2023

PATTERNS & SEQUENCES

Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES *Your Key to Exam Success*

PATTERNS & SEQUENCES (63%): DBE NOV. 2023

QUESTION 2 72%

2.1 Given the arithmetic series: 7 + 12 + 17 + . . .

89%

2.1.1 Determine the value of T_{91} (3)

2.1.2 Calculate S_{91} (2)

2.1.3 Calculate the value of n for which $T_n = 517$ (3)

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Memo

2.1 **A.S.**: 7 + 12 + 17 + . . .

2.1.1 **a** = 7 ; **d** = 5 ; **T_{91}?** ; **n** = 91

$$\mathbf{T_n = a + (n-1)d}$$

$$\therefore T_{91} = 7 + (91-1)(5)$$
$$= \mathbf{457} \;<$$

OR: General term, $\mathbf{T_n = an + b}$
where **a** = 5 and **b** = 2
$\therefore T_{91} = 5(91) + 2$
$= \mathbf{457} \;<$

2.1.2 $\mathbf{S_n = \frac{n}{2}(a + T_n)}$

$$\therefore S_{91} = \frac{91}{2}(7 + T_{91})$$
$$= \frac{91}{2}(7 + 457)$$
$$= \mathbf{21\ 112} \;<$$

OR: $\mathbf{S_n = \frac{n}{2}[2a + (n-1)d]}$
$\therefore S_{91} = \frac{91}{2}[2(7) + (91-1)(5)]$
$= \mathbf{21\ 112} \;<$

2.1.3 **n?** ; $\mathbf{T_n}$ = 517

$\mathbf{T_n = a + (n-1)d}$ ➡ $517 = 7 + (n-1)(5)$
$\therefore 517 = 7 + 5n - 5$
$\therefore 515 = 5n$
$\therefore \mathbf{n = 103} \;<$

OR: $\mathbf{T_n = an + b}$
$\therefore 517 = 5n + 2$
$\therefore 515 = 5n$
$\therefore \mathbf{n = 103} \;<$

Common Errors and Misconceptions

(a) The most common errors in **Q2.1** and its sub-questions were the use of **incorrect formulae** (i.e. $T_n = a(n-1)d$ rather than $T_n = a + (n-1)d$) or **incorrect substitution** of n as T_n or S_n.

(b) Many candidates substituted n = 5 into the given T_n formula in **Q2.1.2** to answer **Q2.1.1** rather than developing ***the quadratic pattern***. The candidates knew how to develop the pattern as this was well done in **Q2.1.2**, when asked to show that the *quadratic pattern* had a general term of $T_n = 6n^2 - 9n + 6$, but the **link** was not made **between** determining **the formula** of a *quadratic pattern* **and generation** of the *quadratic pattern*.

55%

2.2 The following information is given about a quadratic number pattern:

$T_1 = 3, \quad T_2 - T_1 = 9 \quad \text{and} \quad T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is

$T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)

[16]

Memo

2.2.1 General form: $\mathbf{T_n = an^2 + bn + c}$

$T_1 = 3 \ ; \ T_2 - T_1 = 9 \ \text{and} \ T_3 - T_2 = 21$

∴ The pattern:	3		12		33		66		111
1st differences:		9		21		33		45	
2nd differences:			12		12		12		

$\mathbf{T_5 = 66 + 45 = 111}$ ✓

2.2.2 $\mathbf{T_1} = a + b + c = 3$

$\mathbf{2a} = 12$ . . . *Common 2nd difference*

$\therefore \mathbf{a} = 6$

& $\mathbf{3a + b} = 9$ . . . *The first 1st difference*

$\therefore 18 + b = 9$

$\therefore \mathbf{b} = -9$

$\therefore 6 + (-9) + c = 3$

$\therefore \mathbf{c} = 6$

$\therefore \mathbf{T_n = 6n^2 - 9n + 6}$ ✓

2.2.3 $\dfrac{dT_n}{dn} = 12n - 9$

If $n > \frac{3}{4}$ $\quad 12n - 9 > 0$

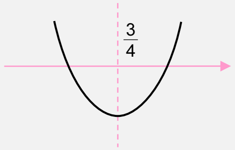
Since $n \in \mathbb{N}$ the derivative will always be positive

$\therefore$ **T_n is always increasing for all $n \in \mathbb{N}$** ✓

OR: $T_n = 6n^2 - 9n + 6$

A of S: $n = -\dfrac{-9}{2(6)}$

$= \dfrac{3}{4}$



$\therefore$ **T_n is always increasing for all $n \in \mathbb{N}$** ✓

OR:

$T_{n+1} - T_n = 6(n+1)^2 - 9(n+1) + 6 - (6n^2 - 9n + 6)$

$= 6(n^2 + 2n + 1) - 9n - 9 + 6 - 6n^2 + 9n - 6$

$= 6n^2 + 12n + 6 - 9 - 6n^2$

$= 12n - 3$

Since $n \geq 1$ and a whole number

. . . *since n = the number of terms*

$12n - 3$ has a minimum value of $12(1) - 3 = 9$, which is positive

$\therefore$ **The difference between consecutive terms T_n and T_{n+1} will always be positive.**

$\therefore$ The pattern is increasing for all $n \in \mathbb{N}$ ✓

Common Errors and Misconceptions

(c) In **Q2.2.3** most candidates **could not show** that the pattern was increasing for all *natural numbers*.

They were **not able to manipulate the *quadratic pattern* as a *quadratic function*** to provide the argument of where the ***axis of symmetry*** of the pattern was.

QUESTION 2: Suggestions for Improvement

(a) Learners must be made aware of **which formulae** on the **information sheet** apply to which **type** of sequence. It is good practice for them to use the information sheet in class so that they become familiar with it.

(b) Teach learners **how to identify whether** the question requires them to calculate the value of the n^{th} term (or) the sum of the first n terms.

(c) Questions must be **read carefully** so that the learners know what is required of them.

(d) Learners should be discouraged from using information provided in later questions to answer earlier questions in an examination. Learners must be encouraged to develop patterns using their properties rather than using their explicit *general terms*. The understanding of where a pattern 'starts' and what the pattern 'does' is important to emphasise. The **basic diagram** was sufficient in answering Q2.2.1.

i.e. 3 12 33 66 111

9 21 33 45

12 12 12

(e) Teachers need to specifically teach the **relationships** between **a *quadratic pattern*** and **a *parabola***, making particular reference to the ***axis of symmetry*** and ***minimum*** or ***maximum*** values of the pattern.

QUESTION 3   48%

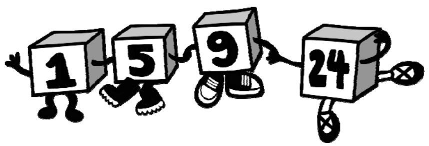
3.1 Given the geometric series: 3 + 6 + 12 + . . . to n terms.

60%

3.1.1 Write down the general term of this series. (1)

3.1.2 Calculate the value of k such that:

$$\sum_{p=1}^{k} \frac{3}{2}(2)^{p} = 98\ 301 \qquad (4)$$



Memo

3.1 **G.S.:** 3 + 6 + 12 + . . . to n terms

3.1.1 $\mathbf{a} = 3$; $\mathbf{r} = 2$

$T_n = ar^{n-1}$ ➡ $T_n = 3.2^{n-1}$ ✓

3.1.2 $\sum_{p=1}^{k} \frac{3}{2}(2)^{p} = 98\ 301$ . . . *i.e.* $\mathbf{S_k} = 98\ 301$

$$\text{LHS} = \frac{3}{2}(2)^1 + \frac{3}{2}(2)^2 + \frac{3}{2}(2)^3 + \ldots + \frac{3}{2}(2)^k$$

$$= 3 + 6 + 12 + \ldots + \frac{3}{2}(2)^k$$

Sum, $\mathbf{S_n}$, of a G.S. with $\mathbf{a} = 3$; $\mathbf{r} = 2$; $\mathbf{n} = k$

$$S_n = \frac{\mathbf{a(r^n - 1)}}{\mathbf{r - 1}}$$

$$S_k = \frac{3(2^k - 1)}{2 - 1}$$

$$= 3(2^k - 1)$$

$\therefore\ 3(2^k - 1) = 98\ 301$

$\therefore\ 2^k - 1 = 32\ 767$

$\therefore\ 2^k = 32\ 768$

$\therefore\ k = \log_2 32\ 768$ . . . *OR, by inspection*

$\therefore\ \mathbf{k = 15}$ ✓

Common Errors and Misconceptions

(a) Candidates were correctly able to determine the general term, however, most candidates **incorrectly simplified** $\mathbf{T_n = 3.2^{n-1}}$ **to** $\mathbf{T_n = 6^{n-1}}$. This did not impact the candidates in Q3.1.1 but did impact the candidates' ability to correctly solve **Q3.1.2**.

(b) Many candidates did not make **the link** between **n** and **k** in the ***sigma notation*** in **Q3.1.2**. The candidates also showed little understanding of how to generate a *series* from ***sigma notation*** to identify the **a** and the **r** values. Further to this, some candidates substituted the calculated information into $\mathbf{T_n}$ rather than $\mathbf{S_n}$.

3.2 A geometric sequence and an arithmetic sequence have the same first term.

35%

- The common ratio of the geometric sequence is $\frac{1}{3}$
- The common difference of the arithmetic sequence is 3
- The sum of 22 terms of the arithmetic sequence is 734 more than the sum to infinity of the geometric sequence.

Calculate the value of the first term. (5)

[10]

Memo

3.2

Arithmetic Sequence

1st term: $T_1 = x$

$d = 3$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{22} = \frac{22}{2}[2x + (22-1)(3)]$

$= 11(2x + 63)$

$= 22x + 693$

Geometric Sequence

$T_1 = x$

$r = \frac{1}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{x}{1-\frac{1}{3}}$

$= \frac{3x}{3-1}$

$= \frac{3x}{2}$

S_{22} of the A.S. $=$ S_∞ of the G.S. $+ 734$

$\therefore\ 22x + 693 = \frac{3x}{2} + 734$

$(\times 2)\quad \therefore\ 44x + 1\,386 = 3x + 1\,468$

$\therefore\ 41x = 82$

$\therefore\ x = 2$

i.e. $T_1 = 2$ ◄

Common Errors and Misconceptions

(c) Most candidates were unable to answer **Q3.2** correctly. The interpretation of *'The sum of 22 terms of the arithmetic series is 734 more than the sum to infinity of the geometric sequence'* was **incorrectly translated** to $S_{22} = 734$.

QUESTION 3: Suggestions for Improvement

(a) Teachers should emphasise the **differences between** the ***term***, ***sum*** and ***sum to infinity*** formulae in *arithmetic* and *geometric patterns*.

(b) The inclusion of word problems in the *patterns* section is important. Teachers need to emphasise how to take the words of a problem and write it in symbolic form to solve an *equation* or *inequality*.

(c) Constant revision of ***exponential laws*** to solve equations correctly is pivotal to candidates' success. Teachers need to emphasise this and revise this thoroughly in all grades.