2023
EUCLIDEAN GEOMETRY

Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES
Your Key to Exam Success

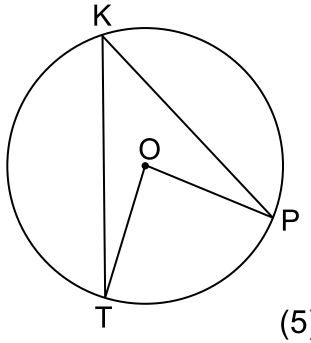
EUCLIDEAN GEOMETRY (41%): DBE NOVEMBER 2023

QUESTION 8 60%

8.1 In the diagram, O is the centre of the circle.

55%

Use the diagram alongside to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$.



(5)

MEMOS

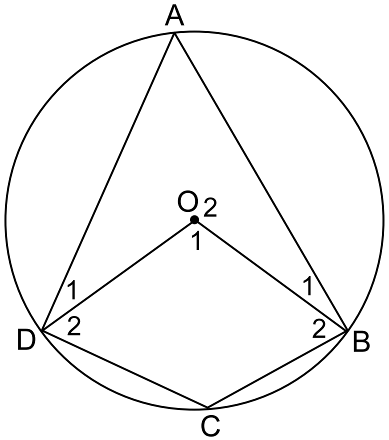
8.1 Theorem proof ➤

8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral.

59%

OB and OD are drawn.

If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x.



(5)

MEMOS

8.2 $\hat{A} = \frac{1}{2}(4x + 100°)$ . . . *∠ at centre = 2 × ∠ at circum*

$= 2x + 50°$

$\hat{A} + \hat{C} = 180°$ . . . *opp ∠s of cyclic quad*

$\therefore 2x + 50° + x + 34° = 180°$

$\therefore 3x + 84° = 180°$

$\therefore 3x = 96°$

$\therefore$ **$x = 32°$** ➤

Common Errors and Misconceptions

(a) **Q8.1** tested bookwork. Some candidates did not show or describe any construction. Some candidates labelled angles inappropriately, e.g. just $\hat{K}$, instead of $\hat{K}_1$ or $\hat{K}_2$.

Some candidates used as reason 'isosceles triangle', instead of 'angles opposite equal sides'.

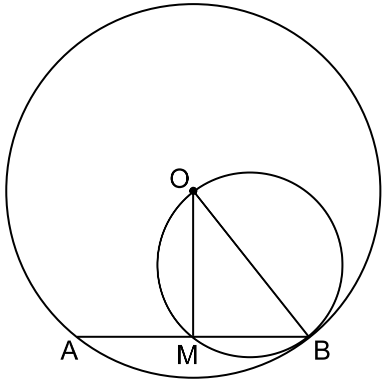
(b) Some candidates made the following **incorrect** statements when answering **Q8.2**:

- **DOBC is a cyclic quadrilateral.**
- **$\hat{A} = 2\hat{O}_1$**
- **$\hat{O}_2 = \frac{1}{2}\hat{C}$**
- **$\hat{A} = \hat{C}$**

QUESTION 8 (cont.)

8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.

65%



8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

8.3.2 If AB = $\sqrt{300}$ units and OM = 5 units, calculate, giving reasons, the length of OB. (4) **[16]**

MEMOS

8.3.1 $O\hat{M}B = 90°$ . . . *∠ in semi-⊙*

8.3.2 $OB^2 = OM^2 + MB^2$ . . . *Pythag*

But $MB = \frac{1}{2}AB$ . . . *line from centre ⊥ to chord*

$= \frac{1}{2}\sqrt{300}$ . . . OR: $5\sqrt{3}$

$\therefore OB^2 = 5^2 + \left(\frac{1}{2}\sqrt{300}\right)^2$

$= 25 + \left(\frac{1}{4}\times 300\right)$

$= 25 + 75$

$= 100$

$\therefore$ **OB = 10 units**

OR: $OB^2 = 5^2 + (\mathbf{5\sqrt{3}})^{\mathbf{2}}$

$= 25 + (25 \times 3)$

$= 25 + 75$

$= 100$, etc.

Common Errors and Misconceptions

(c) When answering **Q8.3.1** many candidates were able to state that $O\hat{M}B = 90°$. However, they provided the following incorrect reasons for their statement:

- radius perpendicular to chord.
- line from centre perpendicular to chord.
- line from centre to midpoint of chord.

(d) In **Q8.3.2** some candidates were unable to provide the correct reason for AM being equal to MB. However, they were able to calculate the length of OB correctly. Some candidates **did not use brackets** when substituting into the expression for the *Theorem of Pythagoras*. They wrote $5\sqrt{3}^{\mathbf{2}}$ instead of $(5\sqrt{3})^{\mathbf{2}}$. Consequently, they went on to enter the same into the calculator and obtained an incorrect final answer.

QUESTION 8: Suggestions for Improvement

(a) Learners should be taught that a construction is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as a breakdown and they get no marks.

(b) Teachers must cover the basic work thoroughly. An explanation of the **theorem** should be accompanied by showing the relationship in a diagram. In addition, learners should be made to prove theorems as part of their informal tasks. A good strategy is to expect learners to write the proof of a theorem as a task the day after the theorem was explained in class.

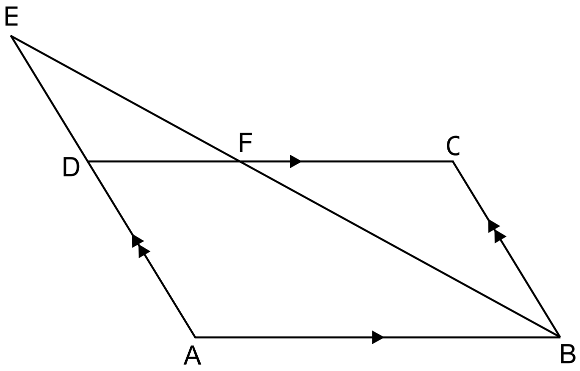
(c) Teachers are encouraged to use the **'Acceptable Reasons'** in the *Examination Guidelines* when teaching. This should start from as early as Grade 8. Learners should be issued with a copy of the 'Acceptable Reasons'.

(d) Learners should be encouraged to scrutinise the given information and the diagram for **clues** about which theorems could be used when answering the question.

(e) Learners should be taught that **all statements must be accompanied by reasons**. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180° or when stating the proportional intercept theorem.

QUESTION 9 44%

In the diagram, ABCD is a parallelogram with AB = 14 units.
AD is produced to E such that AD : DE = 4 : 3.
EB intersects DC in F.
EB = 21 units.



9.1 Calculate, with reasons, the length of FB. (3)

51%

9.2 Prove, with reasons, that ΔEDF ||| ΔEAB. (3)

60%

MEMOS

9.1 $\frac{FB}{FE} = \frac{AD}{DE} \left(= \frac{4}{3} \right)$ . . . *prop thm; DF || AB*

$\therefore$ Let FB = $4x$ & FE = $3x$

$\therefore$ $7x$ = 21 units

$\therefore$ x = 3 units

$\therefore$ **FB = 12 units** ◀

9.2 In Δ^s EDF & EAB

(1) $\hat{E}$ is common

(2) E$\hat{F}$D = E$\hat{B}$A . . . *corresp $\angle^s$; DF || AB*

$\therefore$ **ΔEDF ||| ΔEAB** ◀ . . . $\angle\angle\angle$

Common Errors and Misconceptions

(a) In **Q9.1** many candidates did not get a mark for the reason, because of only stating: 'proportionality theorem', instead of also stating which lines were parallel in the reason. Some candidates equated ratios between sides which were not actually equal, because they did not choose the sides appropriately, e.g. $\frac{FB}{EB} = \frac{DE}{EA}$.

(b) In **Q9.2** many candidates did not label the angles correctly, e.g. $\hat{F}$ and $\hat{B}$ instead of E$\hat{F}$D and E$\hat{B}$A . Some candidates incorrectly gave the reason as 'alternate angles' or 'co-interior angles'. Other candidates correctly gave the reason as 'corresponding angles'. However, they did not state 'the lines parallel' and were not awarded a mark as the reason was incomplete.

QUESTION 9 (cont.)

9.3 Calculate, with reasons, the length of FC. (3) **[9]**

20%

MEMOS

9.3 $\frac{DF}{AB} = \frac{EF}{EB}$ . . . *similar* Δ^s

$\therefore \frac{DF}{14} = \frac{9}{21}$

$\therefore DF = \frac{{}^{3}\cancel{9} \times \cancel{14}^{2}}{\cancel{21}_{\cancel{7}}}$

$= 6$ units

But DC = AB = 14 units . . . *opp sides of* $||^m$

$\therefore$ **FC = 14 – 6 = 8 units** ◄

Common Errors and Misconceptions

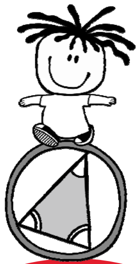
(c) Many candidates **incorrectly** used the *midpoint theorem* to answer **Q9.3**. They should have used the fact that the corresponding sides are in proportion when two triangles are similar.

A few candidates **incorrectly applied the *Theorem of Pythagoras*** even though there was **no right-angled triangle**.

They were not aware of the minimum conditions in which the *Theorem of Pythagoras* could be used.

QUESTION 9: Suggestions for Improvement

(a) Teachers should focus on developing learners' **skills to analyse the question and the diagram for clues** on which **theorems** are required to answer the questions correctly.



(b) Clearly explain to learners the difference between the *midpoint theorem*, the *proportionality theorem* and *similarity* so that they will know which of these concepts can be used in a specific situation.

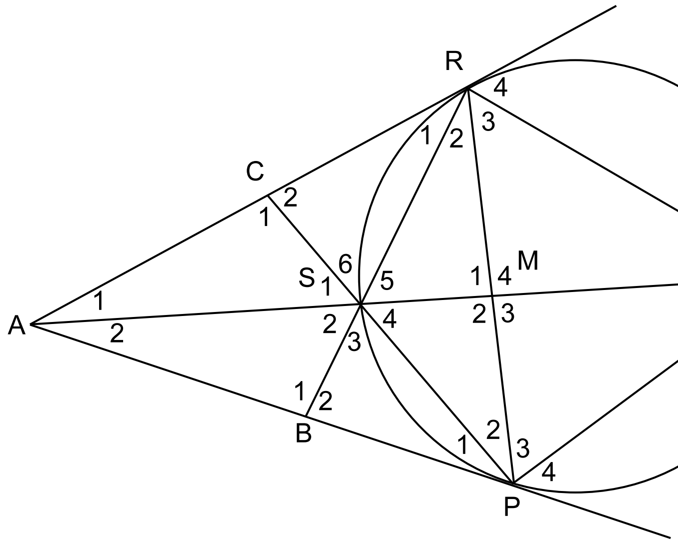
(c) When answering Euclidean Geometry, learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.

(d) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

(e) Teachers should take some time to discuss the naming of angles, for example, the acceptable methods are $\hat{T}$ or $\hat{T}_1$ or $O\hat{T}S$. Teachers should also clarify when it is acceptable to refer to an angle as $\hat{T}$ and when to refer to it as $\hat{T}_1$.

QUESTION 10 18%

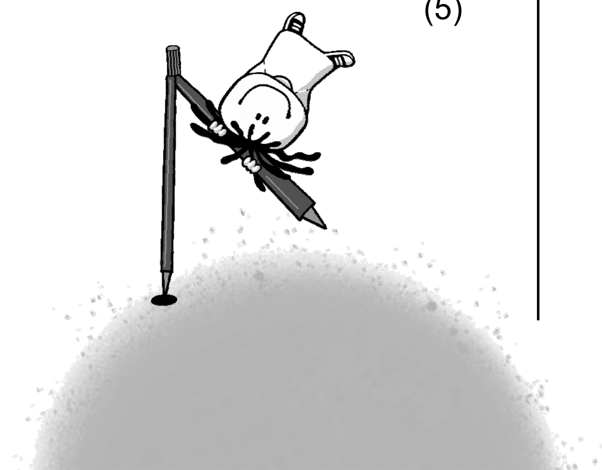
In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

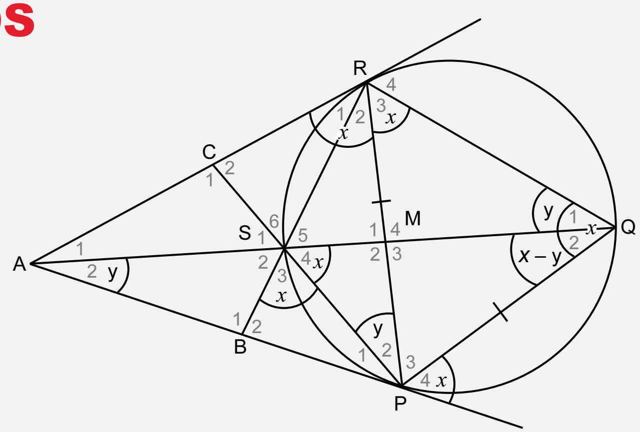
10.1 $\hat{S}_3 = \hat{S}_4$ (5)

29%



MEMOS

10.



10.1 Let $\hat{S}_3 = x$

$\therefore R\hat{Q}P = x$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{R}_3 = x$ . . . *∠^s opp = sides*

$\therefore \hat{S}_4 = x$ . . . *∠^s in the same seg*

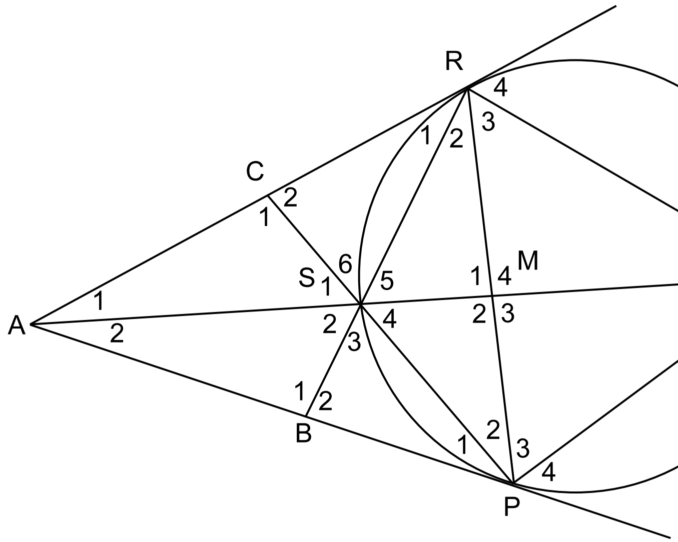
$\therefore \mathbf{\hat{S}_3 = \hat{S}_4}$ ➤

Common Errors and Misconceptions

(a) A fair number of candidates made **incorrect assumptions** when answering **Q10.1**. Among them were that: an exterior angle of the cyclic quadrilateral ($\hat{S}_3$) = the interior opposite angle ($\hat{R}_2$), PQ = RQ and therefore APQR is a kite, RQ || AP and $\hat{M}_1$ = 90º.

QUESTION 10 (cont.)

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



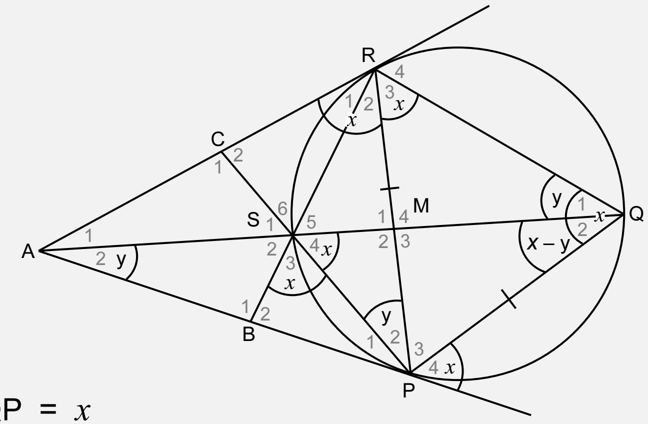
Prove, giving reasons, that:

10.2 SMRC is a cyclic quadrilateral. (4)

16%

MEMOS

10.



10.2 $R\hat{Q}P = x$

$\therefore\ A\hat{R}P = x$ . . . *tan chord theorem*

But $\hat{S}_4 = x$

$\therefore\ \hat{S}_4 = A\hat{R}P$

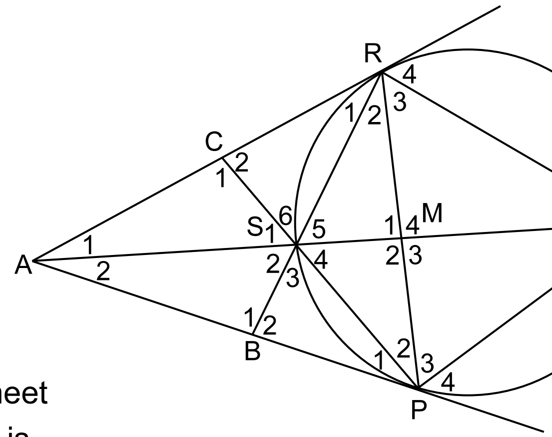
$\therefore$ **SMRC is a cyclic quad** . . . *converse ext ∠ of c.q.*

Common Errors and Misconceptions

(b) Candidates who could not answer **Q10.1** correctly could not understand how to start to answer **Q10.2**. Some candidates used the properties of a cyclic quadrilateral in their attempt to prove that the same quadrilateral is cyclic. Some candidates **did not know the difference** between ***a theorem*** and ***its converse***. They omitted the word 'converse' in the reason: 'exterior angle of cyclic quad'.

QUESTION 10 (cont.)

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.3 RP is a tangent to the circle passing through P, S and A at P. (6)

10%

[15]

MEMOS

10.3 $\hat{P}_4 = x$ ... *tan chord theorem*

Let $\hat{P}_2 = y$

$\therefore \hat{Q}_1 = y$ ... *$\angle^s$ in the same seg*

$\therefore \hat{Q}_2 = x - y$

$\therefore \hat{A}_2 = y$ ... *ext $\angle$ of ΔQAP*

$\therefore \hat{P}_2 = \hat{A}_2$

$\therefore$ RS is a tangent to the circle through P, S and A ✓

... *conv tan chord thm*

OR:

$\hat{P}_4 = x$ ... *tan chord thm*

$\therefore \hat{P}_4 = R\hat{Q}P$

$\therefore$ RQ || AP ... *alt $\angle^s$ =*

Let $\hat{A}_2 = y$

$\therefore \hat{Q}_1 = y$ ... *alt $\angle^s$; RQ || AP*

$\therefore \hat{P}_2 = y$ ... *$\angle^s$ in the same seg*

$\therefore \hat{P}_2 = \hat{A}_2$

$\therefore$ RS is a tangent to the circle through P, S and A ✓

... *converse tan chord thm*

Common Errors and Misconceptions

(c) Very few candidates obtained full marks for **Q10.3**. The main reason for this was that candidates were unable to answer **Q10.1** and **Q10.2** correctly. Poor naming of angles in the answers often led to candidates themselves getting confused about which angle they were referring to.

(d) **Q10.3** required candidates to obtain a proportion from the similar triangles in **Q10.2**, using the *proportional intercept theorem* in ΔRAC to establish a second proportion and then to combine the two. Many candidates failed to establish one or the other proportion and therefore could not make the conclusion.

QUESTION 10: Suggestions for Improvement

(a) **More time needs to be spent on** the teaching of **Euclidean Geometry** in **all grades**. More practice on Grade 11 and 12 Euclidean Geometry will help learners to understand theorems and diagram analysis. They should **read** the given information carefully **without making any assumptions**. The work covered in class must include different activities and all levels of the taxonomy.

(b) Teach learners **not to assume any facts** in a geometry sketch but to **only use what was given and** that which was **proven already** in earlier questions.

(c) Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination.

(d) Consider teaching the approach of **'angle chasing'** where you label one angle as x and then relate other angles to x. In this way, learners should find it easy to identify angles that are equal but moreover, they should find it easier to establish the reasons for the relationships between the angles.