2023
FUNCTIONS & GRAPHS

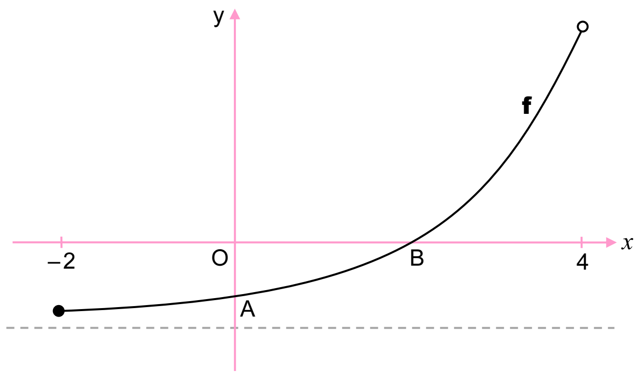
Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES
Your Key to Exam Success

QUESTION 4 55%

Sketched below is the graph of $f(x) = 2^x - 4$ for $x \in [-2; 4)$.

A and B are respectively the y- and x-intercepts of f.



4.1. Write down the equation of the asymptote of f. (1)

70%

4.2 Determine the coordinates of B. (2)

88%

4.3 Determine the equation of k, a straight line passing through A and B in the form $k(x) = \ldots$ (3)

57%

Memo

4.1 $y = -4$ ✓

4.2 At B, $y = 0$

$\therefore 2^x - 4 = 0$

$\therefore 2^x = 4$

$\therefore x = 2$

$\therefore$ **B(2; 0)** ✓

4.3 y-intercept of f:

$f(0) = 2^0 - 4 = 1 - 4 = -3$

$\therefore$ A(0; –3)

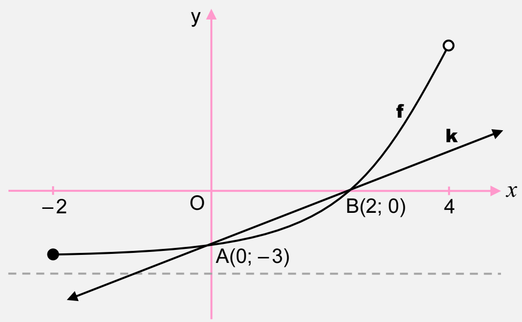
$\therefore$ y-intercept of k, c = –3

Gradient, $m = \frac{3}{2}$ . . . *by inspection*

(or: $m_{AB} = \frac{0-(-3)}{2-0} = \frac{3}{2}$)

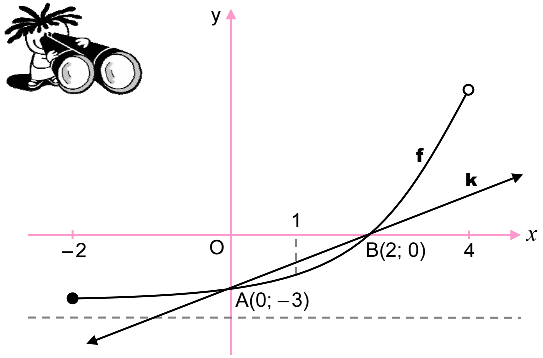
$\therefore$ Eqn of k: $y = \frac{3}{2}x - 3$

i.e. $\mathbf{k(x) = \frac{3}{2}x - 3}$ ✓



Common Errors and Misconceptions

(a) Many candidates did not calculate the coordinates of the x-intercept, A, correctly in **Q4.3** which resulted in an incorrect equation of the *straight line*.



4.4 Calculate the vertical distance between k and f at $x = 1$ (3)

47%

4.5 Write down the equation of g if it is given that $g(x) = f(x) + 4$ (1)

74%

4.6 Write down the domain of g^{-1}. (2)

2%

4.7 Write down the equation of g^{-1} in the form y = . . . (2)

63%

[14]

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Memo

4.4 The vertical distance

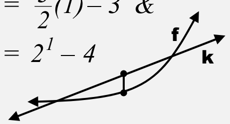
$= k(x) - f(x)$ . . .

$= k(1) - f(1)$

$= -\frac{3}{2} - (-2)$ . . . $k(1) = \frac{3}{2}(1) - 3$ & $f(1) = 2^1 - 4$

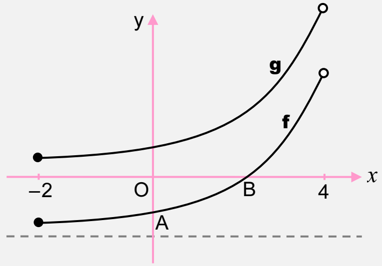
$= \frac{1}{2}$ **unit** ✓

Vertical Distance
The difference between the y-coordinates of **k** and **f** where $x = 1$



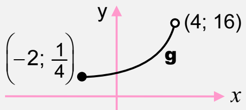
4.5 $g(x) = 2^x - 4 + 4$

$\therefore$ $\mathbf{g(x) = 2^x}$ ✓ for $x \in [-2; 4)$



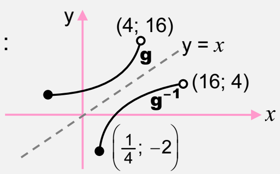
4.6 The domain of g^{-1} = the range of g

range of g: $\frac{1}{4} \leq y < 16$



$\therefore$ domain of g^{-1}: $\mathbf{\frac{1}{4} \leq x < 16}$ ✓

OR:
See the graph of g^{-1}:



4.7 Eqn of g: $y = 2^x$

$\therefore$ Eqn of g^{-1}: $x = 2^y$

$\therefore$ $\mathbf{y = \log_2 x}$ **for** $\mathbf{x \in \left[\frac{1}{4}; 16\right)}$ ✓

Common Errors and Misconceptions

(b) Most candidates did not understand the concept of ***vertical distance*** in **Q4.4**. The candidates substituted incorrect points into the distance formula in an attempt to calculate any distance.

(c) In **Q4.4** some candidates left their answer for the vertical distance between the functions at $x = 1$ as a negative value.

(d) The majority of candidates did not realise that the given function had a restricted domain which in turn meant **the inverse** of the given function would have a restricted domain. The candidates had learnt that the domain of the inverse was $x > 0$, which is true for an unrestricted *logarithm function*.

QUESTION 4: Suggestions for Improvement

(a) Teachers need to emphasise what the **points on a graph** are and help learners **identify what the properties of these points are** before they start answering a question.

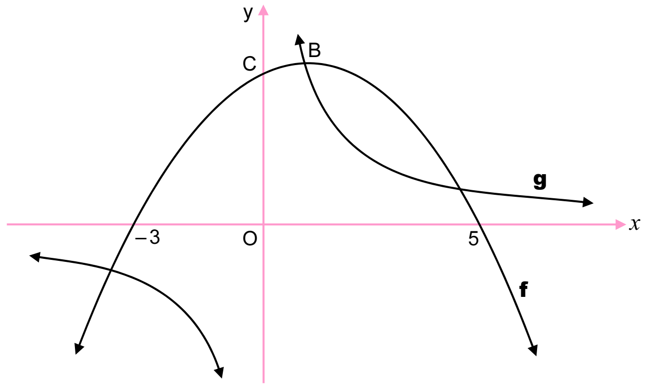
(b) ***Vertical*** *and* ***horizontal distances*** should be explained without the use of the distance formula when interpreting functions. In conjunction with this, teachers must emphasise that distance is a scalar property and cannot be negative.

(c) Teachers should work **with restricted domain** graphs in class to make learners aware that functions can be restricted and **emphasise the effect** this has on their properties.

QUESTION 5 32%

The graphs of $f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$ are drawn below. A point of intersection of f and g is B, the turning point of f.

The graph f has x-intercepts at $(-3; 0)$ and $(5; 0)$ and a y-intercept at C.



5.1 Write down the coordinates of the turning point of f. (2)

80%

5.2 Calculate the coordinates of C. (2)

77%

5.3 Calculate the value of d. (1)

65%

5.4 Write down the range of g. (1)

36%

5.5 For which values of x will $f(x).g(x) \leq 0$? (3)

19%

Memo

5.1 Turning point of f: **B(1; 8)** ✓

5.2 At C, $x = 0$

& $f(0) = -\frac{1}{2}(0-1)^2 + 8 = 7\frac{1}{2}$

$\therefore$ **C$\left(0; 7\frac{1}{2}\right)$** ✓

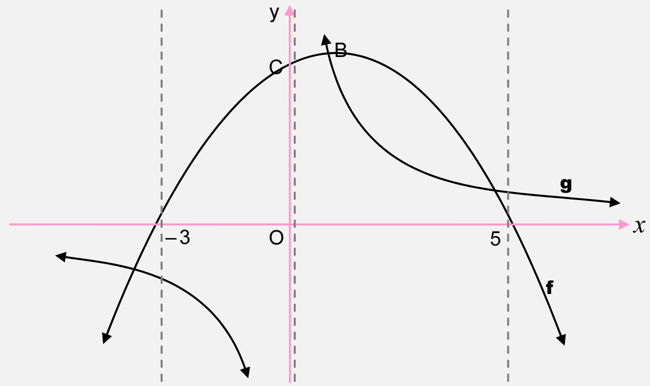
5.3 B(1; 8) a point on g

➡ $g(1) = \frac{d}{1} = 8$ . . . $g(x) = \frac{d}{x}$

$\therefore$ **d = 8** ✓

5.4 **$y \in \mathbb{R}$; $y \neq 0$** ✓

5.5 **$-3 \leq x < 0$ or $x \geq 5$** ✓



$f(x)$:	$f(x) < 0$	$f(x) > 0$	$f(x) > 0$	$f(x) < 0$
$g(x)$:	$g(x) < 0$	$g(x) < 0$	$g(x) > 0$	$g(x) > 0$
$f(x).g(x)$:	+ve	–ve	+ve	–ve

Common Errors and Misconceptions

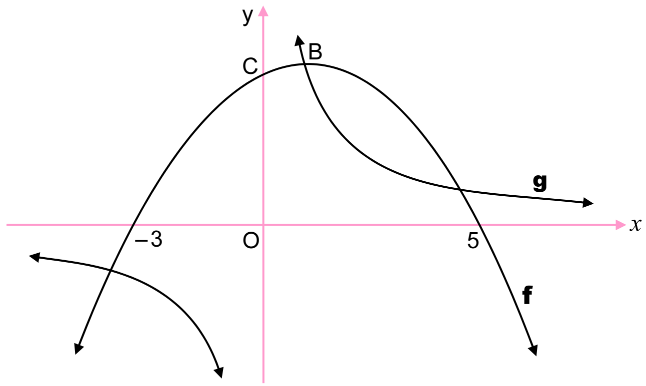
(a) In **Q5.4** many candidates forgot **the restriction of the horizontal asymptote** on the range of the function, possibly because it was the x-axis and visually drawn on the graph as an asymptote.

(b) Most candidates were unable to interpret **Q5.5** correctly from the given graph. Many tried to solve the inequality algebraically and others did not take cognisance of the vertical asymptote of the hyperbola.

The equations of the graphs are:

$f(x) = -\frac{1}{2}(x-1)^2 + 8$ and $g(x) = \frac{d}{x}$

& $g(x) = \frac{8}{x}$ . . . *d = 8 in Q5.3*



5.6 Calculate the values of k so that $h(x) = -2x + k$ will not intersect the graph of g. (5)

14%

5.7 h is a tangent to g at R, a point in the first quadrant. Calculate t such that $y = f(x) + t$ intersects g at R. (4)

8%

[18]

Memo

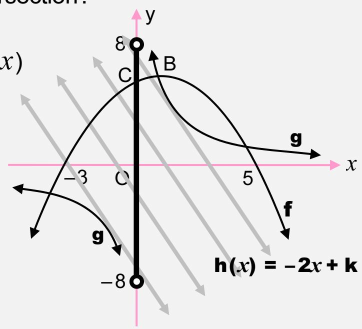
5.6 At any point(s) of intersection:

$h(x) = g(x)$

$\therefore\ -2x + k = \frac{8}{x}$

$(\times x)\quad \therefore\ -2x^2 + kx = 8$

$\times(-1)\quad \therefore\ 2x^2 - kx + 8 = 0$



$\Delta = (-k)^2 - 4(2)(8) \qquad \Delta = b^2 - 4ac$

$= k^2 - 64$

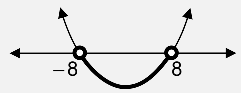
There will be no points of intersection if $\Delta < 0$

For non-real roots:

$k^2 - 64 < 0$

$\therefore\ (k + 8)(k - 8) < 0$

$\therefore\ \mathbf{-8 < k < 8}$ ✓



> We draw the parabola to solve the inequality, to establish the values of k.
>
> But, these values are for k, the y-intercept(s) of h, in the cases where the line (h) does not intersect the hyperbola (g).

5.7 $g'(x) = -\frac{8}{x^2}$ . . . *the* ***gradient*** *of the tangent to g*

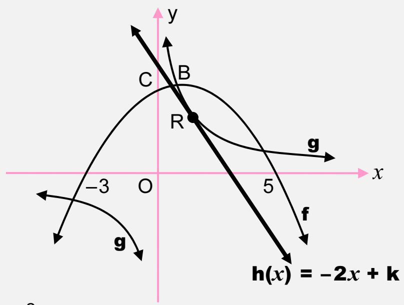
$\therefore\ -\frac{8}{x^2} = -2$

$\therefore\ x^2 = 4$

$\therefore\ x = \pm 2$

$\therefore$ R(2; 4)

$y = f(x) + t$

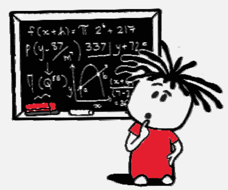


$\therefore\ y = -\frac{1}{2}(x-1)^2 + 8 + t$

$\therefore\ 4 = -\frac{1}{2}(2-1)^2 + 8 + t$ . . . *substitute R(2; 4)*

$\therefore\ 4 = -\frac{1}{2} + 8 + t$

$\therefore\ \mathbf{t = -3\frac{1}{2}}$ ✓



Common Errors and Misconceptions

(c) The majority of candidates **did not understand the terminology** *'will not intersect'* and tried to solve for $-2x + k > \frac{8}{x}$ or $-2x + k \neq \frac{8}{k}$ which led them to **incorrect solutions** and **misinterpretation** of the question. The integration of ***nature of roots*** into functions was not commonly understood by candidates.

(d) **Q5.7** was not understood by many candidates. The **link** between **Q5.6** and **Q5.7** was **not seen** by the candidates which resulted in very few candidates attempting to answer this question.

QUESTION 5: Suggestions for Improvement

(a) Teachers should spend some time on **graphical interpretation** of functions. This can be started with the very first graph that is sketched in Grade 10. The concept of $f(x) > 0$, $f(x) > g(x)$ and $f(x).g(x) > 0$ must be emphasised through the FET phase when teaching functions.

(b) The **link** for learners between the **algebraic work** (i.e. *nature of roots, simultaneous equations* and *inequalities*) and the **graphical representation** must be created by the teacher when working with functions. However, teachers need to emphasise the importance of **understanding these concepts** and teach the learners to **read off the solutions** to the questions **from the graph**. Not all solutions in functions questions need to be algebraic – this practice seems to be the default for most candidates.

(c) The **integration of *calculus*** into *functions* is an important concept for teachers to revise, test and practise with learners.