2023

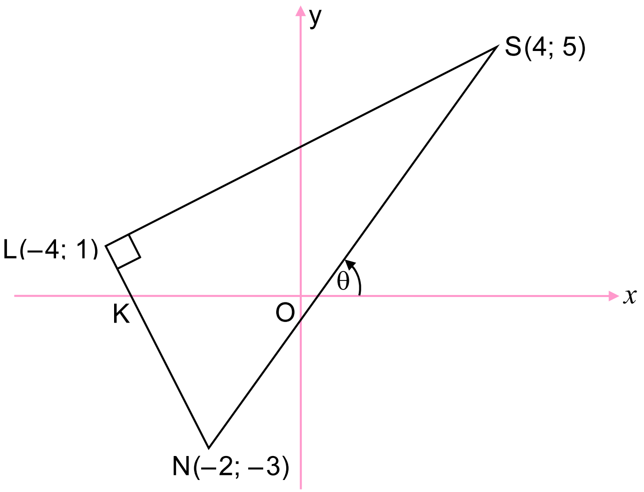
ANALYTICAL GEOMETRY

Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 3 63%

In the figure, L(–4; 1), S(4; 5) and N(–2; –3) are the vertices of a triangle having $S\hat{L}N = 90°$.
LN intersects the x-axis at K.



3.1 Calculate the length of SL. Leave your answer in surd form. (2)
94%

3.2 Calculate the gradient of SN. (2)
94%

3.3 Calculate the size of θ, the **angle** of inclination of SN. (2)
91%

MEMOS

3.1 $SL^2 = (4 + 4)^2 + (5 - 1)^2$
$= 64 + 16$
$= 80$
$\therefore\ SL = \sqrt{80} = 4\sqrt{5}$ **units** ➤

3.2 $m_{SN} = \dfrac{5-(-3)}{4-(-2)} = \dfrac{8}{6} = \mathbf{\dfrac{4}{3}}$ ➤

3.3 $\tan\theta = \dfrac{4}{3}$
$\therefore\ \theta \simeq$ **53,13°** ➤

Common Errors and Misconceptions

(a) In **Q3.1** some candidates swopped the x- and y-values around when substituting into the distance formula. Other candidates calculated the gradient of SL instead of the length of SL.

(b) In **Q3.2** some candidates used the **incorrect** gradient **formula** despite it being given on the information sheet. Other candidates swopped the x- and y-values around when substituting into the gradient formula.

(c) A few candidates only wrote down the value of tan θ when answering **Q3.3**. They did not continue to calculate the size of θ, as was required in the question. Some candidates **incorrectly** assumed that the size of θ was $\dfrac{4}{3}$.

QUESTION 3 (cont.)

3.4 Calculate the size of $L\hat{N}S$. (3)

57%

MEMOS

3.4 $S\hat{N}R = \theta = 53{,}13^{\circ}$ . . . *corresp* $\angle^{s}$; || *lines*

$$m_{LN} = \frac{1-(-3)}{-4-(-2)}$$

$$= \frac{4}{-2}$$

$$= -2$$

correct

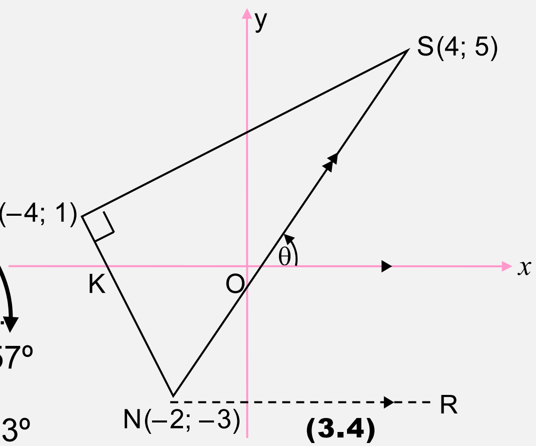
$\therefore \tan L\hat{N}R = -2$

$\therefore L\hat{N}R = 180^{\circ} - 63{,}43\ldots$

$\simeq 116{,}57^{\circ}$

$\therefore L\hat{N}S = 116{,}57^{\circ} - 53{,}13^{\circ}$

$= \mathbf{63{,}44^{\circ}}$ ✓



(3.4)

OR:

$LN^2 = (-4+2)^2 + (1+3)^2$

$= 4 + 16$

$= 20$

$\therefore LN = \sqrt{20} = 2\sqrt{5}$

$\therefore \tan L\hat{N}S = \frac{LS}{LN} = \frac{4\sqrt{5}}{2\sqrt{5}} = 2$

$\therefore \mathbf{L\hat{N}S = 63{,}43^{\circ}}$ ✓ **Note:** difference due to rounding

Common Errors and Misconceptions

(d) In response to **Q3.4** some candidates calculated $N\hat{K}O$, instead of $L\hat{K}O$. Other candidates calculated the gradient correctly but then **incorrectly** calculated the angle of inclination as an acute angle.

$$m_{LN} = \frac{1+4}{-4+2}$$

$$m_{LN} = -2$$

incorrect

$\therefore L\hat{K}O = 63{,}43^{\circ}$

Other candidates incorrectly assumed that ΔKON was an isosceles triangle.

QUESTION 3 (cont.)

3.5 Determine the equation of the line which passes through L and is parallel to SN.

83%

Write your answer in the form $y = mx + c$. (3)

MEMOS

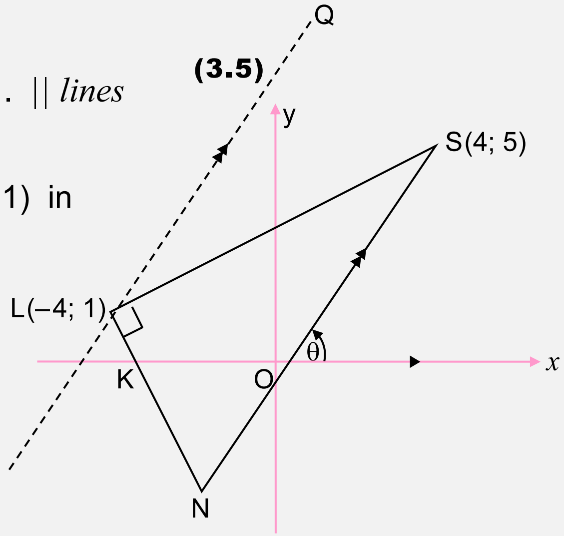
3.5 $m_{QL} = m_{SN} = \frac{4}{3}$ . . . || *lines*

Subst. $m = \frac{4}{3}$ & $(-4;\ 1)$ in

$y - y_1 = m(x - x_1)$

$\therefore\ y - 1 = \frac{4}{3}(x + 4)$

$\therefore\ \mathbf{y = \frac{4}{3}x + \frac{19}{3}}$ ◄



Common Errors and Misconceptions

(e) Although **Q3.5** was well answered by most candidates, some candidates used the gradient of the line perpendicular to SN instead of using the gradient of a line parallel to SN. Some candidates incorrectly assumed that the line passed through S and used the coordinates of S in determining the equation of the line.

3.6 Calculate the area of ΔLSN. (3)

69%

MEMOS

3.6 Area of $\Delta LSN = \frac{1}{2} LN \cdot SL$

$LN^2 = (-4 + 2)^2 + (1 + 3)^2 = 4 + 16 = 20$

$\therefore\ LN = \sqrt{20}$

$\therefore$ Area of ΔLSN

$= \frac{1}{2}\sqrt{20}\sqrt{80} = \frac{1}{2}\sqrt{1\ 600} = \frac{1}{2}(40) = \mathbf{20\ units^2}$ ◄

Common Errors and Misconceptions

(f) Many candidates attempted to answer **Q3.6** by making use of the area rule. In doing so, they made **incorrect** substitutions into the area rule, i.e. they would substitute the length/s of sides that did not form ΔLSN. Candidates **failed to realise** that ΔLSN was right-angled and that they could make use of the area formula:

Area $= \frac{1}{2}$ base $\times$ height to answer the question.

Some candidates incorrectly used SN as the height of the triangle.

QUESTION 3 (cont.)

3.7 Calculate the coordinates of point P, which is equidistant from L, S and N. (3)

17%

3.8 Calculate the size of $L\hat{P}S$. (2)

10%

[20]

MEMOS

3.7 P equidistant from L, S and N

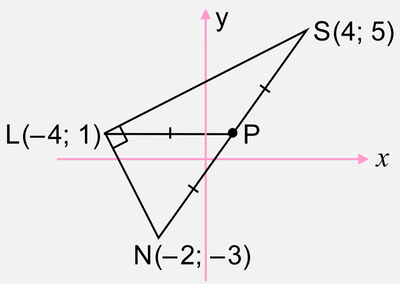
$N\hat{L}S = 90°$

$\therefore$ NS is the diameter of ⊙LSN . . . *conv. ∠ in semi-⊙*

$\therefore$ P is the midpoint of NS

$\therefore P\left(\dfrac{-2+4}{2};\ \dfrac{-3+5}{2}\right)$

$\therefore$ **P(1; 1)** ◄



3.8 $L\hat{P}S = 2L\hat{N}S$ . . . *∠ at centre = 2 × ∠ at circumference*

$= 2(63{,}44°) =$ **126,88°** ◄

OR: PL || x-axis . . . $y_P = y_L$

$L\hat{P}N = 53{,}13°$ . . . *alt ∠s ; || lines*

$\therefore L\hat{P}S = 180° - 53{,}13°$ . . . *∠s on a str line*

$=$ **126,87°** ◄

Note: difference due to rounding

Common Errors and Misconceptions

(g) In **Q3.7** many candidates **assumed** that P would be the midpoint of SN without first stating that SN is a diameter of the circle. It was critical for candidates to establish that SN was a **diameter** for P to be the centre of the circle.

Some candidates **incorrectly assumed** that P was the 4th vertex of a rectangle LNSP and calculated the coordinates of P as (6 ; 1). This was incorrect as this point (6 ; 1) was not equidistant from L, S and N.

(h) Most candidates were unable to answer **Q3.8** because they were unable to calculate the coordinates of P in **Q3.7**.

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QUESTION 3: Suggestions for Improvement

(a) If learners are not sure, they should consult **the information sheet** for the **correct formula**.

(b) Teachers should teach learners to label the coordinates as $(x_1 ; y_1)$ and $(x_2 ; y_2)$ on the diagram. This will prevent learners from making mistakes when substituting the coordinates into a formula. The order of substitution must be consistent, especially when using the gradient formula.

(c) Teachers need to emphasise the relationship between the sign of the gradient and the size of the angle of inclination of the line.

(d) It is important that learners realise that it is **not acceptable to make any assumptions**, e.g. that a certain point is the midpoint of a line. Even if it looks as if the point is the midpoint, it may not just be assumed and used. These need to be proven first before the results can be used in an answer.

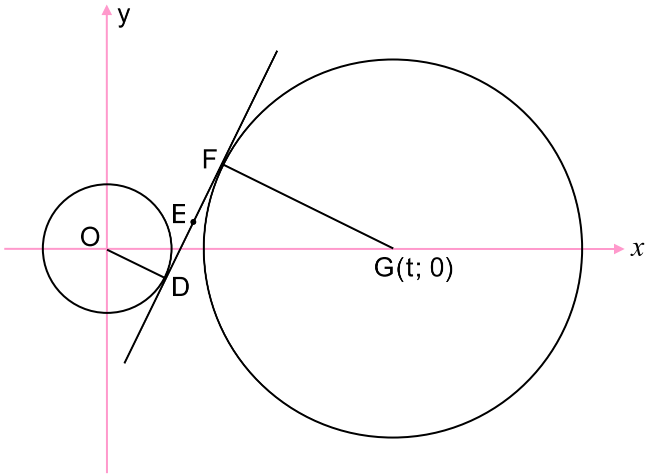
(e) Teach learners to **analyse diagrams** in Analytical Geometry and to use relevant properties to respond to questions.

(f) Teachers should show learners different orientations of the base and the perpendicular height of a triangle. This should give learners more options when calculating the area of a triangle.

(g) Teach learners to expect that **Euclidean Geometry** facts will be integrated into **Analytical Geometry** and will be needed in the answering of some Analytical Geometry questions.

QUESTION 4 59%

In the diagram, the circle with centre O has the equation $x^2 + y^2 = 20$. G(t; 0) is the centre of the larger circle. A common tangent touches the circles at D and F respectively, such that D(p; –2) lies in the 4th quadrant.



4.1 Given that D(p; –2) lies on the smaller circle, show that p = 4. (2)

84%

4.2 E(6; 2) is the midpoint of DF. Determine the coordinates of F. (3)

81%

MEMOS

4.1 D(p; –2) on ⊙O ➡ $p^2 + (-2)^2 = 20$

$\therefore\ p^2 = 16$

$\therefore\ \mathbf{p = 4}$ ✓ . . . *p > 0 in 4th Quadrant*

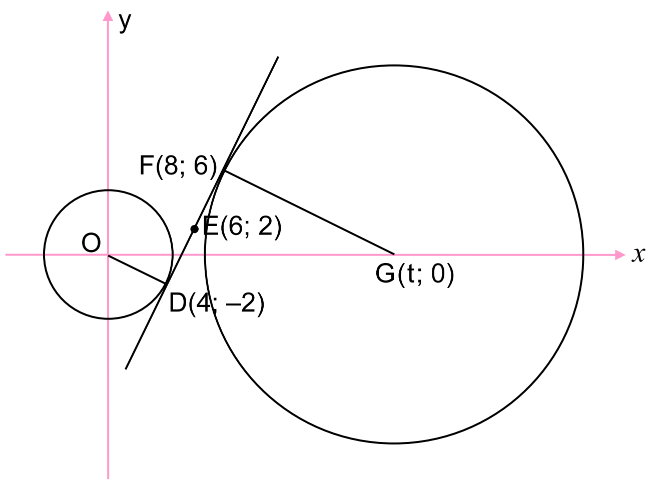
4.2 **F(8; 6)** ✓ . . . ***by inspection***

Common Errors and Misconceptions

(a) In **Q4.1** some candidates solved for x in the equation $x^2 + y^2 = 20$ and did not give the answer in terms of p, as was required by the question. Some candidates gave their answer as p = ± 4, instead of just p = 4.

(b) In **Q4.2** some candidates calculated the midpoint of DE, instead of using the midpoint E to calculate the coordinates of F.

QUESTION 4 (cont.)



4.3 Determine the equation of the common tangent, DF, in the form $y = mx + c$. (4)

84%

MEMOS

4.3 $m_{DE} = \frac{2+2}{6-4} = 2$

$\therefore\ m_{DF} = 2$

OR: $m_{\text{radius OD}} = -\frac{2}{4} = -\frac{1}{2}$

But OD $\perp$ DF . . . *radius $\perp$ tangent*

$\therefore\ m_{DF} = 2$

Subst. m = 2 & D(4; –2) in

$y - y_1 = m(x - x_1)$

$\therefore\ y + 2 = 2(x - 4)$

$\therefore\ \mathbf{y = 2x - 10}$ ◄

OR: $y = mx + c$

$\therefore -2 = (2)(4) + c$

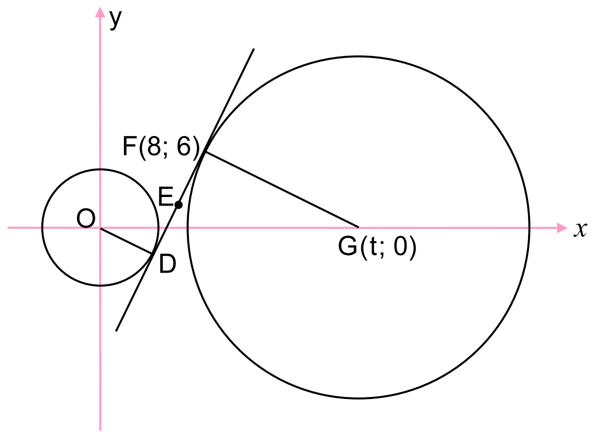
$\therefore\ c = -10$, etc.

Common Errors and Misconceptions

(c) Many candidates answered **Q4.3** correctly.

Those who did not obtain full marks made basic algebraic mistakes, e.g. not changing a sign of a term when transposing it.

QUESTION 4 (cont.)



4.4 Calculate the value of t. Show ALL working. (3)

58%

4.5 Determine the equation of the larger circle in the form

48% $ax^2 + by^2 + cx + dy + e = 0$. (4)

MEMOS

4.4 $m_{FG} = -\frac{1}{2}$ . . . *radius FG ⊥ tangent DF*

$\therefore \frac{6-0}{8-t} = -\frac{1}{2}$

$\times 2(8-t)$ $\therefore 12 = -(8-t)$

$\therefore 12 = -8 + t$

$\therefore$ **t = 20** ◄

OR:

use $m = -\frac{1}{2}$ and (8; 6):

Eqn of DF:

$y - 6 = -\frac{1}{2}(x - 8)$,

& subst. y = 0

4.5 Centre G is (20; 0) & $\mathbf{r^2} = FG^2 = (20-8) + (0-6)^2$

$= 144 + 36$

$= 180$

$\therefore$ Eqn of ⊙G: $(x-20)^2 + (y-0)^2 = 180$

$\therefore x^2 - 40x + 400 + y^2 - 180 = 0$

$\therefore \mathbf{x^2 + y^2 - 40x + 220 = 0}$ ◄

Common Errors and Misconceptions

(d) Most candidates were unable to answer **Q4.4** correctly. Many candidates calculated the x-intercept of DF, namely 5, and gave that as the answer to the x-coordinate of the centre of the bigger circle. Some candidates displayed **poor algebraic skills** when solving for t as shown below:

This question could be answered by using the fact that the tangent is perpendicular to the radius. However, candidates attempted much more complicated methods and **made many incorrect assumptions** and errors in their calculations.

(e) Candidates who could not answer **Q4.4** were unable to attempt **Q4.5** because they did not have the coordinates of G at their disposal. Some candidates incorrectly stated that the radius of the bigger circle was 0.

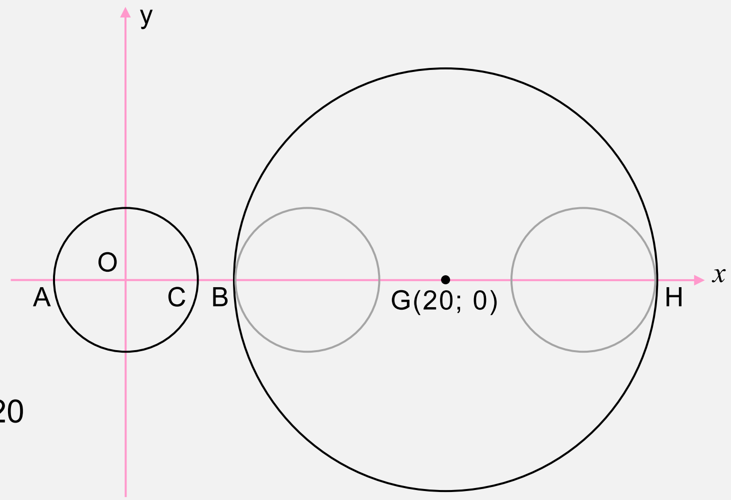
QUESTION 4 (cont.)

4.6 The smaller circle must be translated by k units along the x-axis to touch the larger circle internally. Calculate the possible values of k. (4)

14%

[20]

MEMOS



⊙O:
$x^2 + y^2 = 20$
(given)

4.6 Point A where the small circle cuts the x-axis must move to point B where the large circle cuts the x-axis, or, C to H.

Small ⊙: $r = \sqrt{20} = 2\sqrt{5}$

$\therefore\ x_A = -2\sqrt{5}$ and $x_C = 2\sqrt{5}$

Large ⊙: $R = \sqrt{180} = 6\sqrt{5}$ . . . $\left[\begin{array}{c}\text{In Q4.5: } \odot G \\ (x^2 - 20)^2 + y^2 = 180\end{array}\right]$

$\therefore\ x_B = 20 - 6\sqrt{5}$ and $x_H = 20 + 6\sqrt{5}$

$\therefore$ A ➡ B: $k = x_B - x_A$
$= 20 - 6\sqrt{5} - (-2\sqrt{5})$
$= 20 - 4\sqrt{5}$
$\simeq$ **11,06** ◀

C ➡ H: $k = x_H - x_C$
$= 20 + 6\sqrt{5} - 2\sqrt{5}$
$= 20 + 4\sqrt{5}$
$\simeq$ **28,94** ◀

There are several different approaches to this question.

Common Errors and Misconceptions

(f) Very few candidates were able to answer **Q4.6** correctly. A contributing factor for this was that candidates needed to have answered **Q4.4** and **Q4.5** to attempt **Q4.6**. Some candidates knew that for circles to touch internally the distance between their centres must be equal to the difference between the lengths of their radii. They wrote down this distance. **However**, this question asked for **more than that**, namely, how far to translate the smaller circle to touch the larger circle internally. These candidates could not link the distance that they calculated with the required answer.

QUESTION 4: Suggestions for Improvement

(a) Teachers need to revise the concept of *perpendicular lines* and *gradients*, particularly that the tangent is perpendicular to the radius at the point of contact.

(b) Learners should be reminded to refer to **the information sheet** for the relevant formulae.

(c) Learners should practise using a formula to get an answer (e.g. using the formula to calculate the coordinates of the midpoint), as well as to calculate an unknown variable if the answer has been given (e.g. calculate the coordinates of an endpoint if one endpoint and the midpoint are given) ***by inspection***.

(d) Learners must also be exposed to higher-order questions in class and in school-based assessment tasks. **Questions on intersecting circles and circles touching internally and externally should be included in these tasks**.