Further Studies Mathematics

BOOK 2

Marilyn Buchanan, Gert Esterhuyse, Carl Fourie, Noleen Jakins
& Ingrid Zlobinsky-Roux

GRADE

10-12

ISC

SOLUTIONS

THE ANSWER SERIES *Your Key to Exam Success*

GRADE

10-12

ISC

Further Studies Mathematics

Book 2: Extended Level

Statistics, Finance & Mathematical Modelling, Matrices & Graph Theory

M. Buchanan, G. Esterhuyse, C. Fourie, N. Jakins & I. Zlobinsky-Roux

THIS CLASS TEXT & STUDY GUIDE INCLUDES

1. Notes, Worked Examples, Exercises & Exam Questions
2. Full Solutions in separate booklet

eBook available

THE ANSWER SERIES *Your Key to Exam Success*

2019 publication | ISBN: 978-1-928404-26-2

190224 | TAS

CONTENTS

Solutions to Book 2: Elective Modules

NB The page numbers of this Solutions book coincide with the page numbers in the separate (elective modules) books which contain these solutions.

STATISTICS

Pg. no

Chapter 1 (Grade 10)

Probability Stat.95

Counting methods using Permutations Stat.98

Standard deviation Stat.99

Chapter 2 (Grade 11)

Counting methods using Permutations and Combinations Stat.102

Probability mass functions including binomial and hypergeometric distributions Stat.103

Simple Probability density functions Stat.110

Normal distribution Stat.111

Chapter 3 (Grade 12)

Probability density functions Stat.120

Confidence Intervals Stat.121

Normal approximation to binomial distribution Stat.122

Hypothesis testing Stat.123

FINANCE & MODELLING

Pg. no

FINANCE

Chapter 1 (Grade 10) Fin.51

Chapter 2 (Grade 11) Fin.55

Chapter 3 (Grade 12) Fin.61

MODELLING

Chapter 4 (Grade 10) Mod.53

Chapter 5 (Grade 11) Mod.57

Chapter 6 (Grade 12) Mod.62

MATRICES & GRAPH THEORY

Pg. no

MATRICES

Chapter 1 (Grade 10) Mat.46

Chapter 2 (Grade 11) Mat.50

Chapter 3 (Grade 12) Mat.54

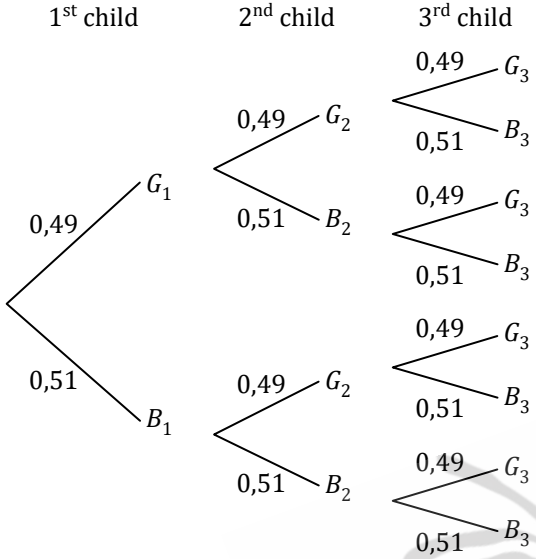
GRAPH THEORY

Chapter 4 (Grade 10) GT.41

Chapter 5 (Grade 11) GT.43

Chapter 6 (Grade 12) GT.47

5.

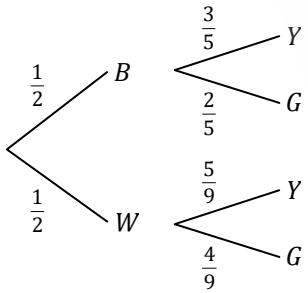


(a) $P(G \geq 1) = 1 - P(G = 0)$
$= 1 - 0{,}51 \times 0{,}51 \times 0{,}51$
$=$ **0,867349**

(b) $1 - P(GGG) - P(BBB)$
$= 1 - 0{,}49^3 - 0{,}51^3$
$=$ **0,74947**

(c) $P(G_3/G_1) = P(G_3) =$ **0,49**

6. (a)



(b) $P(B \cap Y) = \frac{1}{2} \times \frac{3}{5} =$ **$\frac{3}{10}$**

(c) $P(B \cap G) + P(W \cap G) = \frac{1}{2} \times \frac{2}{5} + \frac{1}{2} \times \frac{4}{9}$
$=$ **$\frac{19}{45}$**

(d) $P(Y/W) =$ **$\frac{5}{9}$**

(e) $P(B/Y) = \frac{P(B \cap Y)}{P(Y)}$
$= \frac{\frac{1}{2} \times \frac{3}{5}}{\frac{1}{2} \times \frac{3}{5} + \frac{1}{2} \times \frac{5}{9}}$
$=$ **$\frac{27}{52}$**

COUNTING METHODS

Exercise STAT 1.5

(Questions on Stat.14)

1. (a) $\frac{6!}{2!\,2!} =$ **180** (b) $\frac{7!}{3!} =$ **840**
(c) $\frac{8!}{3!\,2!} =$ **3 360** (d) $\frac{13!}{3!\,3!} =$ **172 972 800**

2. $8!\,2! =$ **80 640**

3. $\frac{10!}{4!\,3!} =$ **25 200**

4. $\frac{9!}{2!\,2!} =$ **90 720**

5. (a) $8! =$ **40 320** (b) $5! \times 2 \times 2 \times 2 =$ **960**

6. $7! =$ **5 040**

7. $10!\,3!\,2! =$ **43 545 600**

8. $4! =$ **24**

9. (a) $4! \times 2 \times 2 \times 2 \times 2 =$ **384**
(b) $4!\,4!\,2! =$ **1 152**

10. $6! =$ **720**

Exercise STAT 1.6

(Questions on Stat.16)

1. (a) $6^4 =$ **1 296**
(b) $6P4 =$ **360**

2. (a) $26^3 \times 10^2 =$ **1 757 600**
(b) $26P3 \times 10P2 =$ **1 404 000**

3. $25P3 =$ **13 800**

4. $8P4 =$ **1 680**

5. $75P25 \approx$ **$8{,}15712 \times 10^{44}$**

6. $10P2 =$ **90**

7. (a) $10^5 =$ **100 000**
(b) $10P5 =$ **30 240**

8. $5P4 =$ **120**

9. $5^3 \times 3 =$ **375**

10. $10P3 =$ **720**

SIMPLE PROBABILITY DENSITY FUNCTIONS

Exercise STAT 2.7

(Questions on Stat.48)

1. (a) $f(x) = \begin{cases} \frac{1}{10}\ ;\ 3 < x < 13 \\ 0\ \ ;\ \text{elsewhere} \end{cases}$

(b) $P(5{,}2 < X < 7{,}1) = (7{,}1 - 5{,}2) \times \frac{1}{10} = \mathbf{0{,}19}$

(c) $P(X > 8{,}7) = (13 - 8{,}7) \times \frac{1}{10} = \mathbf{0{,}43}$

(d) $\frac{m-3}{10} = \frac{1}{2}$

$\therefore\ \boldsymbol{m = 8}$

2. (a) $\frac{1}{k}(10 - 2) = 1$

$\therefore\ \frac{8}{k} = 1$

$\therefore\ \boldsymbol{k = 8}$

(b) $P(X < 3{,}5) = \frac{1}{8}(3{,}5 - 2)$

$= \mathbf{\frac{3}{16}}$

(c) $P(X > 7{,}5) = \frac{1}{8}(10 - 7{,}5)$

$= \mathbf{\frac{5}{16}}$

(d) $P(5{,}5 < X < 6{,}5) = \frac{1}{8}(6{,}5 - 5{,}5)$

$= \mathbf{\frac{1}{8}}$

(e) $\frac{1}{8}(m - 2) = \frac{1}{2}$

$\therefore\ m - 2 = 4$

$\therefore\ \boldsymbol{m = 6}$

3. (a) $\frac{1}{7}(10 - k) = 1$

$\therefore\ 10 - k = 7$

$\therefore\ \boldsymbol{k = 3}$

(b) $P(X > 9{,}3) = \frac{1}{7}(10 - 9{,}3)$

$= \mathbf{\frac{1}{10}}$

(c) $P(X < 5{,}2) = \frac{1}{7}(5{,}2 - 3)$

$= \mathbf{\frac{11}{35}}$

(d) $P(6{,}4 < X < 7{,}1) = \frac{7{,}1 - 6{,}4}{7}$

$= \mathbf{\frac{1}{10}}$

(e) $\frac{m-3}{7} = \frac{1}{2}$

$\therefore\ \boldsymbol{m = \frac{13}{2}}$

4. (a) $\frac{k-5}{8} = 1$

$\therefore\ k - 5 = 8$

$\therefore\ \boldsymbol{k = 13}$

(b) $P(X > 7{,}1) = \frac{13 - 7{,}1}{8}$

$= \mathbf{\frac{59}{80}}$

(c) $P(X < 6{,}7) = \frac{6{,}7 - 5}{8}$

$= \mathbf{\frac{17}{80}}$

(d) $P(7{,}9 < X < 8{,}5) = \frac{8{,}5 - 7{,}9}{8}$

$= \mathbf{\frac{3}{40}}$

(e) $P(X < 4) = \mathbf{0}$

(f) $\frac{m-5}{8} = \frac{1}{2}$

$\therefore\ \boldsymbol{m = 9}$

Exercise STAT 2.8

(Questions on Stat.50)

1. (a) Function is positive in given interval.

$\frac{(4-2)}{2} = 1$ $\quad\therefore$ **is a p.d.f.**

(b) Function is positive in given interval.

$\frac{(3-1)}{2} \cdot \left(\frac{1}{4} + \frac{3}{4}\right) = 1$ $\quad\therefore$ **is a p.d.f.**

(c) Function is positive in given interval.

$\frac{(3-0)}{2} \cdot \left(\frac{0}{6} + \frac{1}{12} + \frac{3}{6} + \frac{1}{12}\right) = 1$ $\quad\therefore$ **is a p.d.f.**

(d) $f(x) = \frac{1}{2}(2x - 1) \quad 0 < x < 2$

$= x - \frac{1}{2}$

Function is negative for $x \in \left(\mathbf{0; \frac{1}{2}}\right)$. $\quad\therefore$ **not a p.d.f.**

NORMAL APPROXIMATION TO BINOMIAL DISTRIBUTION

Exercise STAT 3.4 (Questions on Stat. 80)

1. (a) $P(X \le 2) = P(X = 0) + P(X = 1) + P(X = 2)$

$$= \binom{50}{0}0{,}2^0 0{,}8^{50} + \binom{50}{1}0{,}2^1 0{,}8^{49} + \binom{50}{2}0{,}2^2 0{,}8^{48}$$

$$= \mathbf{0{,}001285}$$

(b) $np = 50 \times 0{,}2 = 10 > 5$

$nq = 50 \times 0{,}8 = 40 > 5$

$\mu = np = 10$; $\sigma = \sqrt{50 \times 0{,}2 \times 0{,}8} \approx 2{,}8284$

$P(X \le 2) \rightarrow P(X < 2{,}5)$

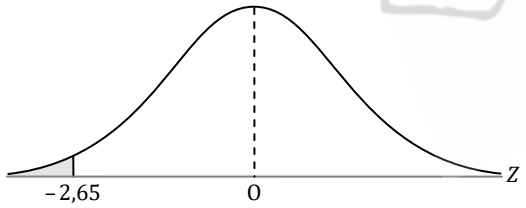
$= P\left(Z < \frac{2{,}5 - 10}{2{,}8284}\right)$

$= P(Z < -2{,}65)$

$= 0{,}5 - H(2{,}65)$

$= 0{,}5 - 0{,}49598$

$= \mathbf{0{,}00402}$



(c) $P(X \ge 10) \rightarrow P(X > 9{,}5)$

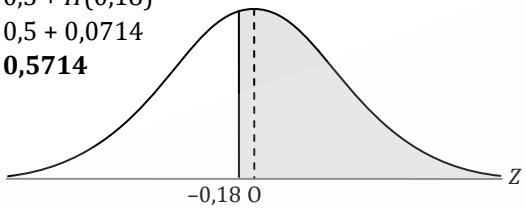
$= P\left(Z > \frac{9{,}5 - 10}{2{,}8284}\right)$

$= P(Z > -0{,}18)$

$= 0{,}5 + H(0{,}18)$

$= 0{,}5 + 0{,}0714$

$= \mathbf{0{,}5714}$



(d) $P(X > 7) \rightarrow P(X > 7{,}5)$

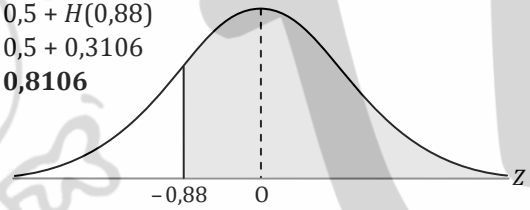
$= P\left(Z > \frac{7{,}5 - 10}{2{,}8284}\right)$

$= P(Z > -0{,}88)$

$= 0{,}5 + H(0{,}88)$

$= 0{,}5 + 0{,}3106$

$= \mathbf{0{,}8106}$



2. (a) $np = 0{,}72 \times 50 = 36 > 5$

$nq = 0{,}28 \times 50 = 14 > 5$

$\mu = 36$; $\sigma \approx 3{,}1749$

$\therefore$ A normal approximation may be used.

$P(X \ge 30) \rightarrow P(X > 29{,}5)$

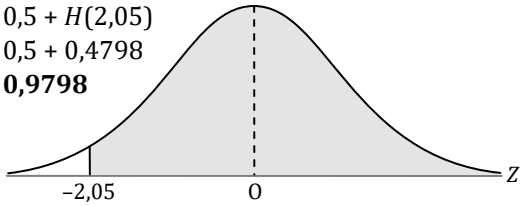
$= P\left(Z > \frac{29{,}5 - 36}{3{,}1749}\right)$

$= P(Z > -2{,}05)$

$= 0{,}5 + H(2{,}05)$

$= 0{,}5 + 0{,}4798$

$= \mathbf{0{,}9798}$



(b) $P(X > 25) \rightarrow P(X > 25{,}5)$

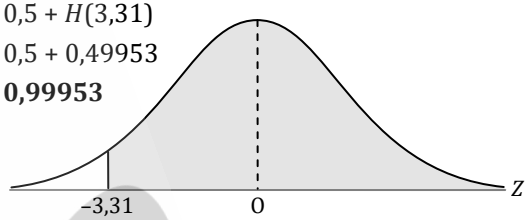
$= P\left(Z > \frac{25{,}5 - 36}{3{,}1749}\right)$

$= P(Z > -3{,}31)$

$= 0{,}5 + H(3{,}31)$

$= 0{,}5 + 0{,}49953$

$= \mathbf{0{,}99953}$



(c) $P(X \le 35) \rightarrow P(X < 35{,}5)$

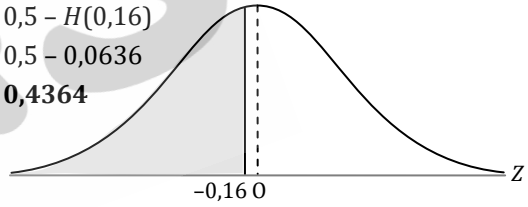
$= P\left(Z < \frac{35{,}5 - 36}{3{,}1749}\right)$

$= P(Z < -0{,}16)$

$= 0{,}5 - H(0{,}16)$

$= 0{,}5 - 0{,}0636$

$= \mathbf{0{,}4364}$



3. $np = 400 \times 0{,}015 = 6 > 5$

$nq = 400 \times 0{,}985 = 394 > 5$

$\therefore \mu = 6$; $\sigma = \sqrt{400 \times 0{,}015 \times 0{,}985} \approx 2{,}431$

$P(X > 5) \rightarrow P(X > 5{,}5)$

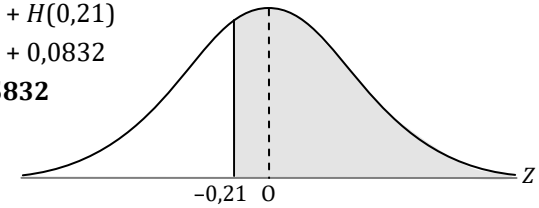
$= P\left(Z > \frac{5{,}5 - 6}{2{,}431}\right)$

$= P(Z > -0{,}21)$

$= 0{,}5 + H(0{,}21)$

$= 0{,}5 + 0{,}0832$

$= \mathbf{0{,}5832}$



4. $np = 325 \times 0{,}11 = 35{,}75 > 5$
$nq = 325 \times 0{,}89 = 289{,}25 > 5$

$\therefore \mu = 35{,}75; \quad \sigma \approx 5{,}6407$

$P(X \leq 25) \rightarrow P(X < 25{,}5)$

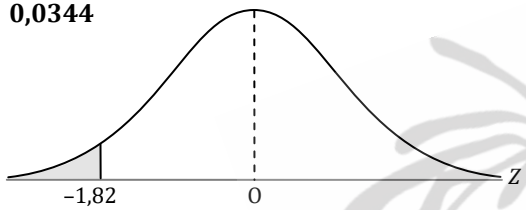
$= P\left(Z < \frac{25{,}5 - 35{,}75}{5{,}6407}\right)$

$= P(Z < -1{,}82)$

$= 0{,}5 - H(1{,}82)$

$= 0{,}5 - 0{,}4656$

$= \mathbf{0{,}0344}$



5. $np = 150 \times 0{,}66 = 99 > 5$
$nq = 150 \times 0{,}34 = 51 > 5$

$\therefore \mu = 99; \quad \sigma \approx 5{,}8017$

$P(X \geq 87) \rightarrow P(X > 86{,}5)$

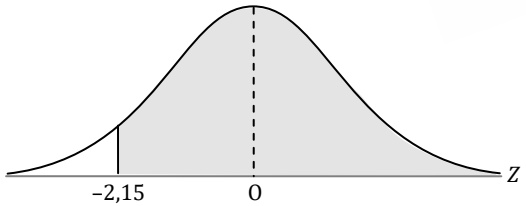
$= P\left(Z > \frac{86{,}5 - 99}{5{,}8017}\right)$

$= P(Z > -2{,}15)$

$= 0{,}5 + H(2{,}15)$

$= 0{,}5 + 0{,}4842$

$= \mathbf{0{,}9842}$



6. $np = 50 \times 0{,}4 = 20 > 5$
$nq = 50 \times 0{,}6 = 30 > 5$

$\therefore \mu = 20; \quad \sigma \approx 3{,}4641$

$P(X > 25) \rightarrow P(X > 25{,}5)$

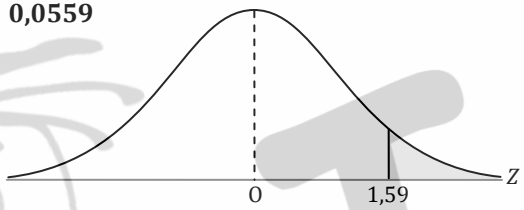
$= P\left(Z > \frac{25{,}5 - 20}{3{,}4641}\right)$

$= P(Z > 1{,}59)$

$= 0{,}5 - H(1{,}59)$

$= 0{,}5 - 0{,}4441$

$= \mathbf{0{,}0559}$



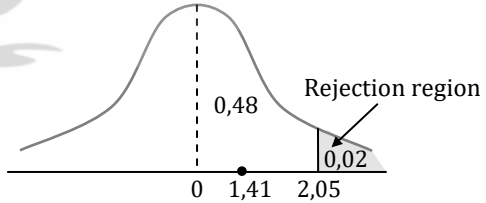
HYPOTHESIS TESTING

Exercise STAT 3.5

(Questions on Stat.85)

1. (a) $H_0: \mu = 14 \qquad H_1: \mu > 14$

$\therefore X \sim N(14; 9)$



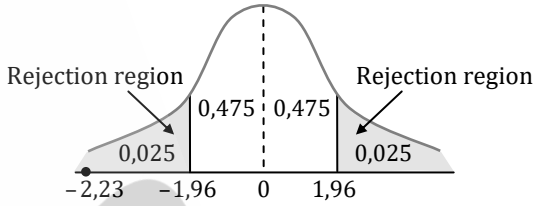
$z = \frac{14{,}3 - 14}{\frac{3}{\sqrt{200}}} \approx 1{,}41$

Does not lie in rejection region (1,41 < 2,05).

$\therefore$ **Do not reject H_0 at a 2% significance level.**
Insufficient evidence to support the claim.

(b) $H_0: \mu = 51 \qquad H_1: \mu \neq 51$

$\therefore X \sim N(51; 5{,}5^2)$



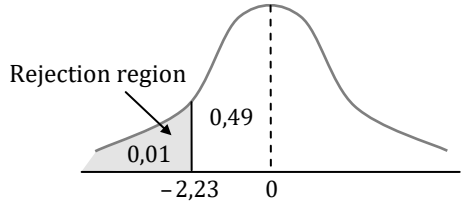
$z = \frac{50 - 51}{\frac{5{,}5}{\sqrt{150}}} \approx -2{,}23$

Lies in rejection region (–2,23 < –1,96).

$\therefore$ **Reject H_0 at a 5% significance level.**
Sufficient evidence to support the claim.

(c) $H_0: \mu = 1{,}75 \qquad H_1: \mu < 1{,}75$

$\therefore X \sim N(1{,}75; 0{,}5^2)$

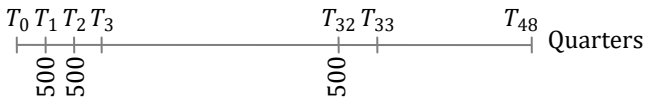


$z = \frac{1{,}5 - 1{,}75}{\frac{0{,}5}{\sqrt{49}}} = -3{,}5$

Lies in rejection region (–3,5 < –2,23).

$\therefore$ **Reject H_0 at a 1% significance level.**
Sufficient evidence to support the claim.

4.



7,4% p.a.c.q.

For the first 8 years we use the formula

$$F = x\frac{(1+i)^n - 1}{i}$$

After that, for the last 4 years we use the formula $F = P(1+i)^n$,

$$1 + \frac{0{,}074}{4} = \frac{2\,037}{2\,000} \quad \rightarrow \text{MEM A}$$

$$F = 500\,\frac{A^{32-1}}{A-1}.A^{16} \approx \textbf{R28 913,14}$$

5. $$1 + \frac{0{,}052}{12} = \frac{3\,013}{3\,000} \quad \rightarrow \text{MEM A}$$

$$205\,772{,}30 = \frac{A^{24\times 12} - 1}{A-1}.A^{6\times 12}$$

$$= x(779{,}4\,405\ldots)$$

$$\therefore\ \boldsymbol{x \approx \text{R264 per month}}$$

6. (a) Depreciated Value $= 420\,000(1 - 0{,}11)^6$

$$\approx \textbf{R208 732,14} \quad \rightarrow \text{MEM B}$$

(b) Cost of new van $= 420\,000(1 + 0{,}055)^6$

$$\approx \textbf{R579 113,98} \quad \rightarrow \text{MEM C}$$

(c) $$1 + \frac{0{,}073}{12} = \frac{12\,073}{12\,000} \quad \rightarrow \text{MEM A}$$

Shortfall $= 370\,381{,}84$

$$= \frac{x\left[A^{6\times 12} - 1\right]}{A-1}$$

$$= x(90{,}008\ldots)$$

$$\therefore\ \boldsymbol{x \approx \text{R4 114,98 per month}}$$

7. (a) Depreciated value $= 12\,000\,000(1 - 0{,}104)^5$

$$\approx \textbf{R6 929 809,47}$$

(b) Replacement Value $= 12\,000\,000(1 + 0{,}062)^5$

$$\approx \textbf{R16 210 776,93}$$

(c) Shortfall $= 16\,210\,776{,}93 - 6\,929\,809{,}47$

$$= \textbf{R9 280 967,47}$$

$$1 + \frac{0{,}046}{4} = \frac{2\,023}{2\,000} \quad \rightarrow \text{MEM A}$$

$$9\,280\,967{,}47 = x.\frac{\left[A^{12} - 1\right]}{A-1}(A)^8$$

$$= x(14{,}013\,899)$$

$$\therefore\ \boldsymbol{x \approx \text{R662 268,74 per quarter}}$$

Exercise FIN 2.4

(Questions on Fin.26)

1. $$1 + \frac{0{,}14}{12} = \frac{607}{600} \quad \rightarrow \text{MEM A}$$

$$\text{At } T_5\text{: } 40\,000A^5 = \frac{x\left[1 - A^{-19}\right]}{A-1}$$

$$\therefore\ 42\,388{,}41\ldots = x(16{,}953\ldots)$$

$$\therefore\ \boldsymbol{x \approx \text{R2 500,30 per month}}$$

2. $$1 + \frac{0{,}08}{12} = \frac{151}{150} \quad \rightarrow \text{MEM A}$$

$$\text{At } T_{23}\text{: } 100\,000A^{23} = \frac{x\left[1 - A^{-37}\right]}{A-1}$$

$$\therefore\ 116\,512{,}0462 = x(32{,}693\ldots)$$

$$\therefore\ \boldsymbol{x \approx \text{R3 563,73 per month}}$$

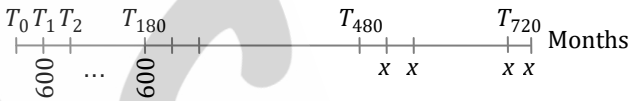
3. $$1 + \frac{0{,}068}{12} = \frac{3\,017}{3\,000} \quad \rightarrow \text{MEM A}$$

$$\text{At } T_3\text{: } P.A^3 = 680.\frac{\left[1 - A^{-57}\right]}{A-1}$$

$$\therefore\ P(1{,}017\,096\ldots) = 33\,043{,}89\,364$$

$$\therefore\ \boldsymbol{P \approx \text{R32 488,45}}$$

4.



$$1 + \frac{0{,}076}{12} = \frac{3\,019}{3\,000} \quad \rightarrow \text{MEM A}$$

$$\text{At } T_{480}\text{: } \frac{600\left[A^{180} - 1\right]}{A-1}.A^{300} = \frac{x\left[1 - A^{-240}\right]}{A-1}$$

$$\therefore\ 1\,331\,985{,}047 = x(123{,}195\ldots)$$

$$\therefore\ \boldsymbol{x \approx \text{R10 811,97}}$$

5. $$1 + \frac{0{,}5}{12} = \frac{25}{24} \quad \rightarrow \text{MEM A}$$

At T_{18}:

Bob's O.B.:

$$50\,000A^{18} - \frac{4\,000\left[A^{18} - 1\right]}{A-1} = 87{,}4\,059\ldots \quad \rightarrow \text{MEM B}$$

Abie's O.B.:

$$50\,000A^{18} - \frac{4\,000\left[A^{12} - 1\right]}{A-1} = 43\,571{,}78\,285 \rightarrow \text{MEM C}$$

Difference ≈ R43 484,38

(b) $1 + \frac{0{,}105}{12} = \frac{807}{800}$ → MEM C

At T_9:

$$\text{O.B.} = \frac{10\,000\left[1 - A^{-231}\right]}{A - 1}$$
$$= 1\,023\,547{,}306 \quad \rightarrow \text{MEM D}$$

$$D = \frac{x\left[1 - C^{-231}\right]}{C - 1}$$ SOLVE: Appendix 1
$$= x(99{,}0103\ldots)$$
$\therefore\ x =$ R10 337,78 per month

$\therefore$ Increase = **R337,78**

(c) $1 + \frac{0{,}115}{12} = \frac{2\,423}{2\,400}$ → MEM E

At T_{24}:

$$\text{O.B.} = \frac{10\,337{,}78\left[1 - C^{-216}\right]}{C - 1}$$
$$= 1\,001\,501{,}84 \quad \rightarrow \text{MEM F}$$

$$F = \frac{10\,337{,}78\left[1 - E^{-n}\right]}{E - 1}$$ SOLVE: Appendix 1

$\therefore\ 1 - E^{-n} = 0{,}92841\ldots$

$\therefore\ E^{-n} = 0{,}07158\ldots$

$\therefore\ -n = \log_E[ANS]$

$= -276{,}46\ldots$

Total no. of payments = 24 + 277 = 301

Extra months: 301 – 240 = 61

i.e. **5 years 1 month**

(d) Interest on O.B. for 1 month

$$= F\left(\frac{0{,}125}{12}\right) = 10\,432{,}31\ldots > 10\,337{,}78$$

Her monthly payments will not cover the interest charges, resulting in her outstanding balance becoming larger each month. She will never be able to pay off the loan, thus the bank cannot grant an extension.

3. $1 + \frac{0{,}115}{12} = \frac{2\,423}{2\,400}$ → MEM A

At T_0:

$$50\,000 = \frac{1\,000\left[1 - A^{-12}\right]}{A - 1}$$ SOLVE: Appendix 1
$$+ \frac{1\,200\left[1 - A^{-12}\right]}{A - 1}.A^{-12}$$
$$+ \frac{1\,500\left[1 - A^{-n}\right]}{A - 1}.A^{-24}$$

$$\therefore \frac{1\,500\left[1 - A^{-n}\right]}{A - 1}.A^{-24} = 26\,637{,}99637$$

$\therefore\ 1 - A^{-n} = 0{,}2139\ldots$

$\therefore\ A^{-n} = 0{,}786\ldots$

$\therefore\ -n = \log_A[ANS]$

$= -25{,}242\ldots$

$\therefore\ n = 25{,}242\ldots$

Total no. of payments = 24 + 26 = **40**

4. (a) $1 + \frac{0{,}09}{12} = \frac{403}{400}$ → MEM A

At T_{120}:

$$F = \frac{200\left[A^{48} - 1\right]}{A - 1}.A^{72} + \frac{500\left[A^{72} - 1\right]}{A - 1}$$
$\approx$ **R67 204,96**

(b) At T_{n+48}:

$$100\,000 = \frac{200\left[A^{48} - 1\right]}{A - 1}.A^{n} + \frac{500\left[A^{n} - 1\right]}{A - 1}$$ SOLVE: Appendix 1

$\therefore\ 100\,000(A - 1) = (86{,}281\ldots)A^n + 500A^n - 500$

$\therefore\ (586{,}628\,1\ldots)A^n = 1\,250$ ← MEM B

$\therefore\ A^n = 2{,}132\ldots$

$\therefore\ n = \log_A[ANS] = 101{,}324\ldots$

Total no. of payments = 48 + 102 = 150

i.e. It will take **150 months**

= 12 years 6 months

Exercise FIN 3.4

(Questions on Fin.42)

1. Fig. 1 - **(b)** Fig. 2 - **(d)**
   Fig. 3 - **(a)** Fig. 4 - **(c)**

2. $1 + \frac{0{,}075}{12} = \frac{161}{160}$ → MEM A

$1 + i_{eff} = A^{12} = 1{,}077\ldots$ → MEM B

At T_{120}: $F = \frac{500\left[A^{120} - 1\right]}{A - 1} + \frac{500\left[B^{10} - 1\right]}{B - 1}$

$\approx$ **R96 127,53**

3. $1 + \frac{0{,}21}{12} = \frac{407}{400}$ → MEM A

$$9\,500 = 1\,000A^{-24} + \frac{300\left[1 - A^{-n}\right]}{A - 1}$$

$\therefore\ 1 - A^{-n} = 0{,}515\ldots$ SOLVE: Appendix 1

$\therefore\ A^{-n} = 0{,}4843\ldots$

$\therefore\ -n = \log_A[ANS]$

$= -41{,}7928\ldots$

$\therefore$ **42 months**

MODELLING SOLUTIONS

CHAPTER 4: GR 10 MODELLING

Exercise MOD 4.1 (Questions on Mod.4)

1. (a) $T_{34} = 87$ (b) $T_2 = T_1 + 8$
   (c) $T_{13} = 2.T_{12} - 3$ (d) $T_n = 3.T_{n-1} + 1$

2. (a) 75, 70, 65, 60, 55
   $T_n = T_{n-1} - 5$, $T_1 = 75$
   (b) 11, 25, 53, 109, 221
   $T_n = 2.T_{n-1} + 3$, $T_1 = 11$
   (c) 144, 84, 54, 39, 31,5
   $T_n = \frac{1}{2}.T_{n-1} + 12$, $T_1 = 144$
   (d) –1, –3, –7, –15, –31
   $T_n = 2.T_{n-1} - 1$, $T_1 = -1$

3. (a) –6, 2, 10, 18, 24
   (b) 3, –11, 31, –95, 283
   (c) –8, 2, 7, 9,5, 10,75
   (d) 3, 2, 0, –4, –12
   (e) 24, –8, 0, –2, –1,5
   (f) 0, 5, 11, 18, 26

4. (a) $T_n = 3n - 1$ $T_n = T_{n-1} + 3$, $T_1 = 2$
   $T_{17} = 50$
   (b) $T_n = 4^n$ $T_n = 4.T_{n-1}$, $T_1 = 4$
   $T_{17} = 17\,179\,869\,184$
   (c) $T_n = (-3)^{n-1}$ $T_n = -3.T_{n-1}$, $T_1 = 1$
   $T_{17} = 43\,046\,721$
   (d) $T_n = 2{,}5.n - 6{,}5$ $T_n = T_{n-1} + 2{,}5$, $T_1 = -4$
   $T_{17} = 36$
   (e) $T_n = 5n - 4$ $T_n = T_{n-1} + 5$, $T_1 = 1$
   $T_{17} = 81$
   (f) $T_n = 5^{n-3}$ $T_n = 5.T_{n-1}$, $T_1 = 0{,}04$
   $T_{17} = 6\,103\,515\,625$

5. (a) $28 = 10k + c$ ❶
   $10 = 4k + c$ ❷
   $18 = 6k$ ❶ – ❷
   $\therefore k = 3$, $c = -2$
   $T_n = 3.T_{n-1} - 2$, $T_1 = 2$ $T_{12} = 177\,148$

   (b) $17 = 9k + c$ ❶
   $9 = 5k + c$ ❷
   $8 = 4k$ ❶ – ❷
   $\therefore k = 2$, $c = -1$
   $T_n = 2.T_{n-1} - 1$, $T_1 = 3$ $T_{12} = 4\,097$

   (c) $6 = 20k + c$ ❶
   $20 = 48k + c$ ❷
   $14 = 28k$ ❷ – ❶
   $\therefore k = \frac{1}{2}$, $c = -4$
   $T_n = \frac{1}{2}.T_{n-1} - 4$, $T_1 = 48$
   $T_{12} \approx -7{,}973$

   (d) $-228 = 60k + c$ ❶
   $60 = -12k + c$ ❷
   $288 = -72k$ ❷ – ❶
   $\therefore k = -4$, $c = 12$
   $T_n = -4.T_{n-1} + 12$, $T_1 = 6$
   $T_{12} = -15\,099\,492$

   (e) $17 = 33k + c$ ❶
   $33 = 81k + c$ ❷
   $16 = 48k$ ❷ – ❶
   $\therefore k = \frac{1}{3}$, $c = 6$
   $T_n = \frac{1}{3}.T_{n-1} + 6$, $T_1 = 225$ $T_{12} \approx 9{,}001$

   (f) $72 = -16k + c$ ❶
   $-16 = 6k + c$ ❷
   $88 = -22k$ ❶ – ❷
   $\therefore k = 4$, $c = 8$
   $T_n = -4.T_{n-1} + 8$, $T_1 = \frac{1}{2}$ $T_{12} = 4\,613\,736$

6. (a) (i) 6, 10, 14, 18, **22, 26**
   (ii) **$T_n = T_{n-1} + 4$, $T_1 = 6$**
   (iii) **$T_n = 4n + 2$**
   (b) (i) 3, 5, 7, 9, **11, 13**
   (ii) **$T_n = T_{n-1} + 2$, $T_1 = 3$**
   (iii) **$T_n = 2n + 1$**
   (c) (i) 5, 8, 11, **14, 17, 20**
   (ii) **$T_n = T_{n-1} + 3$, $T_1 = 5$**
   (iii) **$T_n = 3n + 2$**

7. (a) **$T_n = 1{,}2.T_{n-1}$, $T_1 = 50$**
   (b) **$T_7 = 149{,}2992$ mm**

8. (a) 2, 4, 8, … **$T_n = 2^n$**
   (b) $T_{12} = 2^{12} =$ **4 096**
   (c) **$T_n = 2.T_{n-1}$, $T_1 = 2$**
   (d) **$T_{10} = 1\,024$ $n = 10$**

CHAPTER 6: GR 12 MODELLING

Exercise MOD 6.1 (Questions on Mod.43)

1.1 annual cycle

$\boldsymbol{K = 25\,000}$

$c = \frac{1}{20} = \mathbf{0{,}05}$

$a = 0{,}5 \times 2 \times 0{,}6$ $\quad \boldsymbol{a = 0{,}6}$

$b.P_0.S_0 = 4\,500 = b(6\,000)(1\,500)$ $\quad \boldsymbol{b = 0{,}000\,5}$

$f.b.P_0.S_0 = 100 = f(0{,}000\,5)(6\,000)(1\,500)$

$\boldsymbol{f = 0{,}022}$

1.2 annual cycle

$\boldsymbol{K = 2 \times 500 = 1\,000}$

$c = \frac{1}{5} = \mathbf{0{,}2}$

$a = 0{,}65 \times (7 \times 1) \times 0{,}4$ $\quad \boldsymbol{a = 1{,}82}$

$b.B_0.S_0 = 36 \times 1 \times 52 = 1\,872$

$1\,872 = b(500)(36)$ $\quad \boldsymbol{b = 0{,}104}$

$f.b.B_0.S_0 = (0{,}5 \times 36) \times 6 = 108$

$108 = f(0{,}104)(500)(36)$ $\quad \boldsymbol{f = 0{,}058}$

1.3 annual cycle

$\boldsymbol{K = 45\,000}$

$c = \frac{1}{30} = \mathbf{0{,}033}$

$a = 0{,}65 \times 1 \times 0{,}7$ $\quad \boldsymbol{a = 0{,}455}$

$b.S_0.W_0 = 30 \times 1 \times 52 = 1\,560$

$1\,560 = b(30\,000)(30)$ $\quad \boldsymbol{b = 0{,}001\,733}$

$f.b.S_0.W_0 = (0{,}6 \times 30) \times \left(\frac{4}{3}\right) = 24$

$24 = f(0{,}001\,733)(30\,000)(30)$

$\boldsymbol{f = 0{,}015\,388}$

1.4 annual cycle

$\boldsymbol{K = 180\,000}$

$c = \frac{1}{12} = \mathbf{0{,}083}$

$a = 0{,}5 \times 1 \times 0{,}7$ $\quad \boldsymbol{a = 0{,}35}$

$b.I_0.D_0 = 350 \times 2 \times 12 = 8\,400$

$8\,400 = b(100\,000)(350)$ $\quad \boldsymbol{b = 0{,}000\,24}$

$f.b.I_0.D_0 = (0{,}5 \times 0{,}1 \times 350 \times 10) = 70$

$70 = f(0{,}000\,24)(100\,000)(350)$

$\boldsymbol{f = 0{,}083}$

1.5 two-year cycle

$\boldsymbol{K = 2 \times 500 = 1\,000}$

$c = \frac{2}{10} = \mathbf{0{,}2}$

$a = 0{,}3 \times (3 \times 3) \times 0{,}45$ $\quad \boldsymbol{a = 1{,}215}$

$b.D_0.E_0 = 30 \times 1 \times 24 = 720$

$720 = b(500)(30)$ $\quad \boldsymbol{b = 0{,}048}$

$f.b.D_0.E_0 = 20 = f(0{,}048)(500)(30)$

$\boldsymbol{f = 0{,}028}$

2.1 $28 <$ mouse < 70

2.2 equilibrium: amplitude of each cycle diminishes

2.3 about 35 months

2.4 no: initial drop is not that dramatic

3.1 amplitude of each cycle diminishes

3.2 160 to 200 months

3.3 about 35 months

3.4 deer (prey) population initially quite high (200) when compared to what it is tapering off to (60).

OR

Large amount of deer roaming around makes them ready prey for wolves.

3.5 $18 <$ predator < 48

4.1 badgers ≈ 12, squirrels ≈ 400

4.2 $8 <$ badgers < 20

4.3 about 370

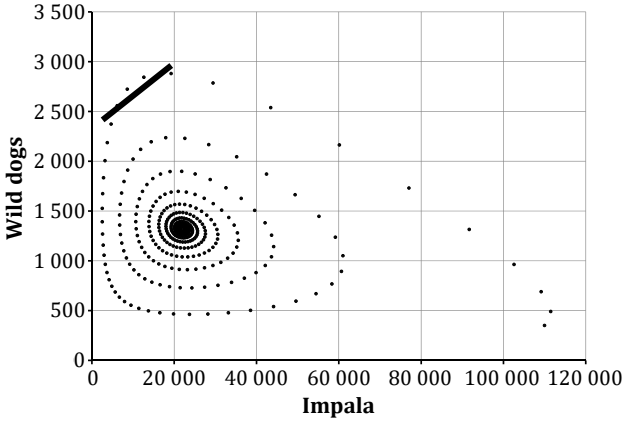
4.4 rapid initial drop (500 to 200) in squirrel population probably caused by great number of badgers, which remains constant for quite a while.

4.5 For equilibrium population for prey: $S_n = \frac{c}{f.b}$

$c = \frac{1}{\text{lifespan}}$ if lifespan increases, c decreases and hence $\boldsymbol{S_n}$ **decreases**.

5.1 $I_0 = 110\,000$ and $D_0 = 350$

5.2



CHAPTER 2: Gr 11 MATRICES

Exercise MAT 2.1 (Identifying Transformations)

(Questions on Mat.21)

1. A rotation of 270° $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

   B reflection across $y = x$ $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

   C shear with y-axis as invariant, by a factor of 2 $\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$

   D reflection across $x = 0$ $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

   E shear with x-axis as invariant, by a factor of 3 $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$

   F stretch with x-axis as invariant, by a factor of -3 $\begin{pmatrix} 1 & 0 \\ 0 & -3 \end{pmatrix}$

   G rotation of 90° anticlockwise $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

   H stretch with y-axis as invariant, by a factor of $-2{,}5$ $\begin{pmatrix} -2{,}5 & 0 \\ 0 & 1 \end{pmatrix}$

   I enlargement by a factor of 3 $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

   J enlargement by a factor of $-2{,}5$ $\begin{pmatrix} -2{,}5 & 0 \\ 0 & -2{,}5 \end{pmatrix}$

   K stretch with y-axis as invariant, by a factor of 2 $\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$

   L shear with y-axis as invariant, by a factor of -3 $\begin{pmatrix} 1 & 0 \\ -3 & 1 \end{pmatrix}$

   M enlargement by a factor of $\frac{1}{2}$ $\begin{pmatrix} 0{,}5 & 0 \\ 0 & 0{,}5 \end{pmatrix}$

   N rotation of 180° $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

   O reflection across $y = 0$ $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

   P stretch with x-axis as invariant, by a factor of 3 $\begin{pmatrix} 1 & 0 \\ 0 & 3 \end{pmatrix}$

   Q shear with x-axis as invariant, by a factor of -2 $\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$

   R reflection across $y = -x$ $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$

2. S translation 5 units left and 3 units up $\begin{pmatrix} -5 & -5 & -5 & -5 \\ +3 & +3 & +3 & +3 \end{pmatrix}$

   T translated 3 units left and 2 units down $\begin{pmatrix} -3 & -3 & -3 & -3 \\ -2 & -2 & -2 & -2 \end{pmatrix}$

   U translated 1 unit right and 6 units down $\begin{pmatrix} +1 & +1 & +1 & +1 \\ -6 & -6 & -6 & -6 \end{pmatrix}$

3. V translated 5 units right and 1 unit up $\begin{pmatrix} +5 & +5 & +5 & +5 \\ +1 & +1 & +1 & +1 \end{pmatrix}$

   then enlarged by a factor of 1 $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

   W translated 1 unit right $\begin{pmatrix} +1 & +1 & +1 & +1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   then enlarged by a factor of 2 $\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$

   X translated 2 units right and 3 units down $\begin{pmatrix} +2 & +2 & +2 & +2 \\ -3 & -3 & -3 & -3 \end{pmatrix}$

   then enlarged by a factor of 3 $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

Exercise MAT 2.2 (Calculating Transformations)

(Questions on Mat.23)

1. (a) $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} \mathbf{-3} & \mathbf{6} & \mathbf{0} \\ \mathbf{-3} & \mathbf{0} & \mathbf{9} \end{pmatrix}$ P'(−3; −3), Q'(6; 0) and R'(0; 9)

   (b) $\begin{pmatrix} 1 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} \mathbf{-1} & \mathbf{2} & \mathbf{0} \\ \mathbf{2} & \mathbf{0} & \mathbf{-6} \end{pmatrix}$ P'(−1; 2), Q'(2; 0) and R'(0; −6)

   (c) $\begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -1 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix} = \begin{pmatrix} \mathbf{-1} & \mathbf{2} & \mathbf{0} \\ \mathbf{-5} & \mathbf{8} & \mathbf{3} \end{pmatrix}$ P'(−1; −5), Q'(2; 8) and R'(0; 3)

2. (a) $\tan\theta = \sqrt{3}$ hence $\theta = 60°$

$$\begin{pmatrix}\cos 2(60°) & \sin 2(60°)\\ \sin 2(60°) & -\cos 2(60°)\end{pmatrix}\begin{pmatrix}\sqrt{3}\\ 1\end{pmatrix} = \begin{pmatrix}\mathbf{0}\\ \mathbf{2}\end{pmatrix}$$

$(\sqrt{3}; 1) \rightarrow (0; 2)$

(b) $\tan\theta = 0{,}5$ hence $\theta = 26{,}565°$

$$\begin{pmatrix}\cos 2(26{,}565°) & \sin 2(26{,}565°)\\ \sin 2(26{,}565°) & -\cos 2(26{,}565°)\end{pmatrix}\begin{pmatrix}3\\ -1\end{pmatrix} = \begin{pmatrix}\mathbf{1}\\ \mathbf{3}\end{pmatrix}$$

$(3; -1) \rightarrow (1; 3)$

3. (a) $$\begin{pmatrix}\cos 53{,}13° & -\sin 53{,}13°\\ \sin 53{,}13° & \cos 53{,}13°\end{pmatrix}\begin{pmatrix}10\\ 15\end{pmatrix} = \begin{pmatrix}\mathbf{-6}\\ \mathbf{17}\end{pmatrix}$$

$(10; 15) \rightarrow (-6; 17)$

(b) $$\begin{pmatrix}\cos 30° & -\sin 30°\\ \sin 30° & \cos 30°\end{pmatrix}\begin{pmatrix}\sqrt{3}\\ -2\end{pmatrix} = \begin{pmatrix}\mathbf{2{,}5}\\ \mathbf{-0{,}87}\end{pmatrix}$$

$(\sqrt{3}; -2) \rightarrow (2{,}5; -0{,}87)$

4. (a) $$\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}\begin{pmatrix}13 & -13 & -52\\ 0 & -26 & 39\end{pmatrix} = \begin{pmatrix}\mathbf{13} & \mathbf{-13} & \mathbf{-52}\\ \mathbf{26} & \mathbf{-52} & \mathbf{-65}\end{pmatrix}$$

(b) $$\begin{pmatrix}\cos 22{,}62° & -\sin 22{,}62°\\ \sin 22{,}62° & \cos 22{,}62°\end{pmatrix}\begin{pmatrix}13 & -13 & -52\\ 0 & -26 & 39\end{pmatrix} = \begin{pmatrix}\mathbf{12} & \mathbf{-2} & \mathbf{-63}\\ \mathbf{5} & \mathbf{-29} & \mathbf{16}\end{pmatrix}$$

(c) $\tan\theta = 0{,}5$ so, $\theta = 26{,}565°$

$$\begin{pmatrix}\cos 2(26{,}565°) & \sin 2(26{,}565°)\\ \sin 2(26{,}565°) & -\cos 2(26{,}565°)\end{pmatrix}\begin{pmatrix}13 & -13 & -52\\ 0 & -26 & 39\end{pmatrix}$$

$$= \begin{pmatrix}\mathbf{7{,}8} & \mathbf{-28{,}6} & \mathbf{0}\\ \mathbf{10{,}4} & \mathbf{5{,}2} & \mathbf{-65}\end{pmatrix}$$

5. (a) $$\begin{pmatrix}1 & 0\\ 0 & \frac{1}{3}\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix} = \begin{pmatrix}\mathbf{3} & \mathbf{0}\\ \mathbf{0} & \mathbf{1}\end{pmatrix}$$

Note the order of matrix multiplication is significant.

(b) $$\begin{pmatrix}3 & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}-1 & -4 & -3 & 0\\ 3 & 4 & 1 & 0\end{pmatrix} = \begin{pmatrix}\mathbf{-3} & \mathbf{-12} & \mathbf{-9} & \mathbf{0}\\ \mathbf{3} & \mathbf{4} & \mathbf{1} & \mathbf{0}\end{pmatrix}$$

6. $$\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} = \begin{pmatrix}\mathbf{2} & \mathbf{1}\\ \mathbf{1} & \mathbf{0}\end{pmatrix}$$

Note the order of matrix multiplication is significant.

7. $\tan\theta = \sqrt{3}$ hence $\theta = 60°$

$$\begin{pmatrix}\cos 2(60°) & \sin 2(60°)\\ \sin 2(60°) & -\cos 2(60°)\end{pmatrix}\begin{pmatrix}-2 & 0\\ 0 & -2\end{pmatrix} = \begin{pmatrix}\mathbf{1} & \mathbf{-1{,}732}\\ \mathbf{-1{,}732} & \mathbf{-1}\end{pmatrix}$$

8. (a) $$\begin{pmatrix}1{,}8 & -2{,}4\\ 2{,}4 & 1{,}8\end{pmatrix}\begin{pmatrix}3 & -2\\ 0 & 5\end{pmatrix} = \begin{pmatrix}\mathbf{5{,}4} & \mathbf{-15{,}6}\\ \mathbf{7{,}2} & \mathbf{4{,}2}\end{pmatrix}$$

(b) $$T^{-1} = \frac{\mathbf{1}}{\mathbf{9}}\begin{pmatrix}\mathbf{1{,}8} & \mathbf{2{,}4}\\ \mathbf{-2{,}4} & \mathbf{1{,}8}\end{pmatrix}$$

(c) $$\frac{1}{9}\begin{pmatrix}1{,}8 & 2{,}4\\ -2{,}4 & 1{,}8\end{pmatrix}\begin{pmatrix}15\\ -30\end{pmatrix} = \begin{pmatrix}\mathbf{-5}\\ \mathbf{-10}\end{pmatrix}$$

(d) $$\begin{pmatrix}\cos 53{,}1° & -\sin 53{,}1°\\ \sin 53{,}1° & \cos 53{,}1°\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & 3\end{pmatrix} = \begin{pmatrix}\mathbf{1{,}8} & \mathbf{-2{,}4}\\ \mathbf{2{,}4} & \mathbf{1{,}8}\end{pmatrix}$$

9. (a) $\cos\theta = 0{,}6$ so $\theta = 53{,}13°$

(b) $x = -\sin 53{,}13° = -0{,}8$ $y = \sin 53{,}13° = 0{,}8$ $z = 0{,}6$

(c) $$\begin{pmatrix}0{,}6 & -0{,}8\\ 0{,}8 & 0{,}6\end{pmatrix}\begin{pmatrix}-5\\ 10\end{pmatrix} = \begin{pmatrix}\mathbf{-11}\\ \mathbf{2}\end{pmatrix}$$

10. $$\begin{pmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{pmatrix}\begin{pmatrix}-10\\ 30\end{pmatrix} = \begin{pmatrix}-18\\ -26\end{pmatrix}$$

$-10\cos\alpha - 30\sin\alpha = -18$ and $30\cos\alpha - 10\sin\alpha = -26$

$\therefore \cos\alpha = -3/5$ $\sin\alpha = 4/5$

$\therefore \alpha = 180° - 53{,}13° = \mathbf{126{,}87°}$

11. $\cos\alpha = -\sqrt{3}/2$ so angle lies in quad II or III

$\sin\alpha = +1/2$ so angle lies in quad I or II

$\therefore \boldsymbol{\alpha} = 180° - 30° \mathbf{= 150°}$

12. $\cos 2\alpha = 0{,}8$ so angle lies in quad I or IV

$\sin 2\alpha = 0{,}6$ so angle lies in quad I or II

$2\alpha = 36{,}87°$ and so $\boldsymbol{\alpha = 18{,}435°}$

13. $$\begin{pmatrix}\cos 2\alpha & \sin 2\alpha\\ \sin 2\alpha & -\cos 2\alpha\end{pmatrix}\begin{pmatrix}3\\ -2\end{pmatrix} = \begin{pmatrix}-0{,}232\\ 3{,}6\end{pmatrix}$$

$3\cos 2\alpha - 2\sin 2\alpha = 0{,}232$ and $2\cos 2\alpha + 3\sin 2\alpha = 3{,}6$

$\cos 2\alpha = 0{,}500$ so $\boldsymbol{\alpha = 29{,}99°}$

OR $\sin 2\alpha = 0{,}866$ so $\boldsymbol{\alpha = 30{,}03°}$

difference due to irrational rounding

CHAPTER 6: Gr 12 GRAPH THEORY

Exercise GT 6.1

(Questions on GT.25)

1. (a) **A D C E G J H I F B A** (many other options)

   (b) DF (19), DH (22), AB (25), FI (27), IJ (29), EG (30), DC (38), CE (57), AD (65) = **312 m**

2. (a) RS (2), QP (3), PW / SV / SU / TU (4) (any order), VW (6) = **R27 000**

   (b) WP (4), PQ (3), WV (6), SV (4), SR (2), SU (4), UT (4) = **R27 000**

3. (a) by choosing all three edges of weight 5, he has created a circuit CBGF
   Vertex A is not linked to the tree

   (b) CD (3), CB (4), CF / FG / BG (5) (any two of the edges), DE (7), HG (8), BA (13) = **45 units**

4. (a) DG (40) + GE (45) + EH (50) + CD (80) + BF (80) + CF (85)
   380 + 225 **= 605 units**

   (b) BF (80) + FC (85) + CD (80) + DG (40) + GE (45) + EH (50) + HGA (215) + AB (150) = **745 units**

   (c) B F C E H G D A B = **700 units**
   OR
   B F H E G D C A B = **725 units**
   OR
   any other route between 515 and 745

5. (a) AD = 5, CFE = 14 72 + 19 = 91 hours

   (b) AD, AE, AB, BC, 5 + 5 = **34 hours**

   (c) Yes, the LB can change if the initial vertex left out is changed.

   (d) A D F E C B A = **43**
   OR
   A F D E C B A = **47** (both acceptable)

   (e) A D E F C B A = **40** (any route between 34 and 43 fine)

6. (a) GE (4), GD (8), GJ (9), JH (6), JK (8), FE (10), AD (12), AB (10), CF (14) = 81 units = **R81 000**

   (b) EG (4), GD (8), DH (10), HJ (6), JK (8), KF (12), FC (14), CA (15), AB (10), BE (25) = 112 units = **112 minutes**

   (c) is quite feasible that a much better "good" route exists
   The first half of the UB (36) is substantially shorter than the second half (76)

7. (a) DE (4), AB (5)/JE (5), JG (7)/EF (7), HG (8), BD (9)/CD (9), KB (6)/KJ (6) = 54 + 12 = **66 units**

   (b) AB (5), BK (6), KJ (6), JE (5), ED (4), DC (9), CKH (15), HG (8), GF (9), FEDBA (25) = **92 units**

   (c) A, B, K, C, D, E, J, F, G, H, A = **74 units**
   OR
   A, B, C, D, E, F, G, J, K, H, A = **74 units**
   OR
   Many other options

Gr 12 Graph Theory Exam Solutions

(Questions on GT.28)

1. (a) Dijkstra (b) Prim
   (c) Fleury (d) Nearest Neighbour

2. (a) 9 edges (b) Kruskal: shortest edges in graph are being chosen
   (c) 6 (d) JE: will complete a mini-circuit
   (e) DE, AB, IJ, DI **EF / JG, HG / CI, BI = 60**

3. (a) PU + UQ < PQ (b) VW
   (c) triangular inequality does not hold
   (d) W V R Q U P S T W or W V R U Q P S T W
   (e) PSTW = 15
   $2 + 3 + x + 3 + 2 < 15$ $x < 5$ so $\boldsymbol{x = 4}$