

(c) In Q2.2.3 most candidates could not show that the pattern was increasing for all *natural numbers*. They were not able to manipulate the *quadratic pattern* as a *quadratic function* to provide the argument of where the *axis of symmetry* of the pattern was.

Suggestions for improvement

(a) Learners must be made aware of which formulae on the information sheet apply to which type of sequence. It is good practice for them to use the information sheet in class so that they become familiar with it.

(b) Teach learners how to identify whether the question requires them to calculate the value of the n^{th} term or the sum of the first n terms.

(c) Questions must be read carefully so that the learners know what is required of them.

(d) Learners should be discouraged from using information provided in later questions to answer earlier questions in an examination. Learners must be encouraged to develop patterns using their properties rather than using their explicit *general terms*. The understanding of where a pattern 'starts' and what the pattern 'does' is important to emphasise. The basic diagram was sufficient in answering Q2.2.1.

i.e. 3 12 33 66 111

9 21 33 45

12 12 12

(e) Teachers need to specifically teach the relationships between a *quadratic pattern* and a *parabola*, making particular reference to the *axis of symmetry* and *minimum* or *maximum* values of the pattern.

QUESTION 3: PATTERNS

Common errors and misconceptions

(a) Candidates were correctly able to determine the general term, however, most candidates incorrectly simplified $T_n = 3.2^{n-1}$ to $T_n = 6^{n-1}$. This did not impact the candidates in Q3.1.1 but did impact the candidates' ability to correctly solve Q3.1.2.

(b) Many candidates did not make the link between n and k in the *sigma notation* in Q3.1.2. The candidates also showed little understanding of how to generate a *series* from *sigma notation* to identify the a and the r values. Further to this, some candidates substituted the calculated information into T_n rather than S_n.

(c) Most candidates were unable to answer Q3.2 correctly. The interpretation of '*The sum of 22 terms of the arithmetic series is 734 more than the sum to infinity of the geometric sequence*' was incorrectly translated to $S_{22} = 734$.

Suggestions for improvement

(a) Teachers should emphasise the differences between the *term*, *sum* and *sum to infinity* formulae in *arithmetic* and *geometric patterns*.

(b) The inclusion of word problems in the *patterns* section is important. Teachers need to emphasise how to take the words of a problem and write it in symbolic form to solve an *equation* or *inequality*.

(c) Constant revision of *exponential laws* to solve equations correctly is pivotal to candidates' success. Teachers need to emphasise this and revise this thoroughly in all grades.

QUESTION 4: FUNCTIONS (EXPONENTIAL AND LOGARITHMIC GRAPH)

Common errors and misconceptions

(a) Many candidates did not calculate the coordinates of the *x*-intercept, A, correctly in Q4.3 which resulted in an incorrect equation of the *straight line*.

(b) Most candidates did not understand the concept of *vertical distance* in Q4.4. The candidates substituted incorrect points into the distance formula in an attempt to calculate any distance.

(c) In Q4.4 some candidates left their answer for the vertical distance between the functions at $x = 1$ as a negative value.

(d) The majority of candidates did not realise that the given function had a restricted domain which in turn meant the inverse of the given function would have a restricted domain. The candidates had learnt that the domain of the inverse was $x > 0$, which is true for an unrestricted *logarithm function*.

Suggestions for improvement

(a) Teachers need to emphasise what the points on a graph are and help learners identify what the properties of these points are before they start answering a question.

(b) *Vertical and horizontal distances* should be explained without the use of the distance formula when interpreting functions. In conjunction with this, teachers must emphasise that distance is a scalar property and cannot be negative.

(c) Teachers should work with restricted domain graphs in class to make learners aware that functions can be restricted and emphasise the effect this has on their properties.

QUESTION 5: FUNCTIONS (HYPERBOLA AND PARABOLA)

Common errors and misconceptions

(a) In Q5.4 many candidates forgot the restriction of the horizontal asymptote on the range of the function, possibly because it was the *x*-axis and visually drawn on the graph as an asymptote.

(b) Most candidates were unable to interpret Q5.5 correctly from the given graph. Many tried to solve the inequality algebraically and others did not take cognisance of the vertical asymptote of the hyperbola.

(c) The majority of candidates did not understand the terminology '*will not intersect'* and tried to solve for $-2x+k>\frac{8}{x}$ or $-2x+k\neq\frac{8}{k}$ which led them to incorrect solutions and misinterpretation of the question. The integration of *nature of roots* into functions was not commonly understood by candidates.

(d) Q5.7 was not understood by many candidates. The link between Q5.6 and Q5.7 was not seen by the candidates which resulted in very few candidates attempting to answer this question.

Suggestions for improvement

(a) Teachers should spend some time on graphical interpretation of functions. This can be started with the very first graph that is sketched in Grade 10. The concepts of $f(x)>0$, $f(x)>g(x)$ and $f(x).g(x)>0$ must be emphasised throughout the FET phase when teaching functions.

(b) The link for learners between the algebraic work (i.e. *nature of roots*, *simultaneous equations* and *inequalities*) and the graphical representation must be created by the teacher when working with functions. However, teachers need to emphasise the importance of understanding these concepts and teach the learners to read off the solutions to the questions from the graph. Not all solutions in functions questions need be algebraic – this practice seems to be the default for most candidates.

(c) The integration of *calculus* into *functions* is an important concept for teachers to revise, test and practise with learners.

QUESTION 6: FINANCE

Common errors and misconceptions

(a) In Q6.1 many of the candidates who did not solve for the value of *r*, did not use the value of *n* as 6. Generally, candidates found it difficult to deal with months if they were not a multiple of 12. Some candidates used $n=\frac{1}{2}$ while many used 72 months.

(b) In Q6.1 it was a common error for candidates not to divide the rate by 12 in the formula to calculate the nominal interest rate correctly.

(c) Many candidates used the incorrect formulae in all questions related to *financial mathematics*. In addition, the candidates commonly did not use their calculator correctly to calculate the final answer.

(d) In Q6.2.2 many candidates used the *present value* formula rather than the *future value* formula.

(e) Most candidates in Q6.3 did not use *logarithms* correctly, and if they did, the candidates rounded their answer of n = 147,8 to n = 148. This indicated a misconception of the number of withdrawals of R20 000 that could have been made.

Suggestions for improvement

(a) Learners need deeper insight into the relevance of each of the formulae and under which circumstances each formula can be used. The variables in each formula must be explained. More practice in Financial Mathematics is necessary so that learners can identify when to use the different formulae.

(b) Teachers should demonstrate all the steps required when using a calculator. This should be done repetitively in class with every example done in Financial Mathematics. In formal assessment tasks at school, learners should be penalised for rounding off early.

(c) The difference between compound interest, future value- and present value annuities must be thoroughly explained.

(d) The correct Financial Mathematics language should be used in class and learners should read the question with understanding.

(e) Teachers need to emphasise – and learners need to practise – using different compounding periods for time intervals other than in years.

QUESTION 7: CALCULUS

Common errors and misconceptions

(a) In Q7.1 some candidates made the following notational errors: $\lim_{h \to 0} = \frac{f(x+h)-f(x)}{h}$ or left out the limit of h altogether. They lost a mark for these errors. Other candidates made errors in substitution.

(b) In Q7.2.2 candidates incorrectly changed $7.\sqrt[3]{x^2}$ to $7x^{\frac{3}{2}}$ instead of $7x^{\frac{2}{3}}$.

(c) When answering Q7.3, many candidates calculated the first derivatives correctly. However, they did not understand how to solve the inequality $x^2 > \frac{4}{3}$, partly because the square root of $\frac{4}{3}$ is not a rational number and partly because of a lack of understanding of how to work with inequalities.

Suggestions for improvement

(a) Emphasis should be placed on the use of the correct notation when determining the derivative, either when using first principles or the rules.

(b) Teachers should revise the rules of *exponents* and *surds* when changing an expression into differentiable format.

(c) The concepts of *gradient of tangent* and *point of inflection* must be understood. Teachers should explain the difference between the two concepts.

(d) Integration and re-emphasis of algebraic concepts, viz fractions, factorising, inequalities and exponential rules, should be emphasised when working with *calculus*.

QUESTION 8: CALCULUS

Common errors and misconceptions

(a) Some candidates did not realise that the function in Q8.1 was already factorised for them. They expanded the factors and equated to the left side. Instead of getting zero, they created another function. These candidates also solved for the *x*-intercepts rather than the *x*-coordinates of the turning points first.

(b) In Q8.1 candidates did not equate the derivative to 0 explicitly. Some candidates only worked out the *x* values without calculating the corresponding *y* values. This led to an incorrect graph in Q8.2. Other candidates attempted to use the quadratic equation principle of $x = -\frac{b}{2a}$ to calculate the *x*-coordinate of the turning point which does not lead to the correct answer in a cubic function.

(c) Many candidates did not label the graph in Q8.2 even though it was explicitly asked of them to do so. Other candidates could not draw the cubic function and drew a version of a parabola. A common mistake was not indicating the *y*-intercept.

(d) In Q8.3 most candidates did not use graphical interpretation to answer this question but rather used quadratic function theory and substituted in $b^2 - 4ac$ which was incorrect.

(e) In Q8.4 many candidates substituted into $m = \frac{\Delta y}{\Delta x}$ to calculate a gradient rather than using calculus methods to solve for the gradient.

(f) Many candidates equated $m = \tan\theta$ where the gradient they had calculated was a negative value. The link that the gradient was positive was not recognised by the candidates.

Suggestions for improvement

(a) Learners should be taught to determine the properties of a graph from Grade 10 to 12 in a progressive manner. The defining property of a turning point having a zero gradient is a way to describe a turning point. Teachers need to prove this link between the definition of a turning point and the Grade 11 concept of determining the axis of symmetry to calculate the *x*-coordinate of the turning point.

(b) When teaching *factorisation of third-degree polynomials*, teachers should include examples where there is only one real root.

(c) Teachers should continue to teach graphical interpretation in cubic graphs as a follow on from the interpretation taught in Grade 10 and 11.

(d) The concept of the point of inflection needs to be taught explicitly.

(e) The application of Calculus lends itself to many applications. Teachers need to expose learners to a wide variety of questions, which include integration of topics including *analytical geometry, measurement* and *trigonometry*.

QUESTION 9: CALCULUS

Common error and misconception

(a) The vast majority of the candidates did not attempt this question because they were unable to formulate the equation required in Q9.1.

(b) Many candidates did not use the given equation in Q9.1 to answer Q9.2.

(c) Some common errors included:

- Candidates simplified the equation required in Q9.2 to $x^{-2} = \frac{1}{576}$ and then incorrectly concluded that $x = \frac{1}{24}$.
- Candidates calculated that $x^2 = 576$ and then concluded that $x = -24$ rather than $x = 24$.

Suggestions for improvement

(a) Learners appear to be dependent on the formulae being given when solving *optimisation* problems. It is advisable that learners interrogate the *optimum* function even when it is given in a question. This should help their conceptual development.

(b) Teachers should ensure that there is enough time for learners to understand the application of Calculus fully.

(c) Reading for understanding should be ongoing if learners are to improve their responses to word problems.

QUESTION 10: PROBABILITY

Common errors and misconceptions

(a) In Q10.1.2 many candidates did not use the probability rule correctly to answer this problem. The theory of P (at least one) = 1 – P(A and B) was incorrectly used.

(b) Most candidates who calculated a probability that was greater than 1 did not realise that this could not be correct.

(c) In answering Q10.2.1 many candidates were unable to correctly draw the tree diagram.

(d) In Q10.3.2 most candidates did not understand that there were 4 positions for the 5 learners to be seated between the 2 youngest learners. The most common error was $5! \times 2!$.

Suggestions for improvement

(a) Teaching basic concepts cannot be overlooked. When learners understand the basic concepts well enough, then the more complex concepts are easier to grasp.

(b) It must be stressed that the probability of an event A lies in the interval $0 \leq P(A) \leq 1$.

(c) Reading for understanding must be a regular practice in the classroom. This should equip learners with the skills to deal with word problems in assessment tasks.

(d) Teachers need to teach both tree diagrams and Venn diagrams thoroughly. These concepts should be examined in school-based assessment tasks throughout the FET phase.

(e) Teach learners the *Fundamental Counting Principle* in such a way that they will be able to base their answers on their reasoning, rather than on the rule.

10.5 OVERVIEW OF CANDIDATES' PERFORMANCE IN PAPER 2

(a) Integration of topics was still a challenge to many candidates. Mathematics should not be studied in compartments. Candidates are expected to apply knowledge from one section to another section of work.

(b) It was evident that many of the errors made by candidates in answering the Trigonometry questions in this paper had their origins in a poor understanding of the basics and the foundational competencies taught in the earlier grades.

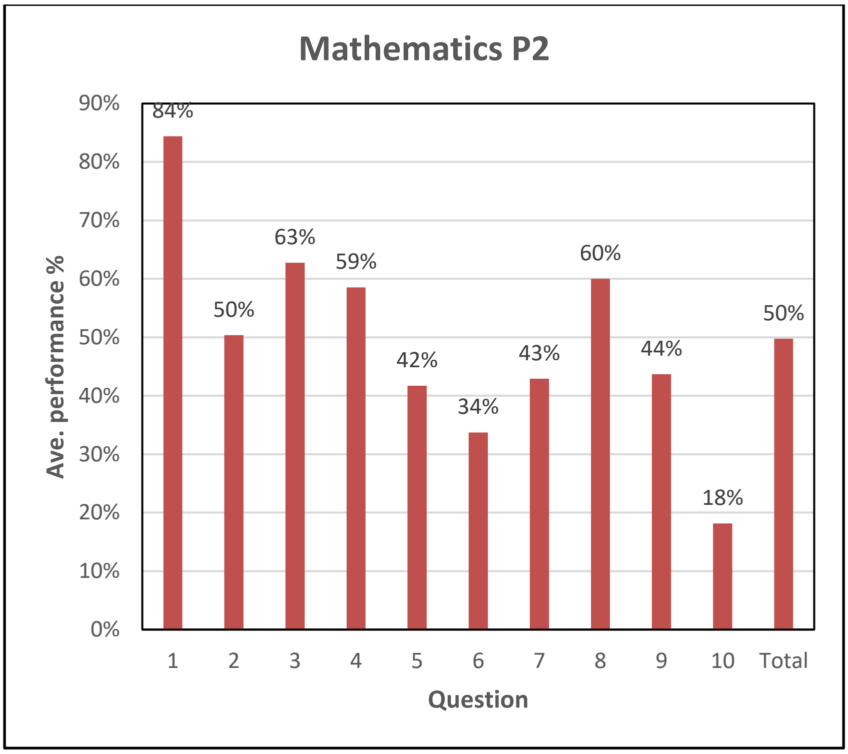
(c) In general, candidates needed to exercise caution with algebraic manipulation skills since overlooking certain basic principles or practices results in the unnecessary loss of marks.

(d) Although the calculator is an effective and necessary tool in Mathematics, learners are of the belief that the calculator provides the answer to all their problems. Some candidates needed to realise that conceptual development and algebraic manipulation were often impeded because of their dependence on a calculator.

10.6 DIAGNOSTIC QUESTION ANALYSIS FOR PAPER 2

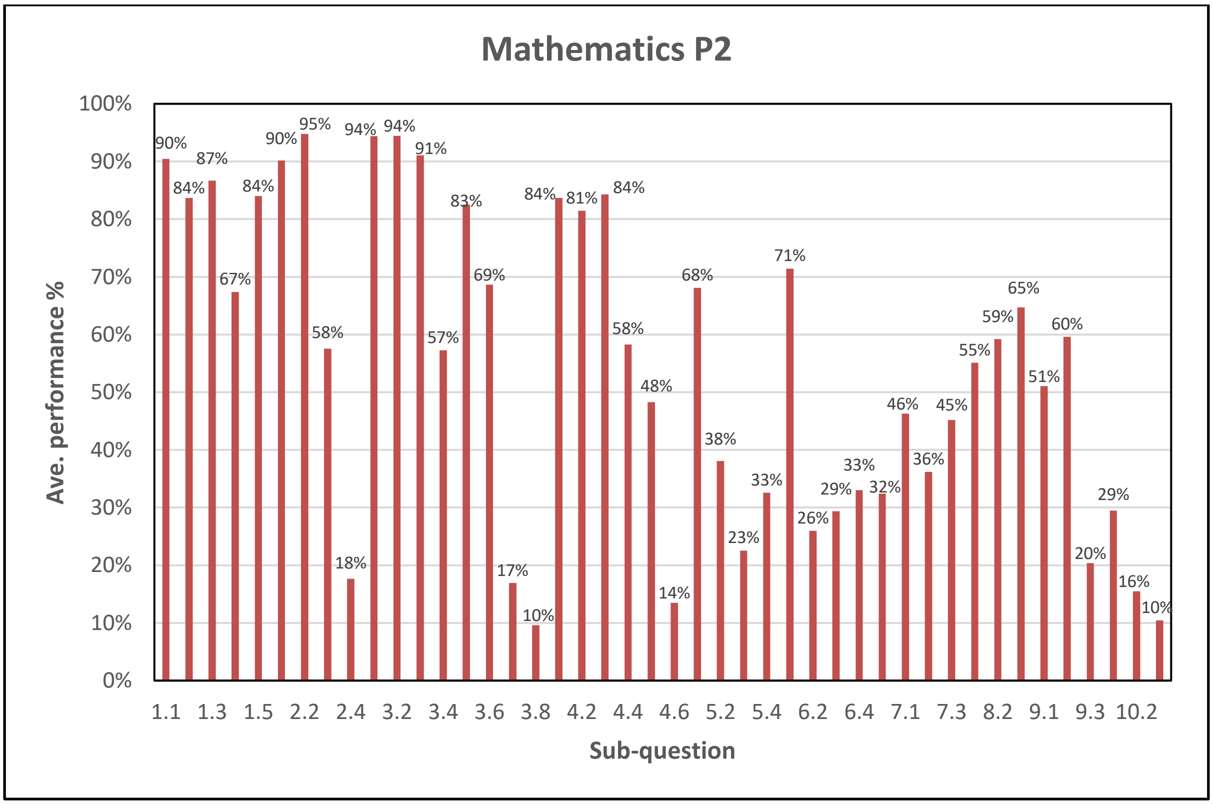
The following graph was based on data from a random sample of candidates' scripts. While this graph might not accurately reflect national averages, it is useful in assessing the relative degrees of challenge of each question as experienced by candidates.

Graph 10.6.1 Average performance per question in Paper 2



Q	Topic(s)
1	Data Handling
2	Data Handling
3	Analytical Geometry
4	Analytical Geometry
5	Trigonometry
6	Trigonometry
7	Trigonometry
8	Euclidean Geometry
9	Euclidean Geometry
10	Euclidean Geometry

Graph 10.6.2 Average performance per sub-question in Paper 2



10.7 ANALYSIS OF CANDIDATES' PERFORMANCE IN EACH QUESTION IN PAPER 2

QUESTION 1: DATA HANDLING

Common errors and misconceptions

(a) When writing the equation in Q1.1 a few candidates interchanged the values of *a* and *b*. Some calculated the values of *a* and *b* correctly but did not write the required equation or they swopped the values of *a* and *b* in the equation. Another common error in this question was that candidates failed to round off their answers for *a* and *b* correctly to two decimal places.

(b) In Q1.2 some candidates substituted 550 for *y* instead of substituting 550 for *x*.

(c) Some candidates rounded off the value of *r* to 1 instead of 0,98 when answering Q1.3. Candidates should have been aware that 0,98 and 1 have very different meanings when analysing *correlation coefficients*.

(d) When answering Q1.4 some candidates either wrote 'strong' or 'positive' as their answer. Both were unacceptable as they only described one attribute of the correct answer.

(e) In Q1.5.2 some candidates did not round off their answer correctly to two decimal places.

(f) Many candidates calculated the standard deviation interval correctly in Q1.5.3 but failed to indicate the number of stops that were less than one standard deviation below the mean. They were not awarded full marks because they did not answer the question completely.

Suggestions for improvement

(a) Teachers should link the equation of the least squares regression line ($y = a + bx$) with the equation of the straight line and emphasise that '*a*' refers to the *y-intercept* and '*b*' refers to the *gradient*.

(b) Learners must be careful not to interchange the *x*- and *y*-values in the table when they input these into the calculator.

(c) When determining the equation of the least squares regression line, it is advisable that learners write down the values of *a* and *b* and then write down the equation of the regression line. In this way, they can get the CA mark for the equation.

(d) When teaching *Statistics*, the focus should not only be the calculations. Teachers should also pay attention to the meaning of the different concepts, e.g. *mean, standard deviation, correlation coefficient,* etc. The values obtained in the calculations should then become more meaningful for learners.

(e) The understanding of statistical terminology is developed by using these terms frequently in the class. The use of diagrams when explaining the concepts of *standard deviation* and *deviation intervals* from the *mean* should help learners in understanding these concepts.

(f) Practise calculator skills with learners. Learners should be familiar with what the symbols on the calculator represent, for example, σ_x represents *population standard deviation* and *r* represents *correlation coefficient*.

(g) Learners should be able to use the values of their calculations to make predictions and comments about the data. Time should be devoted to interpretation questions.

QUESTION 2: DATA HANDLING

Common errors and misconceptions

(a) A few candidates calculated the midpoint of the class intervals when answering Q2.1. These candidates were attempting to calculate the estimated mean of the data instead of the cumulative frequencies.

(b) In Q2.3 many candidates were unable to differentiate between *frequency* and *cumulative frequency*. They gave the answer as 13, the frequency of the interval $4 \le x < 6$, instead of the correct answer of 33.

(c) In response to Q2.4 reading for meaning proved to be a challenge for many candidates. Some of the candidates who were able to understand the question made careless errors when setting up the equation. They left out the brackets and wrote $\left(5\times 13+\frac{k}{2}\right)$ instead of $5\times\left(13+\frac{k}{2}\right)$. Some candidates failed to realise that the total number of teachers was now 40 + *k*. Other candidates incorrectly subtracted the frequencies, i.e. 13 – 5, and arrived at the correct answer of 8. They were not awarded any marks for this attempt. A number of candidates only wrote down 8, the correct answer. These candidates were only awarded one mark as there was no working to substantiate their answer.

Suggestions for improvement

(a) Learners need to understand the difference in the meaning of the concepts: *frequency* and *cumulative frequency*.

(b) Reading for understanding is a fundamental requirement in the *Data Handling* section. This skill needs to be developed in classroom activities.

(c) While the calculator is a useful tool in answering many questions in the *Data Handling* section, teachers cannot overlook performing the calculations manually. This assists learners in understanding the calculation but moreover, it provides learners with the necessary skills to deal with questions that include variables.

QUESTION 3: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) In Q3.1 some candidates swopped the *x*- and *y*-values around when substituting into the distance formula. Other candidates calculated the gradient of SL instead of the length of SL.

(b) In Q3.2 some candidates used the incorrect gradient formula despite it being given on the information sheet. Other candidates swopped the *x*- and *y*-values around when substituting into the gradient formula.

(c) A few candidates only wrote down the value of tan θ when answering Q3.3. They did not continue to calculate the size of θ, as was required in the question. Some candidates incorrectly assumed that the size of θ was $\frac{4}{3}$.

(d) In response to Q3.4 some candidates calculated $N\hat{K}O$, instead of $L\hat{K}O$. Other candidates calculated the gradient correctly but then incorrectly calculated the angle of inclination as a positive angle.

$$m_{LN} = \frac{1+4}{-4+2}$$
$$m_{LN} = -2$$
$$\therefore L\hat{K}O = 63{,}43^\circ.$$

Other candidates incorrectly assumed that ΔKON was an isosceles triangle.

(e) Although Q3.5 was well answered by most candidates, some candidates used the gradient of the line perpendicular to SN instead of using the gradient of a line parallel to SN. Some candidates incorrectly assumed that the line passed through S and used the coordinates of S in determining the equation of the line.

(f) Many candidates attempted to answer Q3.6 by making use of the area rule. In doing so, they made incorrect substitutions into the area rule, i.e. they would substitute the length/s of sides that did not form ΔLSN. Candidates failed to realise that ΔLSN was right- angled and that they could make use of the area formula: Area = $\frac{1}{2}\text{base} \times \text{height}$ to answer the question. Some candidates incorrectly used SN as the height of the triangle.

(g) In Q3.7 many candidates assumed that P would be the midpoint of SN without first stating that SN is a diameter of the circle. It was critical for candidates to establish that SN was a diameter for P to be the centre of the circle. Some candidates incorrectly assumed that P was the 4th vertex of a rectangle LNSP and calculated the coordinates of P as (6 ; 1). This was incorrect as this point (6 ; 1) was not equidistant from L, S and N.

(h) Most candidates were unable to answer Q3.8 because they were unable to calculate the coordinates of P in Q3.7.

Suggestions for improvement

(a) If learners are not sure, they should consult the information sheet for the correct formula.

(b) Teachers should teach learners to label the coordinates as $(x_1 ; y_1)$ and $(x_2 ; y_2)$ on the diagram. This will prevent learners from making mistakes when substituting the coordinates into a formula. The order of substitution must be consistent, especially when using the gradient formula.

(c) Teachers need to emphasise the relationship between the sign of the gradient and the size of the angle of inclination of the line.

(d) It is important that learners realise that it is not acceptable to make any assumptions, e.g. that a certain point is the midpoint of a line. Even if it looks as if the point is the midpoint, it may not just be assumed and used. These need to be proven first before the results can be used in an answer.

(e) Teach learners to analyse diagrams in Analytical Geometry and to use relevant properties to respond to questions.

(f) Teachers should show learners different orientations of the base and the perpendicular height of a triangle. This should give learners more options when calculating the area of a triangle.

(g) Teach learners to expect that Euclidean Geometry facts will be integrated into Analytical Geometry and will be needed in the answering of some Analytical Geometry questions.

QUESTION 4: ANALYTICAL GEOMETRY

Common errors and misconceptions

(a) In Q4.1 some candidates solved for *x* in the equation $x^2 + y^2 = 20$ and did not give the answer in terms of *p,* as was required by the question. Some candidates gave their answer as $p = \pm 4$, instead of just $p = 4$.

(b) In Q4.2 some candidates calculated the midpoint of DE, instead of using the midpoint E to calculate the coordinates of F.

(c) Many candidates answered Q4.3 correctly. Those who did not obtain full marks made basic algebraic mistakes, e.g. not changing a sign of a term when transposing it.

(d) Most candidates were unable to answer Q4.4 correctly. Many candidates calculated the x-intercept of DF, namely 5, and gave that as the answer to the *x*-coordinate of the centre of the bigger circle.
Some candidates displayed poor algebraic skills when solving for *t* as shown below:

$$\frac{0-6}{t-8} = \frac{-1}{2}$$
$$-t-8 = -12$$

This question could be answered by using the fact that the tangent is perpendicular to the radius. However, candidates attempted much more complicated methods and made many incorrect assumptions and errors in their calculations.

(e) Candidates who could not answer Q4.4 were unable to attempt Q4.5 because they did not have the coordinates of G at their disposal. Some candidates incorrectly stated that the radius of the bigger circle was 0.

(f) Very few candidates were able to answer Q4.6 correctly. A contributing factor for this was that candidates needed to have answered Q4.4 and Q4.5 to attempt Q4.6. Some candidates knew that for circles to touch internally the distance between their centres must be equal to the difference between the lengths of their radii. They wrote down this distance. However, this question asked for more than that, namely, how far to translate the smaller circle to touch the larger circle internally. These candidates could not link the distance that they calculated with the required answer.

Suggestions for improvement

(a) Teachers need to revise the concept of *perpendicular lines* and *gradients*, particularly that the tangent is perpendicular to the radius at the point of contact.

(b) Learners should be reminded to refer to the information sheet for the relevant formulae.

(c) Learners should practise using a formula to get an answer (e.g. using the formula to calculate the coordinates of the midpoint), as well as to calculate an unknown variable if the answer has been given (e.g. calculate the coordinates of an endpoint if one endpoint and the midpoint are given).

(d) Learners must also be exposed to higher-order questions in class and in school-based assessment tasks. Questions on intersecting circles and circles touching internally and externally should be included in these tasks.

QUESTION 5: TRIGONOMETRY

Common errors and misconceptions

(a) Many candidates were unable to identify the quadrant correctly, and therefore used $x=+2\sqrt{2}$ instead of $x=-2\sqrt{2}$ throughout Q5.1. Some candidates ignored the instruction to not use a calculator and gave decimal answers to the trigonometric ratios. They were penalised for this.

(b) In Q5.1.2 some candidates were unable to write the expansion for sin 2β correctly despite it being given in the information sheet. Instead they incorrectly wrote the expansion for sin 2β as 2sin β.

(c) When answering Q5.1.3 some candidates were not able to reduce $\cos\left(450°-\beta\right)$ correctly to sin β, i.e. they were unable to deal correctly with an angle greater than 360° as well as a co-ratio.
A common incorrect response that showed a lack of understanding of compound angles was:

$\cos\left(450°\text{-}\beta\right)$

$\cos 450°\text{-}\cos\beta$

$0\text{ - }\cos\beta$

$\text{- }\cos\beta$

(d) Q5.2.1 was poorly answered by many candidates as they failed to realise that $\cos^4 x+\sin^2 x.\cos^2 x$ could be factorised. Some candidates flouted a very basic rule of Algebra by cancelling terms of an expression: $\frac{1-\sin^2 x}{1+\sin x}=1-\sin x$. Although these candidates arrived at the correct answer, they were not awarded any marks. Instead, they should have factorised the numerator and then cancelled factors as shown: $\frac{1-\sin^2 x}{1+\sin x}=\frac{\left(1-\sin x\right)\left(1+\sin x\right)}{1+\sin x}=1-\sin x$.

(e) In Q5.2.2 many candidates left their answers as a general solution, instead of the specific solution in the given interval. Some candidates included the reference angle of 90° as a solution.

(f) Many candidates did not respond to Q5.2.3 because they could not link the given expression to the sine graph.

(g) In Q5.3.1 almost all candidates were unable to use the expansion for cos (A – B) to derive the expansion for sin (A – B).They had no idea how to begin with this derivation.

(h) In Q5.3.2 some candidates incorrectly considered sin48°cos*x* – cos48°sin*x* to be the expansion for the cosine compound angle instead of the sine compound angle. This lead to the incorrect general solution for *x*.

(i) The candidates' response to Q5.3 was poor. Some candidates started their responses incorrectly by indicating that sin 3*x* was equal to sin 2*x* + sin *x*.
Other candidates incorrectly factorised the numerator as:

$$\frac{\sin 3x + \sin x}{\cos 2x + 1}$$

$$\frac{\sin x\left(\sin 2x + 1\right)}{\cos 2x + 1}$$

Candidates also failed to choose the appropriate expansion for cos 2*x*. Consequently, they were unable to simply the expression to a single trigonometric ratio.

Suggestions for improvement

(a) Learners find it difficult to recall the Trigonometry taught in Grades 10 and 11. Teachers should ensure that all learners are able to select the relevant quadrant when drawing sketches in the Cartesian plane to calculate trigonometric ratios.

(b) Remind learners that the same simplification skills used in Algebra also apply to Trigonometry. Revise addition, subtraction and simplification of algebraic fractions with learners before teaching simplification of trigonometric expressions and proving trigonometric identities.

(c) Expose learners to questions on trigonometric ratios, involving combinations of compound angles, angles greater than 360° and co-ratios.

(d) Learners should be encouraged to use sketch graphs of sin *x* and cos *x* when solving equations where either of these ratios is equal to 1, 0 or –1.

(e) Learners should be given exercises to practise simplifying complex trigonometric expressions, proving identities and solving complex trigonometric equations.

QUESTION 6: TRIGONOMETRY (GRAPHS)

Common errors and misconceptions

(a) In Q6.1 some candidates gave the domain (which is an interval) instead of the period (which is a single value). Many candidates incorrectly divided 180° by 2 instead of dividing 360° by 2.

(b) In answering Q6.2 many candidates ignored the domain specified in the question. They gave the range as [–1 ; 1] instead of $\left[-\frac{\sqrt{2}}{2};1\right]$. Some candidates incorrectly excluded the extremities of the interval.

(c) Q6.3.1 is a familiar question, yet many candidates failed to respond to this question. Some included the endpoints of the interval not realising that at the endpoints the two functions are equal. In this instance, their answer was [45° ; 90°] instead of (45° ; 90°). Some candidates showed a lack of understanding of how to write an interval as an inequality. They incorrectly gave the answer as $x > 45°$ or $x < 90°$.

(d) When answering Q6.3.2 some candidates were able to establish the critical values of 105° and 165° but were unable to present the answer correctly as an interval.

(e) Many candidates used algebraic methods to solve the equation in Q6.4 instead of using the graph. They would either leave their answer as a general solution or only provided one solution, i.e. $x = 15°$

(f) As in Q6.4 candidates again resorted to algebraic methods to answer Q6.5 instead of translating the graph and observing the image obtained. It was disturbing that a number of candidates did not respond to this question.

Suggestions for improvement

(a) Although these concepts are discussed in Grade 10, it is necessary for learners to be constantly reminded of the meaning of concepts like *period, domain, amplitude* and *range.*

(b) Learners should be told that the period of a trigonometric function is the length of a function's cycle. Since this value is a length, it is a single number and not an interval of values.

(c) Learners should be shown how to write intervals, using both inequalities and interval notation.

(d) Teachers should make learners aware of the cyclic nature of trigonometric graphs. This is useful in determining the coordinates of other points on the graph.

(e) Teachers should put more emphasis on teaching graphical interpretation, by reading off values from graphs and using the properties of the graphs and transformations rather than using long algebraic methods. This skill is particularly useful when the question is allocated only a few marks.

QUESTION 7: TRIGONOMETRY

Common errors and misconceptions

(a) In Q7.1 many candidates attempted to use the sine formula instead of the area formula. This resulted in them not being able to express the side of the triangle in terms of the required variables.

(b) Q7.2 required candidates to analyse the diagram and create a trigonometric expression for RS and substitute into this expression the result obtained in Q7.1.

Instead, some candidates tried to manipulate the expression given for RS algebraically. These candidates assumed that the given expression was true. They were not awarded any marks for their efforts.

(c) Some candidates did not realise that they could use the expression given in Q7.2 to calculate the size of α. Instead, they tried to formulate other expressions involving α, most common of which was to use the sine formula in ΔSTK. This proved to be fruitless. Some candidates rounded off the answer for sin α. Consequently, they arrived at $\alpha = 53{,}13°$ instead of $\alpha = 53{,}51°$.

Suggestions for improvement

(a) Teachers should devote the appropriate amount of time to this section. This should allow learners to score the accessible marks in this section of work.

(b) Teachers need to develop strategies to be used when solving right-angled triangles and triangles that are not right-angled. Teach learners the conditions that determine which rule should be used to solve the question.

(c) It might be a good idea to give learners an exercise in which they identify which rule is to be used to solve the question. The learners must also substantiate why they think that the rule that they have selected applies to the question.

(d) Learners should be encouraged to highlight the different triangles using different colours.

(e) Initially, expose learners to numeric questions on solving 3D problems. This makes it easier for them to develop strategies on how to solve such questions. Once learners have gained confidence with numeric type questions, they should then be exposed to non-numeric and higher-order questions.

(f) Learners must be reminded that they should not round off intermediate values in their calculations. Early rounding off creates an error in the final answer. Only the final answer should be rounded off to the required number of places.

QUESTION 8: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) Q8.1 tested bookwork. Some candidates did not show or describe any construction. Some candidates labelled angles inappropriately, e.g. just $\hat{K}$, instead of $\hat{K}_1$ or $\hat{K}_2$. Some candidates used as reason 'isosceles triangle', instead of 'angles opposite equal sides'.

(b) Some candidates made the following incorrect statements when answering Q8.2:
- DOBC is a cyclic quadrilateral.
- $\hat{A} = 2\hat{O}_1$
- $\hat{O}_2 = \frac{1}{2}\hat{C}$
- $\hat{A} = \hat{C}$

(c) When answering Q8.3.1 many candidates were able to state that $O\hat{M}B = 90°$. However, they provided the following incorrect reasons for their statement:

- radius perpendicular to chord.
- line from centre perpendicular to chord.
- line from centre to midpoint of chord.

(d) In Q8.3.2 some candidates were unable to provide the correct reason for AM being equal to MB. However, they were able to calculate the length of OB correctly. Some candidates did not use brackets when substituting into the expression for the *Theorem of Pythagoras*. They wrote $5\sqrt{3}^{\,2}$ instead of $\left(5\sqrt{3}\right)^2$. Consequently, they went on to enter the same into the calculator and obtained an incorrect final answer

Suggestions for improvement

(a) Learners should be taught that a construction is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as a breakdown and they get no marks.

(b) Teachers must cover the basic work thoroughly. An explanation of the theorem should be accompanied by showing the relationship in a diagram. In addition, learners should be made to prove theorems as part of their informal tasks. A good strategy is to expect learners to write the proof of a theorem as a task the day after the theorem was explained in class.

(c) Teachers are encouraged to use the 'Acceptable Reasons' in the *Examination Guidelines* when teaching. This should start from as early as Grade 8. Learners should be issued with a copy of the 'Acceptable Reasons'.

(d) Learners should be encouraged to scrutinise the given information and the diagram for clues about which theorems could be used when answering the question.

(e) Learners should be taught that all statements must be accompanied by reasons. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180° or when stating the proportional intercept theorem.

QUESTION 9: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) In Q9.1 many candidates did not get a mark for the reason, because of only stating: 'proportionality theorem', instead of also stating which lines were parallel in the reason. Some candidates equated ratios between sides which were not actually equal, because they did not choose the sides appropriately, e.g. $\frac{\text{FB}}{\text{EB}} = \frac{\text{DE}}{\text{EA}}$.

(b) In Q9.2 many candidates did not label the angles correctly, e.g. $\hat{\text{F}}$ and $\hat{\text{B}}$ instead of $\text{E}\hat{\text{F}}\text{D}$ and $\text{E}\hat{\text{B}}\text{A}$. Some candidates incorrectly gave the reason as 'alternate angles' or 'co-interior angles'. Other candidates correctly gave the reason as 'corresponding angles'. However, they did not state 'the lines parallel' and were not awarded a mark as the reason was incomplete.

(c) Many candidates incorrectly used the *midpoint theorem* to answer Q9.3. They should have used the fact that the corresponding sides are in proportion when two triangles are similar. A few candidates incorrectly applied the *Theorem of Pythagoras* even though there was no right-angled triangle. They were not aware of the minimum conditions in which the *Theorem of Pythagoras* could be used.

Suggestions for improvement

(a) Teachers should focus on developing learners' skills to analyse the question and the diagram for clues on which theorems are required to answer the questions correctly.

(b) Clearly explain to learners the difference between the *midpoint theorem*, the *proportionality theorem* and *similarity* so that they will know which of these concepts can be used in a specific situation.

(c) When answering Euclidean Geometry, learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.

(d) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

(e) Teachers should take some time to discuss the naming of angles, for example, the acceptable methods are $\hat{T}$ or $\hat{T}_1$ or $O\hat{T}S$. Teachers should also clarify when it is acceptable to refer to an angle at $\hat{T}$ and when to refer to it as $\hat{T}_1$.

QUESTION 10: EUCLIDEAN GEOMETRY

Common errors and misconceptions

(a) A fair number of candidates made incorrect assumptions when answering Q10.1. Among them were that: an exterior angle of the cyclic quadrilateral ($\hat{B}_3$) = the interior opposite angle ($\hat{R}_2$), RP = RQ and therefore APQR is a kite, RQ || AP and $\hat{M}_1 = 90°$.

(b) Candidates who could not answer Q10.1 correctly could not understand how to start to answer Q10.2. Some candidates used the properties of a cyclic quadrilateral in their attempt to prove that the same quadrilateral is cyclic. Some candidates did not know the difference between a *theorem* and its *converse*. They omitted the word 'converse' in the reason: 'exterior angle of cyclic quad'.

(c) Very few candidates obtained full marks for Q10.3. The main reason for this was that candidates were unable to answer Q10.1 and Q10.2 correctly. Poor naming of angles in the answers often led to candidates themselves getting confused about which angle they were referring to.

(d) Q10.3 required candidates to obtain a proportion from the similar triangles in Q10.2, using the *proportional intercept theorem* in ΔRAC to establish a second proportion and then to combine the two. Many candidates failed to establish one or the other proportion and therefore could not make the conclusion.

Suggestions for improvement

(a) More time needs to be spent on the teaching of Euclidean Geometry in all grades. More practice on Grade 11 and 12 Euclidean Geometry will help learners to understand theorems and diagram analysis. They should read the given information carefully without making any assumptions. This work covered in class must include different activities and all levels of the taxonomy.

(b) Teach learners not to assume any facts in a geometry sketch but to only use what was given and that which was proven already in earlier questions.

(c) Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination.

(d) Consider teaching the approach of ‘angle chasing’ where you label one angle as x and then relate other angles to x. In this way, learners should find it easy to identify angles that are equal but moreover, they should find it easier to establish the reasons for the relationships between the angles.