2023
ALGEBRA

Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES
Your Key to Exam Success

ALGEBRA (71%): DBE NOVEMBER 2023

QUESTION 1   71%

1.1 Solve for x:

84%

1.1.1 $x^2 + x - 12 = 0$ (3)

1.1.2 $3x^2 - 2x = 6$ *(correct to TWO decimal places)* (4)

1.1.3 $\sqrt{2x+1} = x - 1$ (4)

Memo

1.1.1 $x^2 + x - 12 = 0$

$\therefore (x+4)(x-3) = 0$

$\therefore x + 4 = 0$ or $x - 3 = 0$

$\therefore \boldsymbol{x = -4}$ ◄ $\therefore \boldsymbol{x = 3}$ ◄

1.1.2 $3x^2 - 2x = 6$

$\therefore 3x^2 - 2x - 6 = 0$

$\therefore x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(-6)}}{2(3)}$ . . . $\boldsymbol{x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$

$= \dfrac{2 \pm \sqrt{4+72}}{6}$

$\approx$ **1,79 or –1,12** ◄

1.1.3 $\sqrt{2x+1} = x - 1$

$\therefore \left(\sqrt{2x+1}\right)^2 = (x-1)^2$

$\therefore 2x + 1 = x^2 - 2x + 1$

$\therefore 0 = x^2 - 4x$

$\therefore x(x-4) = 0$

$\therefore x = 0$ or 4

Only $\boldsymbol{x = 4}$ ◄ . . . *For $x = 0$, $\sqrt{\ }$ is neg*

Common Errors and Misconceptions

(a) In **Q1.1.2** some candidates did not write the equation correctly in **standard form**, so they incorrectly substituted '6' as the value of c instead of '–6'.

(b) In **Q1.1.3** many candidates were able to square the equation correctly, but then factorised $x^2 - 4x = 0$ incorrectly as $(x-2)(x+2) = 0$. Some divided $x^2 - 4x = 0$ through by x which **meant** that the candidate **lost a solution** to the equation.

1.1.4 $x^2 - 3 > 2x$ (4)

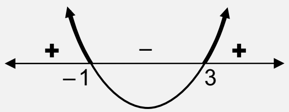
Memo

1.1.4 $x^2 - 3 > 2x$

$\therefore x^2 - 2x - 3 > 0$

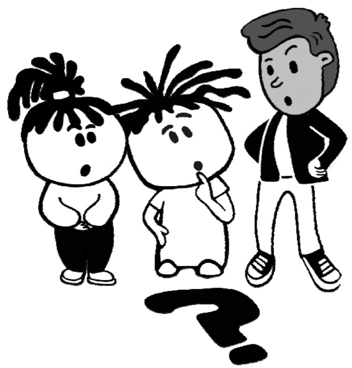
$\therefore (x - 3)(x + 1) > 0$

$\therefore x < -1$ or $x > 3$



Common Errors and Misconceptions

(c) Many candidates **struggled to solve the inequality** in **Q1.1.4** after correctly calculating the critical values. Some candidates drew a **sketch but were unable to use it to write down the required answer**. Another common error was the **incorrect notation** in the answer. Candidates wrote the answer as $x > -1$ or $x > 3$ instead of $x < -1$ or $x > 3$.



1.2 Solve for x and y simultaneously:

62%

$x + 2 = 2y$ and $\frac{1}{x} + \frac{1}{y} = 1$ (5)

Memo

1.2 $x + 2 = 2y$

$\therefore x = 2y - 2$ ... ❶

$\frac{1}{x} + \frac{1}{y} = 1$

$(\times xy)$ $\therefore y + x = xy$ ... ❷

❶ in ❷: $\therefore y + (2y - 2) = y(2y - 2)$

$\therefore 3y - 2 = 2y^2 - 2y$

$\therefore 0 = 2y^2 - 5y + 2$

$\therefore (2y - 1)(y - 2) = 0$

$\therefore y = \frac{1}{2}$ or $y = 2$

For $y = \frac{1}{2}$: $x = 2\left(\frac{1}{2}\right) - 2 = -1$

& For $y = 2$: $x = 2(2) - 2 = 2$

$\therefore$ Solution: $\left(-1; \frac{1}{2}\right)$ **or** **(2; 2)**

OR:

$x + 2 = 2y$

$\therefore x = 2y - 2$ ... ❶

$\frac{1}{x} + \frac{1}{y} = 1$ ... ❷

❶ in ❷: $\therefore \frac{1}{2y-2} + \frac{1}{y} = 1$ $\times y(2y - 2)$

$\therefore y + 2y - 2 = y(2y - 2)$

$\therefore y + 2y - 2 = 2y^2 - 2y$

$\therefore 2y^2 - 5y + 2 = 0$

$\therefore (2y - 1)(y - 2) = 0$

$\therefore y = \frac{1}{2}$ or $y = 2$

$\therefore x = -1$ or $x = 2$

$\therefore \left(-1; \frac{1}{2}\right)$ **or** **(2; 2)**

Common Errors and Misconceptions

(d) The simultaneous equation given in fraction form in **Q1.2** proved challenging for most candidates. The notion of isolating the one variable and substituting this into the second equation was well understood. However, candidates changed $\frac{1}{x} + \frac{1}{y} = 1$ to $x^{-1} + y^{-1} = 1$ and then after substitution **incorrectly worked** with the exponent of -1.

1.3 Given: $2^{m+1} + 2^m = 3^{n+2} - 3^n$ where m and n are integers. Determine the value of $m + n$. (4)

32%

[24]

Memo

1.3 $2^{m+1} + 2^m = 3^{n+2} - 3^n$

$\therefore\ 2^m.2 + 2^m = 3^n.3^2 - 3^n$

$\therefore\ 2^m(2+1) = 3^n(3^2 - 1)$

$\therefore\ 3.2^m = 8.3^n$

$(\div 3.8)\quad \frac{2^m}{8} = \frac{3^n}{3}$

$\therefore\ \frac{2^m}{2^3} = \frac{3^n}{3}$

$\therefore\ 2^{m-3} = 3^{n-1}$

only possible if $m - 3 = 0$ and $n - 1 = 0$

$\therefore\ m = 3 \qquad \therefore\ n = 1$

$\therefore$ **m + n = 4** ✓

OR: $2^m(2+1) = 3^n(3^2 - 1)$

$\therefore\ 2^m.3 = 3^n.8$

$\frac{2^m}{3^n} = \frac{8}{3}$

$\therefore\ \frac{2^m}{3^n} = \frac{2^3}{3}$

$\therefore\ m = 3$ and $n = 1$

$\therefore$ **m + n = 4** ✓

Common Errors and Misconceptions

(e) In **Q1.3** many candidates were able to write the exponents with separate bases, i.e. $2^m.2 + 2^m = 3^n.3^2 - 3^n$. However, some **failed to factorise** the resulting expression correctly, i.e. $2^m.(2 + 1) = 3^n.(3^2 - 1)$. The majority of the candidates just worked with the exponents and this resulted in an equation $m + 1 + m = n + 2 - n$ which did not lead them to solving the given problem.

QUESTION 1: Suggestions for Improvement

(a) Learners must ensure that they understand what correct ***standard form*** is in a quadratic equation.

(b) Learners must be taught to **check their solutions** when using the squaring technique to solve an equation that is not originally quadratic.

(c) Emphasis must be given to working with lowest common denominators to solve fraction-based problems.

(d) When dealing with ***surd* equations**, learners should be reminded that they need to isolate the *radical* **before** they can **square both sides** of the equation. Teachers must emphasise that **implicit restrictions** are placed **on *surd* equations** and that learners should continue to test whether their answers satisfy the original equation.

(e) Learners need to be exposed to complex questions involving *surds* and *exponents*. Correct use of *exponential rules* must be revised, tested and re-explained from Grade 9 through to the final Grade 12 examination.

(f) Regular revision and emphasis on working with ***prime bases*** in *exponents* is important.

(g) Learners need to be exposed to problem-solving questions involving **algebraic manipulation in all areas of algebra**. This requires regular revision and testing of non-routine questions in the algebra section, focusing on a range of skills that candidates need to have mastered from Grade 10.

(h) As suggested in previous reports:

- Teachers should not take for granted that learners know how to **round off a number** to the required number of places. Where necessary, this skill should be retaught in Grades 11 and 12. Teachers should penalise learners in SBA tasks when they do not round off to the correct number of places.

- Teachers should take some time, preferably in Grade 10, to focus on teaching learners how to represent **inequalities** (e.g. $-3 < x < 5$; $x < -3$ or $x > 5$) on a number line and **also how to write an inequality** from the illustration on a number line. This will benefit learners as they are required to write inequality solutions for a number of questions in both examination papers. Emphasis on the **correct notation is essential** when writing down the solutions to inequalities.

- Linked to this, teachers should explain the **difference** between ***and*** and ***or*** in the context of inequalities. Learners cannot use these words interchangeably as **they have different meanings**.