Further Studies Mathematics

BOOK 1

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& Ingrid Zlobinsky-Roux

GRADE

10-12

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SOLUTIONS

THE ANSWER SERIES *Your Key to Exam Success*

GRADE

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Further Studies Mathematics

Book 1: Standard Level

Calculus & Algebra

**Marilyn Buchanan, Anne Eadie, Carl Fourie,
Noleen Jakins & Ingrid Zlobinsky-Roux**

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Solutions to Book 1: Compulsory Modules

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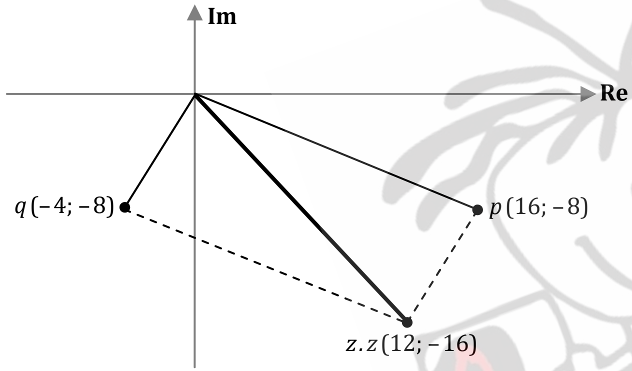
(j) $z^2 = z.z$
$= (-4 + 2i)(-4 + 2i)$
$= -4(-4 + 2i) + 2i(-4 + 2i)$
$= (16 - 8i) + (-8i + 4i^2)$
$= (16 - 8i) + (-4 - 8i)$
$= \mathbf{12 - 16}\boldsymbol{i}$

The product of two complex numbers is rewritten as the sum of two different numbers. The result is the diagonal from the origin of the rectangle. The product of a complex conjugate pair is always real.



Gr 10 Complex Numbers Exam Solutions

(Questions – p. 40 in Book)

1. $x^3 - 1 = (x - 1)(x^2 + x + 1) = 0$

$x = 1$ **or** $x = \dfrac{-1 \pm \sqrt{1^2 - 4.1.1}}{2.1} = \dfrac{\mathbf{-1 \pm \sqrt{3}}\boldsymbol{i}}{\mathbf{2}}$

2. $(a + 3i)bi = (-11 - 13i)(2 - 5i)$
$\therefore\ abi + 3bi^2 = -22 + 55i - 26i + 65i^2$
$\therefore\ -3b + abi = -87 + 29i$
$\therefore\ -3b = -87$ and $ab = 29$
$\therefore\ \boldsymbol{a} \mathbf{= 1,}\ \boldsymbol{b} \mathbf{= 29}$

3. $2z - iw = 2(5 - 2i) - i(6i - 1)$
$= 10 - 4i - 6i^2 + i$
$= \mathbf{16 - 3}\boldsymbol{i}$

4. $\dfrac{a + bi}{a - bi} \times \dfrac{a + bi}{a + bi} = \dfrac{a^2 + 2abi + b^2i^2}{a^2 - b^2i^2} = \dfrac{a^2 - b^2 + 2abi}{a^2 + b^2}$

real $= \dfrac{\boldsymbol{a^2 - b^2}}{\boldsymbol{a^2 + b^2}}$

5. Let $\alpha = q + \sqrt{3}i$, and its conjugate $\beta = q - \sqrt{3}i$

$\alpha + \beta = 2q = -\dfrac{b}{a} = -(-2)$ $\quad\therefore\ \boldsymbol{q} \mathbf{= 1}$

$\alpha\beta = q^2 - 3i^2 = q^2 + 3 = \dfrac{c}{a} = p$ $\quad\therefore\ \boldsymbol{p} \mathbf{= 4}$

6. (a) $b^2 - 4ac < 0$ for non-real roots
$\therefore\ p^2 - 4p(1) < 0$
$\therefore\ p(p - 4) < 0$
$\therefore\ \mathbf{0 <} \boldsymbol{p} \mathbf{< 4}$

(b) $i + i^2 + i^3 + \dots + i^{2017}$
$= (i - 1 - i + 1) + (i - 1 - i + 1) + \dots i = \boldsymbol{i}$

7. $\dfrac{a + bi}{-b + ai} \times \dfrac{-b - ai}{-b - ai} = \dfrac{-ab - b^2i - a^2i - abi^2}{b^2 - a^2i^2}$
$= \dfrac{-ab - i(b^2 + a^2) + ab}{b^2 + a^2}$
$= \boldsymbol{-i}$

8. (a) $m - 2n^* = (4 + 2i) - 2(-2 + i) = 4 + 2i + 4 - 2i$
$= \mathbf{8}$

(b) $\dfrac{m}{n} = \dfrac{4 + 2i}{-2 - i} \times \dfrac{-2 + i}{-2 + i} = \dfrac{-8 + 2i^2}{4 - i^2} = -\dfrac{10}{5}$
$= \mathbf{-2}$

9. (a) $x^2 + 8x + 16 + 9 = (x + 4)^2 - 9i^2$
$= \boldsymbol{(x + 4 - 3i)(x + 4 + 3i)}$

(b) Let $\alpha = 2 + 3i$, and its conjugate $\beta = 2 - 3i$

$\alpha + \beta = 4 = -\dfrac{b}{a}$ and $\alpha\beta = 13 = \dfrac{c}{a}$

If $ax^2 + bx + c = 0$ then $a\left(x^2 + \dfrac{b}{a}x + \dfrac{c}{a}\right) = 0$

$\boldsymbol{a(x^2 - 4x + 13) = 0}$ will produce the desired roots.

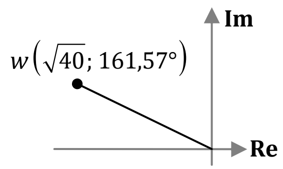
10. (a) $b^2 - 4ac = (-4)^2 - 4(1)(-8)$
$= \mathbf{48}$

(b) **two real, irrational roots**

(c) **12** by trial and error **or**
$x^2 - 4x - 8 + t = 0$
$b^2 - 4ac = (-4)^2 - 4(-8 + t) = 0$ for one real root
Hence, $\boldsymbol{t} \mathbf{= 12.}$

11. $r^2 = 36 + 4 = 40$
$\therefore\ r = \sqrt{40} \approx 6{,}32$

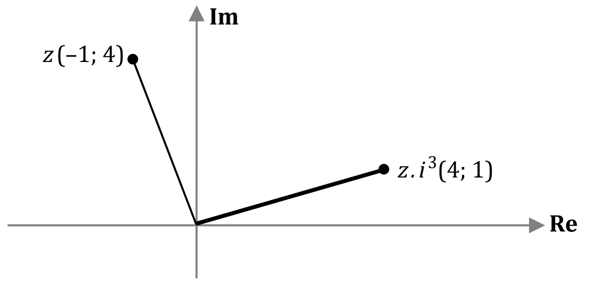
$\tan\theta = -\dfrac{2}{6}$



$(-6; 2)$ lies in second quadrant
$\theta = 180° - 18{,}43° = 161{,}57°$
$\boldsymbol{w}\left(\sqrt{\mathbf{40}}; \mathbf{161{,}57°}\right)$

12. (a) $z.i^3 = z.(-i)$
$= -z.i$
$= -(-1 + 4i)i$
$= -(-1i + 4i^2)$
$= i - 4i^2$
$= \mathbf{4 +} \boldsymbol{i}$

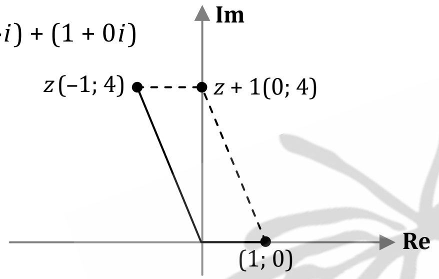
Multiplying by i rotates z by 90° anticlockwise about the origin. Multiplying by (−1) rotates the result by 180° about the origin. Hence the overall result is z rotated by 270° anti-clockwise about the origin.



(b) $z + 1$

$= (-1 + 4i) + (1 + 0i)$

$= \mathbf{4i}$

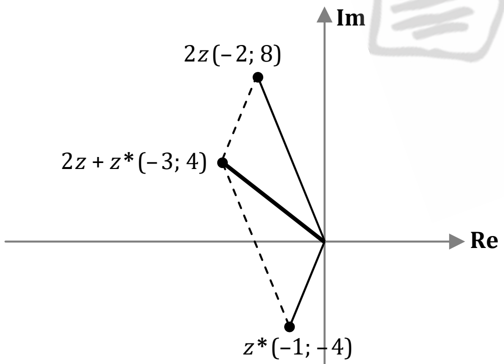


The sum of two complex numbers is the diagonal from the origin of the parallelogram.

(c) $2z + z^* = (-2 + 8i) + (-1 - 4i)$

$= \mathbf{-3 + 4i}$

The sum of two complex numbers is the diagonal from the origin of the parallelogram.



(d) $z.z^* = (-1 + 4i)(-1 - 4i)$

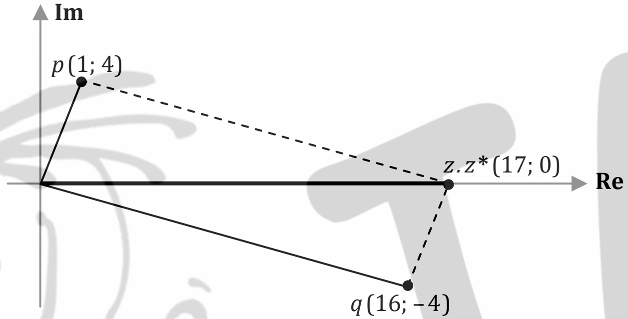
$= -1(-1 - 4i) + 4i(-1 - 4i)$

$= (1 + 4i) + (-4i - 16i^2)$

$= (1 + 4i) + (16 - 4i)$

$= \mathbf{17}$

The product of two complex numbers is rewritten as the sum of two different numbers. The result is the diagonal from the origin of the rectangle. The product of a complex conjugate pair is always real.



13. $3p + 3qi + pi + qi^2 = -4 + 2i$

$\therefore\ (3p - q) + (3q + p)i = -4 + 2i$

$\therefore\ 3p - q = -4$ and $p + 3q = 2$

$\therefore\ \boldsymbol{q = 1}$ and $\therefore\ \boldsymbol{p = -1}$

solve simultaneously

14. For $x^2 + kx + t = 0$, $-\frac{b}{a} = -\frac{k}{1}$ and $\frac{c}{a} = \frac{t}{1}$.

$(a + bi)(a - bi) = a^2 - b^2i^2 = a^2 + b^2 = \frac{t}{1} = t$

$(a + bi) + (a - bi) = 2a = -\frac{k}{1}$ so $\mathbf{2a + k = 0}$.

Exercise 3.4

(Questions – p. 43 in Book)

1. (a) $2x^2 - \frac{13x}{3} + 2 = \frac{6x^2 - 13x + 6}{3}$

$= \frac{(3x - 2)(2x - 3)}{3} = 0$

$\therefore\ \boldsymbol{x = \frac{2}{3}}$ **or** $\boldsymbol{\frac{3}{2}}$

(b) $\frac{2x^2}{3} + \frac{x}{3} - 1 = \frac{2x^2 + x - 3}{3} = \frac{(2x + 3)(x - 1)}{3} = 0$

$\therefore\ \boldsymbol{x = -\frac{3}{2}}$ **or** $\mathbf{1}$

(c) $4x^4 + 3x^2 - 1 = (4x^2 - 1)(x^2 + 1) = 0$

$\therefore\ \boldsymbol{x = \pm\frac{1}{2}}$ or $\boldsymbol{x = \pm i}$

(d) $x^4 - x^2 - 12 = (x^2 - 4)(x^2 + 3) = 0$

$\boldsymbol{x = \pm 2}$ **or** $\boldsymbol{x = \pm\sqrt{3}i}$

(e) $x^4 + 3x^2 - 28 = (x^2 - 4)(x^2 + 7) = 0$

$\boldsymbol{x = \pm 2}$ **or** $\boldsymbol{x = \pm\sqrt{7}i}$

(f) $3x^4 + 25x^2 - 18 = (3x^2 - 2)(x^2 + 9) = 0$

$\boldsymbol{x = \pm\sqrt{\frac{2}{3}}}$ **or** $\boldsymbol{x = \pm 3i}$

2. (a) $x^3 + x^2 - x - 1 = x^2(x + 1) - (x + 1)$

$= (x + 1)(x^2 - 1) = 0$

$\therefore\ \boldsymbol{x = \pm 1}$

(b) $x^3 + 4x^2 + 9x + 36 = x^2(x + 4) + 9(x + 4)$

$= (x + 4)(x^2 + 9) = 0$

$\therefore\ \boldsymbol{x = -4},\ \boldsymbol{x = \pm 3i}$

(c) $x^3 + 2x^2 + 6x + 12 = x^2(x + 2) + 6(x + 2)$

$= (x + 2)(x^2 + 6) = 0$

$\therefore\ \boldsymbol{x = -2},\ \boldsymbol{x = \pm\sqrt{6}i}$

7. $\dfrac{9x-24}{x^2-6x+8}$

$= \dfrac{9x-24}{(x-4)(x-2)} \equiv \dfrac{A}{x-4} + \dfrac{B}{x-2}$

$A = \dfrac{12}{2} = 6 \qquad B = \dfrac{-6}{-2} = 3$

$\dfrac{9x-24}{x^2-6x+8} = \mathbf{\dfrac{6}{x-4} + \dfrac{3}{x-2}}$

8. $\dfrac{1}{x^2-2x-8}$

$= \dfrac{1}{(x-4)(x+2)} \equiv \dfrac{A}{x-4} + \dfrac{B}{x+2}$

$A = \dfrac{1}{6} \qquad B = \dfrac{1}{-6}$

$\dfrac{1}{x^2-2x-8} = \mathbf{\dfrac{1}{6(x-4)} - \dfrac{1}{6(x+2)}}$

9. $\dfrac{23x-1}{6x^2+x-1}$

$= \dfrac{23x-1}{(3x-1)(2x+1)} \equiv \dfrac{A}{3x-1} + \dfrac{B}{2x+1}$

$A = \dfrac{\frac{23}{3}-1}{\frac{2}{3}+1} = 4 \qquad B = \dfrac{\frac{-23}{2}-1}{\frac{-3}{2}-1} = 5$

$\dfrac{23x-1}{6x^2+x-1} = \mathbf{\dfrac{4}{3x-1} + \dfrac{5}{2x+1}}$

10. $\dfrac{6x^2-22x+18}{(x-1)(x-2)(x-3)} \equiv \dfrac{A}{x-1} + \dfrac{B}{x-2} + \dfrac{C}{x-3}$

$A = \dfrac{2}{2} = 1 \qquad B = \dfrac{-2}{-1} = 2 \qquad C = \dfrac{6}{2} = 3$

$\dfrac{6x^2-22x+18}{(x-1)(x-2)(x-3)} = \mathbf{\dfrac{1}{x-1} + \dfrac{2}{x-2} + \dfrac{3}{x-3}}$

Exercise 5.3

(Questions – p. 57 in Book)

1. (a) $\dfrac{3x+4}{(x+3)^2} = \dfrac{A}{(x+3)} + \dfrac{B}{(x+3)^2}$

$\therefore\ 3x+4 \equiv A(x+3) + B$

Let $x = -3$: $\therefore\ -5 = B \qquad \Rightarrow B = -5$

Let $x = 0$: $\therefore\ 4 = 3A - 5$

$\therefore\ 9 = 3A \qquad \Rightarrow A = 3$

$\therefore\ \dfrac{3x+4}{(x+3)^2} = \mathbf{\dfrac{3}{(x+3)} - \dfrac{5}{(x+3)^2}}$

(b) $\dfrac{x^2-7x+12}{x(x-2)^2} = \dfrac{A}{x} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$

$\therefore\ x^2-7x+12 \equiv A(x-2)^2 + Bx(x-2) + Cx$

Let $x = 0$: $\therefore\ 12 = 4A \qquad \Rightarrow A = 3$

Let $x = 2$: $\therefore\ 2 = 2C \qquad \Rightarrow C = 1$

Let $x = 1$: $\therefore\ 6 = 3 + B(-1) + 1 \qquad \Rightarrow B = -2$

$\therefore\ \dfrac{x^2-7x+12}{x(x-2)^2} = \mathbf{\dfrac{3}{x} - \dfrac{2}{(x-2)} + \dfrac{1}{(x-2)^2}}$

(c) $\dfrac{8x-12}{(x+3)(x^2-6x+9)} = \dfrac{8x-12}{(x+3)(x-3)^2}$

$= \dfrac{A}{(x+3)} + \dfrac{B}{(x-3)} + \dfrac{C}{(x-3)^2}$

$\therefore\ 8x-12 \equiv A(x-3)^2 + B(x+3)(x-3) + C(x+3)$

Let $x = -3$: $\therefore\ -36 = 36A \qquad \Rightarrow A = -1$

Let $x = 3$: $\therefore\ 12 = 6C \qquad \Rightarrow C = 2$

Let $x = 0$: $\therefore\ -12 = -9 - 9B + 6 \qquad \Rightarrow B = 1$

$\therefore\ \dfrac{8x-12}{(x+3)(x^2-6x+9)} = \mathbf{\dfrac{-1}{(x+3)} + \dfrac{1}{(x-3)} + \dfrac{2}{(x-3)^2}}$

(d) $\dfrac{2x^2-9x+16}{(x+1)(x-2)^2} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-2)} + \dfrac{C}{(x-2)^2}$

$\therefore\ 2x^2-9x+16 \equiv A(x-2)^2 + B(x+1)(x-2) + C(x+1)$

Let $x = -1$: $\therefore\ 27 = 9A \qquad \Rightarrow A = 3$

Let $x = 2$: $\therefore\ 6 = 3C \qquad \Rightarrow C = 2$

Let $x = 0$: $\therefore\ 16 = 12 - 2B + 2 \qquad \Rightarrow B = -1$

$\therefore\ \dfrac{2x^2-9x+16}{(x+1)(x-2)^2} = \mathbf{\dfrac{3}{(x+1)} - \dfrac{1}{(x-2)} + \dfrac{2}{(x-2)^2}}$

(e) $\dfrac{2x^2+1}{x^2(x^2-2x+1)} = \dfrac{2x^2+1}{x^2.(x-1)^2}$

$= \dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{(x-1)} + \dfrac{D}{(x-1)^2}$

$\therefore\ 2x^2+1 \equiv Ax(x-1)^2 + B(x-1)^2 + Cx^2(x-1) + Dx^2$

Let $x = 1$: $\therefore\ 3 = D$

Let $x = 0$: $\therefore\ 1 = B$

Let $x = -1$: $\therefore\ 3 = -4A + 4 - 2C + 3$

$\therefore\ 4A + 2C = 4 \qquad$ ... ❶

Let $x = 2$: $\therefore\ 9 = 2A + 1 + 4C + 12$

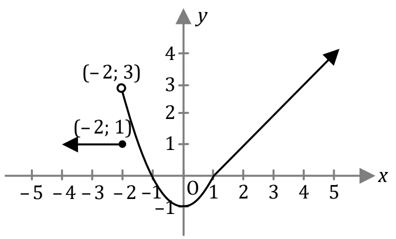
$\therefore\ 2A + 4C = -4 \qquad$ ... ❷

Solve equations ❶ and ❷ simultaneously, using the calculator:

$\therefore\ A = 2$ and $C = -2$

$\therefore\ \dfrac{2x^2+1}{x^2(x^2-2x+1)} = \mathbf{\dfrac{2}{x} + \dfrac{1}{x^2} - \dfrac{2}{(x-1)} + \dfrac{3}{(x-1)^2}}$

4. (a)



(b) Domain: $\boldsymbol{x \in \mathbb{R}}$ Range: $\boldsymbol{y \geq -1}$

(c) $f(-3) = \mathbf{1}$ $f(-1) = \mathbf{0}$ $f(3) = \mathbf{2}$

(d) (i) **No solution** (ii) $\boldsymbol{x = 4}$

(iii) $\boldsymbol{x \leq -2}$ or $\boldsymbol{x = 2}$

5. $f(x) = x + \dfrac{1}{x}, \ x \neq 0$

$$y = \frac{x^2 + 1}{x}$$

$\therefore\ yx = x^2 + 1$

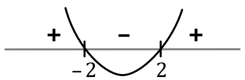
$\therefore\ 0 = x^2 - yx + 1$ quadratic equation

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{+y \pm \sqrt{y^2 - 4}}{2}$$

Restrictions placed on the range such that the domain is real:

$y^2 - 4 \geq 0$

$\therefore\ (y-2)(y+2) \geq 0$



$\therefore\ -\infty < y \leq -2$ or $2 \leq y < \infty$

Exercise 7.3

(Questions – p. 75 in Book)

1. (a) $f(g(5))$
$g(5) = 2$
$f(2) = 4$
$\therefore\ \boldsymbol{f(g(5)) = 4}$

(b) $g(f(5))$
$f(5) = 25$
$g(25) = 22$
$\therefore\ \boldsymbol{g(f(5)) = 22}$

Outer function

(c) $f(g(x))$

Inner function

$= (g(x))^2$
$= \boldsymbol{(x-3)^2}$

Outer function

(d) $g(f(x))$

Inner function

$= (f(x)) - 3$
$= (x^2) - 3$
$= \boldsymbol{x^2 - 3}$

Outer function

(e) $f(f(x))$

Inner function

$= (x^2)^2$
$= \boldsymbol{x^4}$

Outer function

(f) $g(g(x))$

Inner function

$= (x-3) - 3$
$= \boldsymbol{x - 6}$

2. (a) $f(g(9))$
$= f(2)$
$= \mathbf{2}$

(b) $g(f(9))$
$= g(9)$
$= \sqrt{9} - 1 = \mathbf{2}$

(c) $f(g(0))$
$= f(-1)$
$= \mathbf{1}$

(d) $g(f(0))$
$= g(0)$
$= \mathbf{-1}$

(e) $f(g(-4))$
undefined

(f) $g(f(-4))$
$= g(4)$
$= \sqrt{4} - 1$
$= \mathbf{1}$

(g) $f(g(x))$

$$= \begin{cases} (g(x)) & \text{if } (g(x)) \geq 0 \\ -(g(x)) & \text{if } (g(x)) < 0 \end{cases}$$

$$= \begin{cases} \left(\sqrt{x} - 1\right) & \text{if } \sqrt{x} - 1 \geq 0 \\ -\left(\sqrt{x} - 1\right) & \text{if } \sqrt{x} - 1 < 0 \end{cases}$$

$\sqrt{x} \geq 1$
$x \geq 1$

$$= \begin{cases} \boldsymbol{\sqrt{x} - 1} & \textbf{if } \boldsymbol{x \geq 1} \\ \boldsymbol{-\sqrt{x} + 1} & \textbf{if } \boldsymbol{x < 1} \end{cases}$$

(h) $g(f(x))$

$$= \begin{cases} \boldsymbol{\sqrt{(x)} - 1} & \textbf{if } \boldsymbol{x \geq 0} \\ \boldsymbol{\sqrt{-(x)} - 1} & \textbf{if } \boldsymbol{x < 0} \end{cases}$$

3. (a) $f(x) = 3x - 2$ $g(x) = 2 - x$

$f(g(x)) = 3(g(x)) - 2$
$= 3(2-x) - 2$
$= \boldsymbol{4 - 3x}$
Domain: $\boldsymbol{x \in \mathbb{R}}$

$g(f(x)) = 2 - (f(x))$
$= 2 - (3x - 2)$
$= \boldsymbol{4 - 3x}$
Domain: $\boldsymbol{x \in \mathbb{R}}$

(b) $f(x) = x^2$ $g(x) = x + 2$

$f(g(x)) = (g(x))^2$
$= \boldsymbol{(x+2)^2}$
Domain: $\boldsymbol{x \in \mathbb{R}}$

$g(f(x)) = (f(x)) + 2$
$= \boldsymbol{x^2 + 2}$
Domain: $\boldsymbol{x \in \mathbb{R}}$

(c) $f(x) = x^2 - 4$ $g(x) = \sqrt{x}$

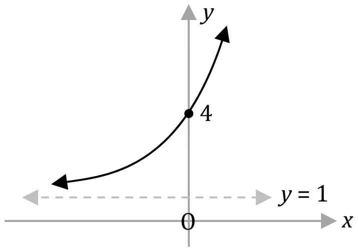
$f(g(x)) = \left(\sqrt{x}\right)^2 - 4$
$= \boldsymbol{x - 4}$

$g(f(x)) = \sqrt{x^2 - 4}$

Domain $g(x)$: $x \geq 0$
Domain $f \circ g$: $\boldsymbol{x \geq 0}$

Domain: $x^2 - 4 \geq 0$
$\therefore\ \boldsymbol{x \leq -2}$ **or** $\boldsymbol{x \geq 2}$

7. $y = |3^{x+1} + 1| = 3^{x+1} + 1$



Note:
$|3^{x+1} + 1|$
$= 3^{x+1} + 1$
for all real values of x

Domain: $\boldsymbol{x \in \mathbb{R}}$

Range: $\boldsymbol{y > 1}$

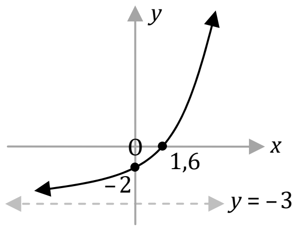
8. $y = |2^{1+x-1} - 3| = |2^x - 3|$

y-intercept ($x = 0$): $y = |1 - 3| = 2$

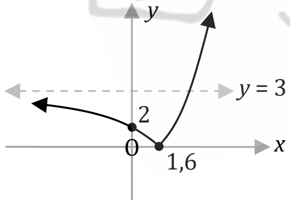
x-intercept ($y = 0$): $2^x = 3$
$\therefore\ x \approx 1{,}6$

SOLVE: Appendix 1

$y = 2.2^{x-1} - 3$:



$y = |2.2^{x-1} - 3|$:



Domain: $\boldsymbol{x \in \mathbb{R}}$

Range: $\boldsymbol{y \geq 0}$

Exercise 8.4

(Questions – p. 88 in Book)

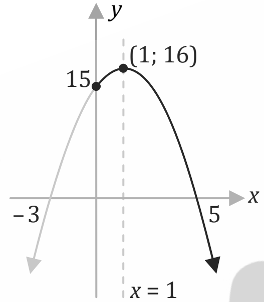
1. (a) $y = -|x|^2 + 2|x| + 15$

For $x \geq 0$:
$y = -x^2 + 2x + 15$
$\therefore\ y = -(x^2 - 2x - 15)$
$\therefore\ y = -(x + 3)(x - 5)$

y-intercept: $y = 15$

x-intercept(s): $x = 5$; $x \neq -3$

Turning point: $(1; 16)$

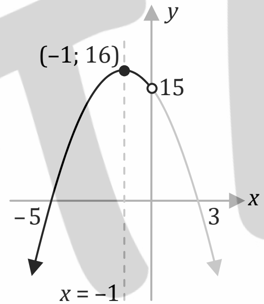


For $x < 0$:
$y = -x^2 - 2x + 15$
$\therefore\ y = -(x^2 + 2x - 15)$
$\therefore\ y = -(x + 5)(x - 3)$

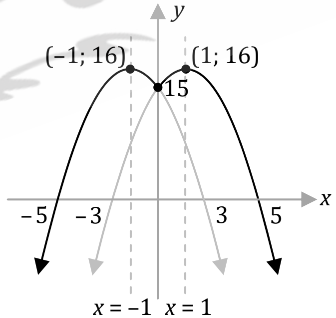
y-intercept: $y = 15$

x-intercept(s): $x = -5$; $x \neq 3$

Turning point: $(-1; 16)$



$\therefore$ The graph of $y = -|x|^2 + 2|x| - 15$ for ALL values of x:



Domain: $\boldsymbol{x \in \mathbb{R}}$

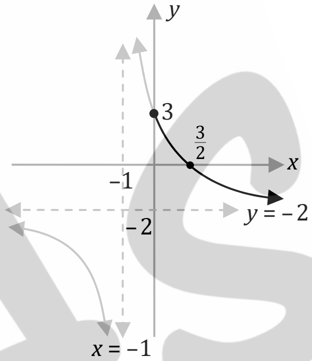
Range: $\boldsymbol{y \leq 16}$

In examples (a) → (d), the graph of $y = f(|x|)$ for $x \geq 0$ can be reflected in the y-axis to obtain the other part (where $x < 0$).

(b) $y = \dfrac{5}{|x| + 1} - 2$

For $x \geq 0$:

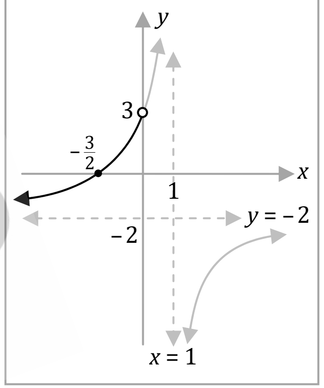
$y = \dfrac{5}{x + 1} - 2$



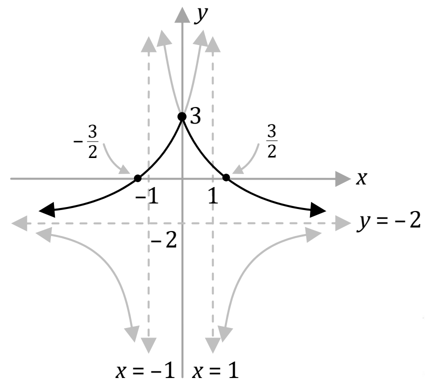
For $x < 0$:

$y = \dfrac{5}{-x + 1} - 2$

$\therefore\ y = \dfrac{-5}{x - 1} - 2$



$\therefore$ The graph of $y = \dfrac{5}{|x| + 1} - 2$ for ALL values of x:



Domain: $\boldsymbol{x \in \mathbb{R}}$

Range: $\boldsymbol{-2 < y \leq 3}$

Note that the final graphs in (a), (b), (c) and (d) are all symmetrical about the y-axis.

(e) $f(x) = 3x - \frac{1}{\sqrt{x}} = 3x - x^{-\frac{1}{2}}$

$f'(x) = 3 + \frac{1}{2} . x^{-\frac{3}{2}} = \mathbf{3 + \frac{1}{2\sqrt{x^3}}}$

(f) $f(x) = x^3 - 6x^2 + 9x - 4$

$f'(x) = \mathbf{3x^2 - 12x + 9}$

(g) $f(x) = \frac{x^3}{3} + x^2 - 5x + 1$

$f'(x) = \mathbf{x^2 + 2x - 5}$

(h) $f(x) = \frac{x^2 - 4x}{x} = x - 4$

$f'(x) = \mathbf{1}$

(i) $f(x) = \frac{3x^2 + x - 1}{x} = 3x + 1 - x^{-1}$

$f'(x) = 3 + x^{-2} = \mathbf{3 + \frac{1}{x^2}}$

2. (a) $y = 3x^3 + 5x^2 - 4x - 3$

$\frac{dy}{dx} = \mathbf{9x^2 + 10x - 4}$

(b) $g(x) = \frac{4x^2 - 1}{2x + 1}$

$= \frac{(2x+1)(2x-1)}{2x+1}$

$= 2x - 1; \ x \neq -\frac{1}{2}$

$g'(x) = \mathbf{2}$

3. (a) $D_x\left[x^2 - \frac{1}{x^3}\right]$

$D_x\,[x^2 - x^{-3}]$

$= 2x + 3x^{-4}$

$= \mathbf{2x + \frac{3}{x^4}}$

(b) $\frac{d}{dx}\left(\frac{1 + x^{\frac{3}{2}}}{x^{\frac{1}{2}}}\right)$

$= \frac{d}{dx}\left(x^{-\frac{1}{2}} + x\right)$

$= -\frac{1}{2}x^{-\frac{3}{2}} + 1$

$= \mathbf{-\frac{1}{2x^{\frac{3}{2}}} + 1}$

(c) $D_t\left[\frac{\sqrt{t} - 3t}{\sqrt{t}}\right]$

$= D_t\left[1 - 3t^{\frac{1}{2}}\right]$

$= -\frac{3}{2}t^{-\frac{1}{2}}$

$= \mathbf{\frac{-3}{2\sqrt{t}}}$

(d) $\frac{d}{ds}\left(\frac{2s - s^2 + 3s^3}{s^2}\right)$

$= \frac{d}{ds}(2s^{-1} - 1 + 3s)$

$= -2s^{-2} + 3$

$= \mathbf{\frac{-2}{s^2} + 3}$

(e) $f(x) = \frac{2x^3 - x^2 - 8x + 4}{x - 2}$

$= \frac{x^2(2x-1) - 4(2x-1)}{x-2}$

$= \frac{(2x-1)(x-2)(x+2)}{x-2}$

$= 2x^2 + 3x - 2, \ x \neq 2$

$\therefore \mathbf{f'(x) = 4x + 3}$

Chapter 11: Grade 11 TRIGONOMETRY

Exercise 11.1

(Questions – p. 113 in Book)

1. (a) $\text{cosec}\,\theta . \tan\theta$

$= \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \mathbf{\sec\theta}$

(b) $\text{cosec}\,\theta . \frac{1}{\cot\theta}$

$= \frac{1}{\sin\theta} \times \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\cos\theta} = \mathbf{\sec\theta}$

(c) $\sec\theta . \frac{1}{\tan\theta} . \frac{1}{\text{cosec}\,\theta}$

$= \frac{1}{\cos\theta} \times \frac{\cos\theta}{\sin\theta} \times \sin\theta = \mathbf{1}$

(d) $\cot\theta . \text{cosec}\,\theta . \tan\theta$

$= \frac{\cos\theta}{\sin\theta} \times \frac{1}{\sin\theta} . \frac{\sin\theta}{\cos\theta}$

$= \frac{1}{\sin\theta} = \mathbf{\text{cosec}\,\theta}$

(e) $\tan\theta . \cos\theta . \cot\theta . \sec\theta$

$= \tan\theta . \cos\theta . \frac{1}{\tan\theta} . \frac{1}{\cos\theta} = \mathbf{1}$

2. (a) R.T.P.: $\tan A . \text{cosec}^2 A . \cos^2 A = \cot A$

LHS $= \frac{\sin A}{\cos A} \times \frac{1}{\sin^2 A} \times \cos^2 A$

$= \frac{\cos A}{\sin A} = \cot A =$ RHS

(b) R.T.P.: $\cot^2 A - \cos^2 A = \cot^2 A . \cos^2 A$

LHS $= \frac{\cos^2 A}{\sin^2 A} - \cos^2 A$

$= \frac{\cos^2 A - \cos^2 A \sin^2 A}{\sin^2 A}$

$= \frac{\cos^2 A(1 - \sin^2 A)}{\sin^2 A}$

$= \frac{\cos^2 A}{\sin^2 A} \times \cos^2 A = \cot^2 A . \cos^2 A =$ RHS

(b) Area segment $= \frac{1}{2}(2)^2 \times \frac{\pi}{3} - \frac{1}{2}(2)^2 \sin\frac{\pi}{3}$

$= \frac{2\pi}{3} - 2 \times \frac{\sqrt{3}}{2}$

$= \frac{2\pi - 3\sqrt{3}}{3}$

Area triangle $= \frac{1}{2}(2)^2 \sin\frac{\pi}{3} = \sqrt{3}$

Area of coin $= 3 \times \left(\frac{2\pi - 3\sqrt{3}}{3}\right) + \sqrt{3}$

$= \mathbf{2\pi - 2\sqrt{3}\ cm^2}$

Exercise 11.4

(Questions – p. 124 in Book)

1. $2\tan\left(x - \frac{\pi}{12}\right) = 1{,}45$

$\therefore \tan\left(x - \frac{\pi}{12}\right) = 0{,}725$

$\therefore x - \frac{\pi}{12} = 0{,}627\ldots + \pi k, \quad k \in \mathbb{Z}$

$\therefore \boldsymbol{x \approx 0{,}889 + \pi k}$

2. (a) $3\sin\left(2x + \frac{\pi}{6}\right) = 1{,}5$

$\therefore \sin\left(2x + \frac{\pi}{6}\right) = \frac{1}{2}$

$\therefore 2x + \frac{\pi}{6} = \frac{\pi}{6} + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore \boldsymbol{x = \pi k}$

or $2x + \frac{\pi}{6} = \pi - \frac{\pi}{6} + 2\pi k$

$\therefore 2x = \frac{2\pi}{3} + 2\pi k$

$\therefore \boldsymbol{x = \frac{\pi}{3} + \pi k}$

(b) $\boldsymbol{x \in \left\{-\pi;\ 0;\ \pi;\ 2\pi;\ -\frac{2\pi}{3};\ \frac{\pi}{3};\ \frac{4\pi}{3}\right\}}$

3. $\cos\left(3x - \frac{\pi}{36}\right) = -\cos\left(x + \frac{\pi}{36}\right)$

$\therefore \cos\left(3x - \frac{\pi}{36}\right) = \cos\left(\pi - \left(x + \frac{\pi}{36}\right)\right)$

$\therefore 3x - \frac{\pi}{36} = \pm\left(\frac{35\pi}{36} - x\right) + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore 3x - \frac{\pi}{36} = \left(\frac{35\pi}{36} - x\right) + 2\pi k$

$\therefore 4x = \pi + 2\pi k$

$\therefore \boldsymbol{x = \frac{\pi}{4} + \frac{\pi}{2}k}$

or $3x - \frac{\pi}{36} = -\left(\frac{35\pi}{36} - x\right) + 2\pi k$

$\therefore 2x = -\frac{34\pi}{36} + 2\pi k$

$\therefore \boldsymbol{x = -\frac{17\pi}{36} + \pi k} \quad k \in \mathbb{Z}$

4. (a) $\sin\left(2x + \frac{\pi}{6}\right) = \sin x$

$\therefore 2x + \frac{\pi}{6} = x + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore \boldsymbol{x = -\frac{\pi}{6} + 2\pi k}$

or $2x + \frac{\pi}{6} = \pi - x + 2\pi k$

$\therefore 3x = \frac{5\pi}{6} + 2\pi k$

$\therefore \boldsymbol{x = \frac{5\pi}{18} + \frac{2\pi}{3}k}$

(b) $\boldsymbol{x \in \left\{-\frac{\pi}{6};\ -\frac{7\pi}{18};\ \frac{5\pi}{18};\ \frac{17\pi}{18}\right\}}$

5. $\tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = -\tan x$

$\therefore \tan\left(\frac{x}{2} + \frac{\pi}{4}\right) = \tan(-x)$

$\therefore \frac{x}{2} + \frac{\pi}{4} = -x + \pi k, \quad k \in \mathbb{Z}$

$\therefore \frac{3x}{2} = -\frac{\pi}{4} + \pi k$

$\therefore \boldsymbol{x = -\frac{\pi}{6} + \frac{2\pi k}{3}}$

6. (a) $\cos 2x = \sin x$

$\therefore \cos 2x = \cos\left(\frac{\pi}{2} - x\right)$

$\therefore 2x = \pm\left(\frac{\pi}{2} - x\right) + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore 2x = \left(\frac{\pi}{2} - x\right) + 2\pi k$

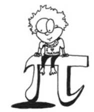
$\therefore 3x = \frac{\pi}{2} + 2\pi k$

$\therefore \boldsymbol{x = \frac{\pi}{6} + \frac{2\pi}{3}k}$

or $2x = -\left(\frac{\pi}{2} - x\right) + 2\pi k$

$\therefore \boldsymbol{x = -\frac{\pi}{2} + 2\pi k}$

(b) $\boldsymbol{x \in \left\{-\frac{11\pi}{6};\ -\frac{7\pi}{6};\ -\frac{\pi}{2};\ \frac{\pi}{6};\ \frac{5\pi}{6}\right\}}$



7. $2\cos 2x = 1{,}3$

$\therefore \cos 2x = 0{,}65$

$\therefore 2x = \pm 0{,}8632\ldots + 2\pi k, \quad k \in \mathbb{Z}$

$\therefore \boldsymbol{x \approx \pm 0{,}432 + \pi k}$

Ch 11: Gr 11 TRIGONOMETRY

(h) **RTP:**

$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}$ → Ⓐ

Proof:

For $n = 1$: LHS $= 1$ RHS $= 1$

$\therefore$ Ⓐ is true for $n = 1$

Assume Ⓐ is true for $n = k$, $k \in \mathbb{N}$

i.e. $1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \ldots + k\left(\frac{1}{2}\right)^{k-1} = 4 - \frac{k+2}{2^{k-1}}$

For $n = k + 1$:

$$\text{LHS} = 1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + \ldots + k\left(\frac{1}{2}\right)^{k-1} + (k+1)\left(\frac{1}{2}\right)^k$$

$$= 4 - \frac{k+2}{2^{k-1}} + (k+1)\left(\frac{1}{2}\right)^k$$

$$= 4 - \frac{2k + 4 - k - 1}{2^k}$$

$$= 4 - \frac{k+3}{2^k}$$

$$\text{RHS} = 4 - \frac{k+1+2}{2^{k+1-1}} = 4 - \frac{k+3}{2^k} = \text{LHS}$$

$\therefore$ If Ⓐ is true for $n = k$, then it is also true for $n = k + 1$.

Ⓐ is true for $n = 1$.

$\therefore$ **By Mathematical Induction, Ⓐ is true for $n \in \mathbb{N}$.**

3. (a) **Assume the statement is true for $n = k$, $k \in \mathbb{N}$**

(b) $\text{LHS} = [1 + 8 + 27 + \ldots + k^3] + [(k+1)^3]$

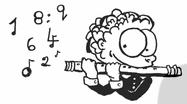
$$= \frac{k^2(k+1)^2}{4} + (k+1)^3$$

$$= \frac{k^2(k+1)^2 + 4(k+1)^3}{4}$$

$$= \frac{(k+1)^2\,[k^2 + 4k + 4]}{4}$$

$$= \frac{(k+1)^2(k+2)^2}{4}$$

$= \text{RHS}$



Exercise 13.2

(Questions – p. 143 in Book)

1. **RTP:** $n^2 + n$ is an even number

Proof:

For $n = 1$: $1^2 + 1 = 2$ which is an even number.

$\therefore$ The statement is true for $n = 1$.

Assume that the statement is true for $n = k$, $k \in \mathbb{N}$

i.e. $k^2 + k = 2p$, $p \in \mathbb{N}$

For $n = k + 1$:

$$(k+1)^2 + (k+1) = k^2 + 2k + 1 + k + 1$$
$$= (k^2 + k) + 2k + 2$$
$$= 2p + 2(k+1)$$
$$= 2(p + k + 1) \text{ which is divisible by 2.}$$

$\therefore$ If the statement is true for $n = k$, then it is also true for $n = k + 1$.

The statement is true for $n = 1$.

$\therefore$ **by Mathematical Induction the statement is true for $n \in \mathbb{N}$.**

2. **RTP:** $n^3 + 2n$ is divisible by 3

Proof:

For $n = 1$: $1^3 + 2(1) = 3(1)$, which is divisible by 3

$\therefore$ The statement is true for $n = 1$

Assume the statement is true for $n = k$, $k \in \mathbb{N}$

i.e. $k^3 + 2k = 3p$, $p \in \mathbb{N}$

For $n = k + 1$:

$$(k+1)^3 + 2(k+1) = k^3 + 3k^2 + 3k + 1 + 2k + 2$$
$$= (k^3 + 2k) + 3k^2 + 3k + 3$$
$$= 3p + 3(k^2 + k + 1)$$
$$= 3(p + k^2 + k + 1) \text{ which is divisible by 3.}$$

$\therefore$ If the statement is true for $n = k$, then it is also true for $n = k + 1$.

The statement is true for $n = 1$.

$\therefore$ **Statement is true for all $n \in \mathbb{N}$, $n \geq 1$ by the principle of Mathematical Induction.**

3. **RTP:**

$6n^2 + 2n$ is divisible by 4

Proof:

For $n = 1$: $6(1)^2 + 2(1) = 8 = 4(2)$, which is divisible by 4

$\therefore$ The statement is true for $n = 1$.

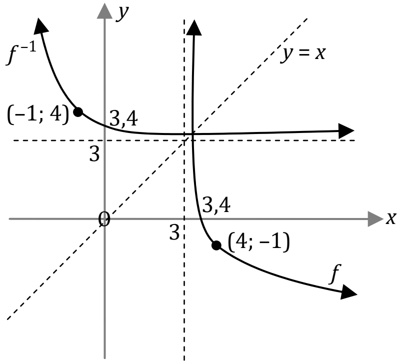
Assume the statement is true for $n = k$, $k \in \mathbb{N}$

i.e. $6k^2 + 2k = 4p$, $p \in \mathbb{N}$

For $n = k + 1$:

$$6(k+1)^2 + 2(k+1) = 6k^2 + 12k + 6 + 2k + 2$$
$$= (6k^2 + 2k) + 12k + 8$$
$$= 4p + 4(3k + 2)$$
$$= 4(p + 3k + 2) \text{ which is divisible by 4.}$$

(c)



2. (a)

$$f(x) = -e^{x+1} - 1$$
$$\therefore\ x = -e^{y+1} - 1$$
$$\therefore\ x + 1 = -e^{y+1}$$
$$\therefore\ -x - 1 = e^{y+1}$$
$$\therefore\ y + 1 = \ln(-x-1)$$
$$\therefore\ \mathbf{f^{-1}(x) = \ln(-x-1) - 1}$$

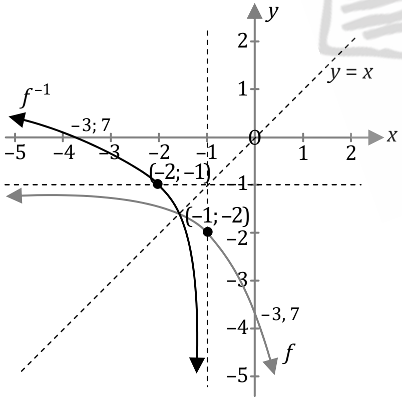
(b) $f(x)$

Domain: $\boldsymbol{x \in \mathbb{R}}$ Range: $\boldsymbol{y < -1}$

$f^{-1}(x)$

Domain: $\boldsymbol{x < -1}$ Range: $\boldsymbol{y \in \mathbb{R}}$

(c)



3. (a)

$$f(x) = 2e^{x} - 2$$
$$\therefore\ x = 2e^{y} - 2$$
$$\therefore\ x + 2 = 2e^{y}$$
$$\therefore\ \frac{x}{2} + 1 = e^{y}$$
$$\therefore\ y = \ln\left(\frac{x}{2} + 1\right)$$
$$\therefore\ \mathbf{f^{-1}(x) = \ln\left(\frac{x}{2} + 1\right)}$$

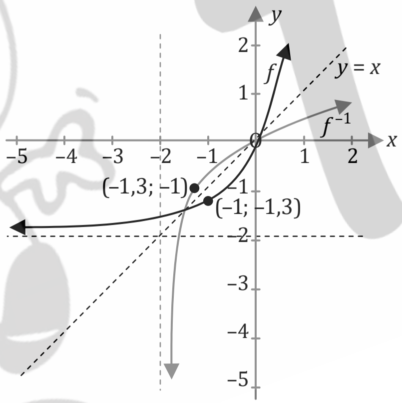
(b) $f(x)$

Domain: $\boldsymbol{x \in \mathbb{R}}$ Range: $\boldsymbol{y > -2}$

$f^{-1}(x)$

Domain: $\boldsymbol{x > -2}$ Range: $\boldsymbol{y \in \mathbb{R}}$

(c)



4. (a)

$$f(x) = 2\ln(x+2) - 1$$
$$\therefore\ x = 2\ln(y+2) - 1$$
$$\therefore\ x + 1 = 2\ln(y+2)$$
$$\therefore\ \frac{x+1}{2} = \ln(y+2)$$
$$\therefore\ y + 2 = e^{\frac{x+1}{2}}$$
$$\therefore\ \mathbf{f^{-1}(x) = e^{\frac{x+1}{2}} - 2}$$

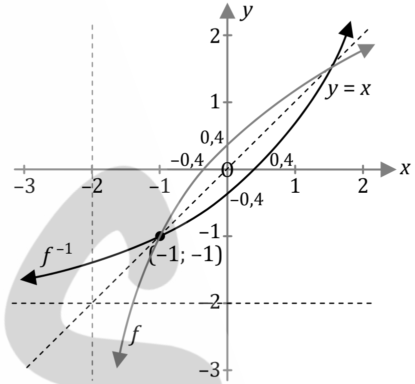
(b) $f(x)$

Domain: $\boldsymbol{x > -2}$ Range: $\boldsymbol{y \in \mathbb{R}}$

$f^{-1}(x)$

Domain: $\boldsymbol{x \in \mathbb{R}}$ Range: $\boldsymbol{y > -2}$

(c)



5. (a)

$$f(x) = \ln(x+4) - 2$$
$$\therefore\ x = \ln(y+4) - 2$$
$$\therefore\ x + 2 = \ln(y+4)$$
$$\therefore\ y + 4 = e^{x+2}$$
$$\therefore\ \mathbf{f^{-1}(x) = e^{x+2} - 4}$$

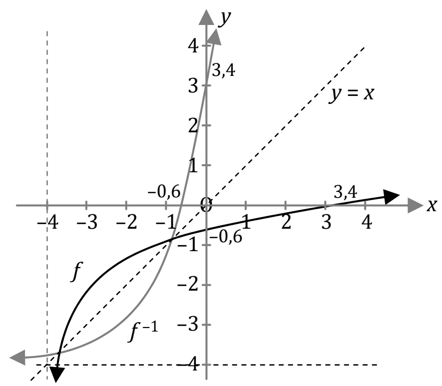
(b) $f(x)$

Domain: $\boldsymbol{x > -4}$ Range: $\boldsymbol{y \in \mathbb{R}}$

$f^{-1}(x)$

Domain: $\boldsymbol{x \in \mathbb{R}}$ Range: $\boldsymbol{y > -4}$

(c)



(b) $y - 0 = -\frac{5}{2}(x - 8)$

$\therefore\ y = \mathbf{-\frac{5}{2}x + 20}$

2. $D_x(3y^4 + 4x - x^2\sin y - 4) = D_x(0)$

$\therefore\ 12y^3\frac{dy}{dx} + 4 - \left(2x\sin y + x^2\cos y\frac{dy}{dx}\right) = 0$

$\therefore\ 12y^3\frac{dy}{dx} - x^2\cos y\frac{dy}{dx} = 2x\sin y - 4$

$\therefore\ \frac{dy}{dx} = \frac{2x\sin y - 4}{12y^3 - x^2\cos y}$

At (1; 0) $m_T = \frac{2(1)\sin 0 - 4}{12(0)^3 - (1)^2\cos 0} = 4$

$\therefore\ m_N = -\frac{1}{4}$

$y - 0 = -\frac{1}{4}(x - 1)$

$\therefore\ \boldsymbol{y = -\frac{1}{4}x + \frac{1}{4}}$

3. (a) LHS $= 1^3 + 2^3 - (1)(2^2) = 5$
RHS $= 5 =$ LHS

Point lies on curve.

(b) $D_x(x^3 + y^3 - xy^2) = D_x(5)$

$\therefore\ 3x^2 + 3y^2\frac{dy}{dx} - \left(y^2 + 2xy\frac{dy}{dx}\right) = 0$

$\therefore\ 3y^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = y^2 - 3x^2$

$\therefore\ \frac{dy}{dx} = \boldsymbol{\frac{y^2 - 3x^2}{3y^2 - 2xy}}$

(c) At (1; 2): $m_T = \frac{2^2 - 3(1)^2}{3(2)^2 - 2(1)(2)} = \frac{1}{8}$; $\therefore m_N = -8$

$y - 2 = -8(x - 1)$

$\therefore\ \boldsymbol{y = -8x + 10}$

4. (a) $D_x(\sin y) = D_x(x)$

$\therefore\ \cos y \times \frac{dy}{dx} = 1$

$\therefore\ \frac{dy}{dx} = \boldsymbol{\frac{1}{\cos y}}$

(b) $\sin y = 0{,}5$

$\therefore\ y = \frac{\pi}{6}$ $\left(0{,}5;\ \frac{\pi}{6}\right)$

$m_T = \frac{1}{\cos\frac{\pi}{6}} = \frac{2\sqrt{3}}{3}$; $\therefore\ m_N = -\frac{\sqrt{3}}{2}$

$y - \frac{\pi}{6} = -\frac{\sqrt{3}}{2}(x - 0{,}5)$

$\therefore\ \boldsymbol{y = -\frac{\sqrt{3}}{2}x + 0{,}96}$

5. $f(x) = e^{4x^2}$

$f'(x) = e^{4x^2} \times 8x$

$m_T = f'\left(\frac{1}{4}\right) = e^{4\left(\frac{1}{4}\right)^2} \times 8\left(\frac{1}{4}\right) = 2{,}568\ldots$;

$m_N \approx -0{,}39$

$f\left(\frac{1}{4}\right) = e^{4\left(\frac{1}{4}\right)^2} \approx 1{,}28$

$y - 1{,}28 = -0{,}39\left(x - \frac{1}{4}\right)$

$\therefore\ \boldsymbol{y = -0{,}39x + 1{,}38}$

6. $y = e^x$; $y' = e^x$

At $x = 0$: $m = 1$

$y = e^{-x}$; $y' = -e^x$

At $x = 0$: $m = -1$

$m_T \times m_N = -1$

$\therefore$ **Graphs are orthogonal**

Gr 12 Exam Solutions (Questions – p. 176 in Book)

1. (a) $f(x) = \frac{1}{1 - 2x}$

$f'(x) = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0}\left(\frac{1}{1 - 2(x+h)} - \frac{1}{1 - 2x}\right) \div h$

$= \lim_{h \to 0}\frac{1}{h}\cdot\frac{1 - 2x - (1 - 2x - 2h)}{(1 - 2x - 2h)(1 - 2x)}$

$= \lim_{h \to 0}\frac{1}{h}\cdot\frac{2h}{(1 - 2x - 2h)(1 - 2x)}$

$= \boldsymbol{\frac{2}{(1 - 2x)^2}}$

(b) $f'(x) = \lim_{h \to 0}\frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0}\left(\frac{1}{\sqrt{2x + 2h + 1}} - \frac{1}{\sqrt{2x + 1}}\right) \times \frac{1}{h}$

$= \lim_{h \to 0}\frac{\left(\sqrt{2x+1} - \sqrt{2x+2h+1}\right)\left(\sqrt{2x+1} + \sqrt{2x+2h+1}\right)}{h\sqrt{2x+1}\sqrt{2x+2h+1}\left(\sqrt{2x+1} + \sqrt{2x+2h+1}\right)}$

$= \lim_{h \to 0}\frac{2x + 1 - (2x + 2h + 1)}{h\sqrt{2x+1}\sqrt{2x+2h+1}\left(\sqrt{2x+1} + \sqrt{2x+2h+1}\right)}$

$= \lim_{h \to 0}\frac{-2h}{h\sqrt{2x+1}\sqrt{2x+2h+1}\left(\sqrt{2x+1} + \sqrt{2x+2h+1}\right)}$

$= \frac{-2}{(2x+1)\left(2\sqrt{2x+1}\right)}$

$= \boldsymbol{\frac{-1}{(2x+1)^{\frac{3}{2}}}}$

7. $f(x) = \dfrac{2x^2 - 5x + 1}{x - 1}$

Vertical: $\boldsymbol{x = 1}$

Oblique: $f(x) = \dfrac{2x(x-1) - 3x + 1}{x - 1}$

$= \dfrac{2x(x-1) - 3(x-1) - 2}{x - 1}$

$= (2x - 3) - \dfrac{2}{x - 1}$

$\therefore$ O.A.: $\boldsymbol{y = 2x - 3}$

8. Vertical: $2x^2 + 9x - 5 = (2x - 1)(x + 5) = 0$

$\therefore$ V.A.: $\boldsymbol{x = \frac{1}{2}\,;\ x = -5}$

Horizontal: $\lim\limits_{x \to \infty} \dfrac{\cancel{x^2}\left(\frac{1}{x} - \frac{1}{x^2}\right)}{\cancel{x^2}\left(2 + \frac{9}{x} - \frac{5}{x^2}\right)} = 0$

$\therefore$ H.A.: $\boldsymbol{y = 0}$

9. $f(x) = \dfrac{3x - 5}{5 - 2x}$

Vertical: $\boldsymbol{x = \frac{5}{2}}$

Horizontal: $\lim\limits_{x \to \infty} \dfrac{\cancel{x}\left(3 - \frac{5}{x}\right)}{\cancel{x}\left(-2 + \frac{5}{x}\right)} = -\dfrac{3}{2}$

H.A.: $\boldsymbol{y = -\frac{3}{2}}$

10. $f(x) = \dfrac{x^2 + 5x + 1}{x^3 - 7x^2 + 10x}$

Vertical: $x(x^2 - 7x + 10) = x(x-5)(x-2) = 0$

V.A.: $\boldsymbol{x = 0;\quad x = 2;\quad x = 5}$

Horizontal: $\lim\limits_{x \to \infty} \dfrac{\cancel{x^3}\left(\frac{1}{x} + \frac{5}{x^2} + \frac{1}{x^3}\right)}{\cancel{x^3}\left(1 - \frac{7}{x} + \frac{10}{x^2}\right)} = 0$

$\therefore$ H.A.: $\boldsymbol{y = 0}$

Exercise 17.9

(Questions – p. 205 in Book)

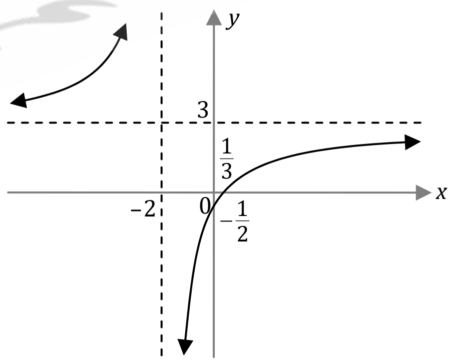
1. (a) $f(x) = \dfrac{3x - 1}{x + 2}$

y–intercept: $\left(0;\ -\frac{1}{2}\right)$ $\quad x$-intercept(s): $\left(\frac{1}{3};\ 0\right)$

$f(x) = \dfrac{3x - 1}{x + 2} = \dfrac{3(x+2) - 7}{(x+2)} = 3 - \dfrac{7}{(x+2)}$

V.A.: $x = -2$ $\quad$ H.A.: $y = 3$

x	$x < -2$	$x = -2$	$-2 < x < \frac{1}{3}$	$x = \frac{1}{3}$	$x > \frac{1}{3}$
$f(x)$	+	u/d	–	0	+



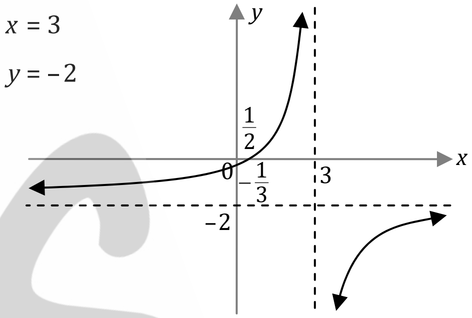
(b) $f(x) = \dfrac{1 - 2x}{x - 3}$

Intercepts: y-int.: $\left(0;\ -\frac{1}{3}\right)$ $\quad x$-int: $\left(\frac{1}{2};\ 0\right)$

$f(x) = \dfrac{-2(x-3) - 5}{x - 3} = -2 - \dfrac{5}{x - 3}$

V.A.: $x = 3$

H.A.: $y = -2$



(c) $f(x) = \dfrac{x^2 - 4}{x + 1} = \dfrac{(x-2)(x+2)}{(x+1)}$

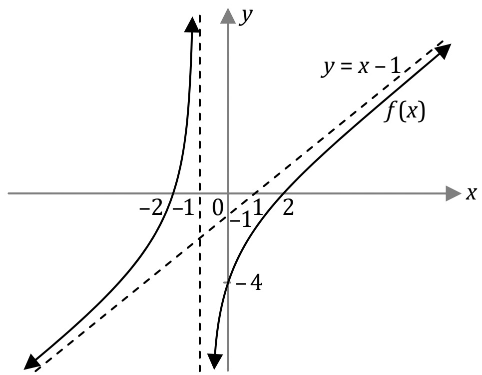
y–int.: $(0; -4)$ $\quad x$-int.: $(-2; 0)$; $(2; 0)$

$f(x) = \dfrac{x(x+1) - x - 4}{x + 1} = \dfrac{x(x+1) - (x+1) - 3}{x + 1}$

$= (x - 1) - \dfrac{3}{(x+1)}$

V.A.: $x = -1$ $\quad$ O.A.: $y = x - 1$

x	$x < -2$	$x = -2$	$-2 < x < -1$	$x = -1$	$-1 < x < 2$	$x = 2$	$x > 2$
$f(x)$	–	0	+	u/d	–	0	+



3. $\int 2x(x+2)^9 dx$

$u = x + 2 \quad x = u - 2 \qquad du = dx$

$\int 2(u-2)u^9 du$

$= 2\int (u^{10} - 2u^9)du$

$= 2\left(\frac{u^{11}}{11} - \frac{2u^{10}}{10}\right) + c$

$= 2u^{10}\left(\frac{u}{11} - \frac{1}{5}\right) + c$

$= 2u^{10} \cdot \frac{5u - 11}{55} + c$

$= \frac{2(x+2)^{10}(5x + 10 - 11)}{55} + c$

$= \mathbf{\frac{2(x+2)^{10}(5x-1)}{55} + c}$

4. $\int 9x(3x+2)^3 dx$

$u = 3x + 2 \quad x = \frac{u-2}{3} \qquad du = 3dx$

$\int 9 \cdot \frac{u-2}{3} \cdot u^3 \frac{du}{3} = \int (u^4 - 2u^3)du$

$= \frac{u^5}{5} - \frac{2u^4}{4} + c$

$= u^4\left(\frac{u}{5} - \frac{1}{2}\right) + c$

$= u^4 \cdot \frac{2u - 5}{10} + c$

$= \frac{(3x+2)^4(2(3x+2) - 5)}{10} + c$

$= \mathbf{\frac{(3x+2)^4(6x-1)}{10} + c}$

5. $\int \frac{3x}{\sqrt{2x+3}} dx$

$u = 2x + 3 \quad x = \frac{u-3}{2} \qquad du = 2dx$

$\int 3 \cdot \frac{u-3}{2} \cdot u^{-\frac{1}{2}} \cdot \frac{du}{2} = \frac{3}{4}\int \left(u^{\frac{1}{2}} - 3u^{-\frac{1}{2}}\right)du$

$= \frac{3}{4}\left(\frac{2u^{\frac{3}{2}}}{3} - \frac{3 \cdot 2u^{\frac{1}{2}}}{1}\right) + c$

$= \frac{3u^{\frac{1}{2}}}{2}\left(\frac{u}{3} - 3\right) + c$

$= \frac{3u^{\frac{1}{2}}}{2} \cdot \frac{u-9}{3} + c$

$= \frac{\sqrt{2x+3}(2x+3-9)}{2} + c$

$= \mathbf{\sqrt{2x+3}(x-3) + c}$

6. $\int_1^2 x^2(x-1)^5 dx$

$u = x - 1 \qquad x = u + 1$

$du = dx \qquad x^2 = (u+1)^2 = u^2 + 2u + 1$

$x = 1,\ u = 0 \qquad x = 2,\ u = 1$

$\int_0^1 (u^2 + 2u + 1)u^5 du = \int_0^1 (u^7 + 2u^6 + u^5)du$

$= \left[\frac{u^8}{8} + \frac{2u^7}{7} + \frac{u^6}{6}\right]_0^1$

$= \left(\frac{1}{8} + \frac{2}{7} + \frac{1}{6}\right) - 0$

$= \mathbf{\frac{97}{168}}$

Exercise 18.12

(Questions – p. 229 in Book)

1. $\int \frac{x-1}{x+1} dx$

$\frac{x-1}{x+1} = \frac{x+1-2}{x+1} = 1 - \frac{2}{x+1}$

$\int \left(1 - \frac{2}{x+1}\right)dx = \mathbf{x - 2\ln|x+1| + c}$

2. $\int \frac{x+3}{x-5} dx$

$\frac{x+3}{x-5} = \frac{x-5+8}{x-5} = 1 + \frac{8}{x-5}$

$\int \left(1 + \frac{8}{x-5}\right)dx = \mathbf{x + 8\ln|x-5| + c}$

3. $\int \frac{x^2 - 2x + 3}{x} dx$

$\frac{x^2 - 2x + 3}{x} = x - 2 + \frac{3}{x}$

$\int \left(x - 2 + \frac{3}{x}\right)dx = \mathbf{\frac{x^2}{2} - 2x + 3\ln|x| + c}$

4. $\int \frac{x^3}{x^2 - 4} dx$

$\frac{x^3}{x^2-4} = \frac{x(x^2-4) + 4x}{x^2-4} = x + \frac{4x}{x^2-4}$

$\int \left(x + \frac{4x}{x^2-4}\right)dx = \int x\, dx + \int \frac{4x}{x^2-4} dx$

$= \frac{x^2}{2} + 4\int \frac{2x}{2} \cdot \frac{1}{x^2-4} dx$

$= \mathbf{\frac{x^2}{2} + 2\ln|x^2 - 4| + c}$