2023 TRIGONOMETRY

Questions ○ Memos ○ Diagnostic Report

THE ANSWER SERIES
Your Key to Exam Success

QUESTION 5 42%

5.1 Given: $\sin\beta = \frac{1}{3}$, where $\beta \in (90°; 270°)$.
68%

Without using a calculator, determine each of the following:

5.1.1 $\cos\beta$ (3)

5.1.2 $\sin 2\beta$ (3)

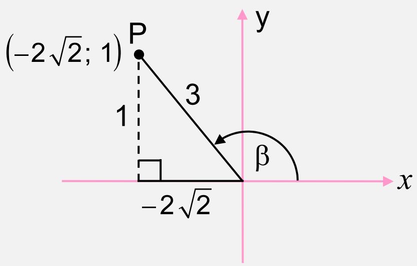
MEMOS

5.1.1 $x_P = -\sqrt{9-1}$

$= -\sqrt{8}$

$= -2\sqrt{2}$

$\therefore \cos\beta = \frac{x}{r} = -\frac{2\sqrt{2}}{3}$ ✓



5.1.2 $\sin 2\beta = 2\sin\beta\cos\beta = 2\left(\frac{1}{3}\right)\left(-\frac{2\sqrt{2}}{3}\right)$

$= -\frac{4\sqrt{2}}{9}$ ✓

Common Errors and Misconceptions

(a) Many candidates were unable to identify the quadrant correctly, and therefore used $x = +2\sqrt{2}$ instead of $x = -2\sqrt{2}$ throughout **Q5.1**. Some candidates ignored the instruction to not use a calculator and gave decimal answers to the trigonometric ratios. They were penalised for this.

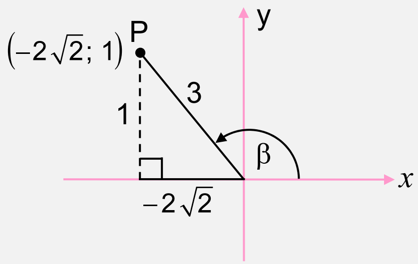
(b) In **Q5.1.2** some candidates were unable to write the expansion for sin 2β correctly despite it being given in the information sheet. Instead they **incorrectly** wrote the expansion for **sin 2**β as **2 sin** β.

QUESTION 5 (cont.)

5.1.3 cos(450º – β) (3)

MEMOS

5.1.3 **Method 1**



$\cos(450^\circ - \beta) = \cos(90^\circ - \beta)$

$= \sin\beta$

$= \mathbf{\frac{1}{3}}$ ✓

Method 2

$\cos(450^\circ - \beta) = \cos 450^\circ \cos\beta + \sin 450^\circ \sin\beta$

$= \cos 90^\circ \cos\beta + \sin 90^\circ \sin\beta$

$= (0)(\cos\beta) + (1)\sin\beta$

$= \sin\beta$

$= \mathbf{\frac{1}{3}}$ ✓

Common Errors and Misconceptions

(c) When answering **Q5.1.3** some candidates were not able to reduce cos(450º – β) correctly to sin β, i.e. they were unable to deal correctly with an angle greater than 360º as well as a co-ratio.

A common **incorrect** response that showed a lack of understanding of compound angles was:

$\cos(450^\circ - \beta)$

$= \cos 450^\circ - \cos\beta$

$= 0 - \cos\beta$

$= -\cos\beta$

QUESTION 5 (cont.)

5.2 Given: $\dfrac{\cos^4 x + \sin^2 x . \cos^2 x}{1 + \sin x}$

38%

5.2.1 Prove that $\dfrac{\cos^4 x + \sin^2 x . \cos^2 x}{1 + \sin x} = 1 - \sin x$ (4)

MEMOS

5.2.1 $\dfrac{(\cos^2 x)^2 + \sin^2 x \cos^2 x}{1 + \sin x}$

$= \dfrac{\cos^2 x(\cos^2 x + \sin^2 x)}{1 + \sin x}$

$= \dfrac{(1 - \sin^2 x)(1)}{1 + \sin x}$

$= \dfrac{(1 + \sin x)(1 - \sin x)}{1 + \sin x}$

$= \mathbf{1 - \sin x}$ ✓

Common Errors and Misconceptions

(d) **Q5.2.1** was poorly answered by many candidates as they failed to realise that $\cos^4 x + \sin^2 x . \cos^2 x$ could be factorised. Some candidates **flouted** a very **basic rule of Algebra** by cancelling terms of an expression: $\dfrac{1 - \sin^2 x}{1 + \sin x} = 1 - \sin x$.

Although these candidates arrived at the correct answer, they were not awarded any marks.

Instead, they should have factorised the numerator and then cancelled factors as shown:

$$\frac{1 - \sin^2 x}{1 + \sin x} = \frac{(1 - \sin x)(1 + \sin x)}{1 + \sin x} = 1 - \sin x.$$

QUESTION 5 (cont.)

5.2.2 For what value(s) of x in the interval $x \in [0°; 360°]$ is

$\dfrac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ undefined? (2)

5.2.3 Write down the minimum value of the function defined by

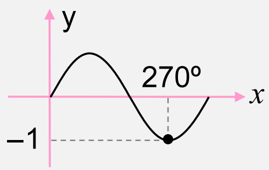
$y = \dfrac{\cos^4 x + \sin^2 x \cdot \cos^2 x}{1 + \sin x}$ (2)

MEMOS

5.2.2 Undefined when $1 + \sin x = 0$

$\therefore \sin x = -1$

$\therefore$ $\boldsymbol{x = 270°}$ ✓



5.2.3 $-1 \leq \sin x \leq 1$

The minimum value of $1 - \sin x$

$= 1 - 1$

$= \mathbf{0}$ ✓

[The minimum occurs when $\sin x = 1$]

Common Errors and Misconceptions

(e) In **Q5.2.2** many candidates left their answers as a general solution, instead of the specific solution in the given interval. Some candidates included the reference angle of 90º as a solution.

(f) Many candidates did not respond to **Q5.2.3** because they could not link the given expression to the sine graph.

QUESTION 5 (cont.)

5.3 Given: $\cos(A - B) = \cos A \cos B + \sin A \sin B$

23%

5.3.1 Use the above identity to deduce that
$\sin(A - B) = \sin A \cos B - \cos A \sin B$ (3)

MEMOS

5.3.1
$$\begin{aligned}\sin(A - B) &= \cos[90^\circ - (A - B)] \\ &= \cos[(90^\circ - A) - (-B)] \\ &= \cos(90^\circ - A)\cos(-B) + \sin(90^\circ - A)\sin(-B) \\ &= \sin A \cos B + \cos A(-\sin B) \\ &= \mathbf{\sin A \cos B - \cos A \sin B} \ \prec\end{aligned}$$

Common Errors and Misconceptions

(g) In **Q5.3.1** almost all candidates were unable to use the expansion for cos (A – B) to derive the expansion for sin (A – B). They had no idea how to begin with this derivation.

5.3.2 Hence, or otherwise, determine the general solution of the equation $\sin 48^\circ \cos x - \cos 48^\circ \sin x = \cos 2x$ (5)

MEMOS

5.3.2 From 5.3.1 $\sin A \cos B - \cos A \sin B = \sin(A - B)$

$\therefore \sin 48^\circ \cos x - \cos 48^\circ \sin x = \sin(48^\circ - x)$

$$\begin{aligned}\therefore \sin(48^\circ - x) &= \cos 2x \\ &= \sin(90^\circ - 2x)\end{aligned}$$

$\therefore 48^\circ - x = 90^\circ - 2x + n360^\circ$

$\therefore$ $\mathbf{x = 42^\circ + n360^\circ;\ n \in \mathbb{Z}}$ $\prec$

OR:

$$\begin{aligned}48^\circ - x &= 180^\circ - (90^\circ - 2x) + n360^\circ \\ \therefore 48^\circ - x &= 90^\circ + 2x + n360^\circ \\ \therefore -3x &= 42^\circ + n360^\circ \\ \div(-3)\ \therefore x &= \mathbf{-14^\circ + n120^\circ;\ n \in \mathbb{Z}} \ \prec\end{aligned}$$

Common Errors and Misconceptions

(h) In **Q5.3.2** some candidates incorrectly considered $\sin 48^\circ \cos x - \cos 48^\circ \sin x$ to be the expansion for the cosine compound angle instead of the sine compound angle. This lead to the incorrect general solution for x.

QUESTION 5 (cont.)

5.4 Simplify $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$ to a single trigonometric ratio. (6)

33%

[31]

Common Errors and Misconceptions

(i) The candidates' response to **Q5.4** was poor.

Some candidates started their responses incorrectly by indicating that sin $3x$ was equal to sin $2x$ + sin x.

Other candidates **incorrectly** factorised the numerator as:

$$\frac{\sin 3x + \sin x}{\cos 2x + 1}$$

$$= \frac{\sin x(\sin 2x + 1)}{\cos 2x + 1}$$

Candidates also failed to choose the appropriate expansion for cos $2x$. Consequently, they were unable to simplify the expression to a single trigonometric ratio.

MEMOS

5.4 $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \dfrac{\sin(2x + x) + \sin x}{2\cos^2 x - 1 + 1}$

$= \dfrac{\sin 2x \cos x + \cos 2x \sin x + \sin x}{2\cos^2 x}$

$= \dfrac{2\sin x \cos^2 x + (2\cos^2 x - 1)\sin x + \sin x}{2\cos^2 x}$

$= \dfrac{2\sin x \cos^2 x + 2\sin x \cos^2 x - \sin x + \sin x}{2\cos^2 x}$

$= \dfrac{4\sin x \cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin x}$ ✓

OR: $\dfrac{\sin 3x + \sin x}{\cos 2x + 1}$

$= \dfrac{\sin(2x + x) + \sin(2x - x)}{2\cos^2 x - 1 + 1}$

$= \dfrac{\sin 2x \cos x + \cos 2x \sin x + \sin 2x \cos x - \cos 2x \sin x}{2\cos^2 x}$

$= \dfrac{2\sin 2x \cos x}{2\cos^2 x}$

$= \dfrac{2(2\sin x \cos x)\cos x}{2\cos^2 x}$

$= \dfrac{4\sin x \cos^2 x}{2\cos^2 x}$

$= \mathbf{2\sin x}$ ✓

QUESTION 5: Suggestions for Improvement

(a) Learners find it difficult to recall the Trigonometry taught in Grades 10 and 11. Teachers should ensure that all learners are able to select the relevant quadrant when drawing sketches in the Cartesian plane to calculate trigonometric ratios.

(b) Remind learners that the same simplification **skills** used in **Algebra** also apply to **Trigonometry**. Revise addition, subtraction and simplification of algebraic fractions with learners before teaching simplification of trigonometric expressions and proving trigonometric identities.

(c) Expose learners to questions on trigonometric ratios, involving combinations of compound angles, angles greater than 360º and co-ratios.

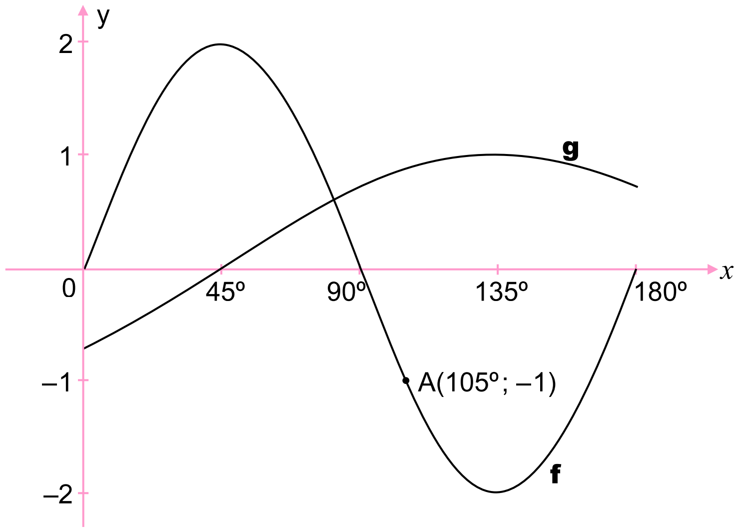
(d) Learners should be encouraged to use sketch graphs of sin x and cos x when solving equations where either of these ratios is equal to 1, 0 or –1.

(e) Learners should be given exercises to practise simplifying complex trigonometric expressions, proving identities and solving complex trigonometric equations.

QUESTION 6 34%

In the diagram, the graphs of $f(x) = 2 \sin 2x$ and $g(x) = -\cos(x + 45°)$ are drawn for the interval $x \in [0°; 180°]$.

A(105°; –1) lies on f.

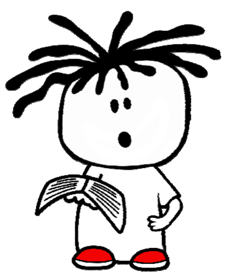


6.1 Write down the period of f. (1)

71%

6.2 Determine the range of g in the interval $x \in [0°; 180°]$ (2)

26%



MEMOS

6.1 **180°** ◂ . . . $\frac{1}{2} \times 360°$

6.2 y-int: Subt. $x = 0$ in $y = -\cos(x + 45°)$

$\therefore\ y = -\cos 45°$

$\therefore\ y = -\frac{1}{\sqrt{2}}$

$\therefore$ The range of g: $\mathbf{-\frac{1}{\sqrt{2}} \leq y \leq 1}$ ◂

Common Errors and Misconceptions

(a) In **Q6.1** some candidates gave the domain (which is **an interval**) instead of the period (which is **a single value**). Many candidates incorrectly divided 180° by 2 instead of dividing 360° by 2.

(b) In answering **Q6.2** many candidates ignored the domain specified in the question. They gave the range as [–1 ; 1] instead of $\left[-\frac{\sqrt{2}}{2}; 1\right]$.

Some candidates incorrectly excluded the extremities of the interval.

QUESTION 6 (cont.)

6.3 Determine the values of x, in the interval $x \in [0°; 180°]$, for which:

29%

6.3.1 $f(x).g(x) > 0$ (2)

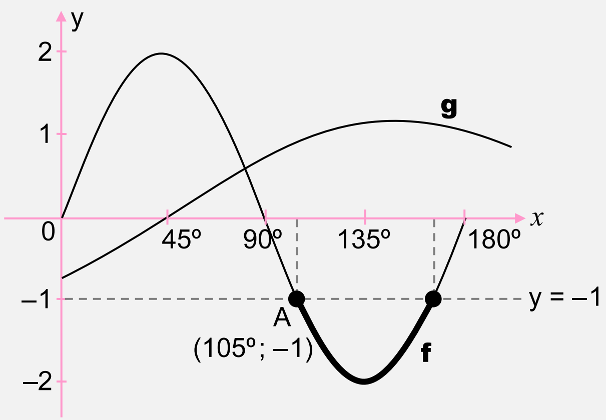
6.3.2 $f(x) + 1 \leq 0$ (2)

MEMOS

6.3.1 **$45° < x < 90°$** ✓ . . . *both graphs have the same sign*

6.3.2 $f(x) \leq -1$

$\therefore$ **$105° \leq x \leq 165°$** ✓ . . . *by symmetry*



Common Errors and Misconceptions

(c) **Q6.3.1** is a familiar question, yet many candidates failed to respond to this question. Some included the endpoints of the interval not realising that at the endpoints the two functions are equal. In this instance, their answer was [45°; 90°] instead of **(**45°; 90°**)**. Some candidates showed a lack of understanding of how to write an interval as an inequality. They **incorrectly gave** the answer as $x > 45°$ or $x < 90°$ **instead of $45° < x < 90°$**.

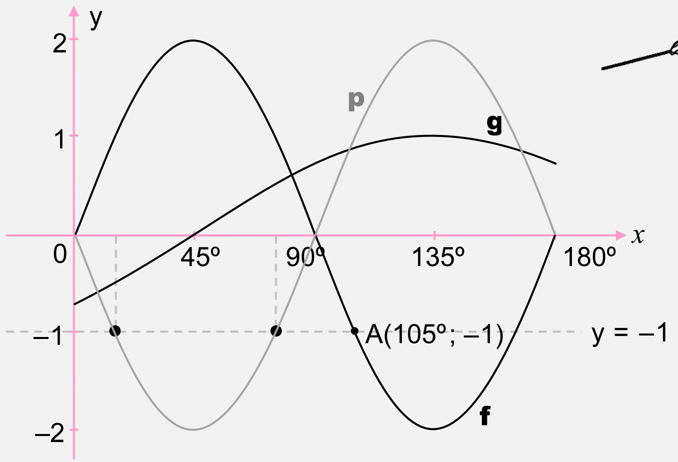
(d) When answering **Q6.3.2** some candidates were able to establish the critical values of 105° and 165° but were **unable to present** the answer correctly as **an interval**.

QUESTION 6 (cont.)

6.4 Another graph p is defined as $p(x) = -f(x)$.
33% D(k; –1) lies on p. Determine the value(s) of k in the interval $x \in [0°; 180°]$. (3)

MEMOS

6.4 This can be done by inspection:



k = 15º or 75º ✓

Eqn of p: $y = -2\sin 2x$

D(k; –1) on p ➡ $-2\sin 2k = -1$

$\therefore \sin 2k = \frac{1}{2}$

$\therefore 2k = 30°$ or $150°$

$\therefore$ **k = 15º or 75º** ✓

Common Errors and Misconceptions

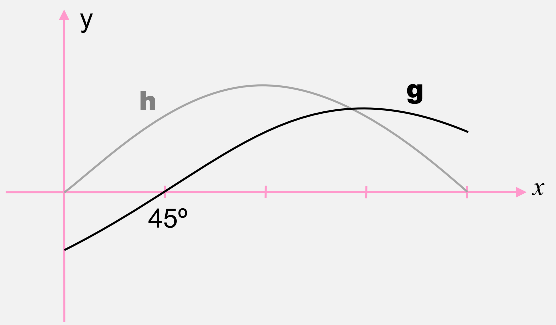
(e) Many candidates used algebraic methods to solve the equation in **Q6.4 instead of using the graph**. They would either leave their answer as a general solution or only provided one solution, i.e. $x = 15°$.

QUESTION 6 (cont.)

6.5 Graph h is obtained when g is translated 45º to the left.
32% Determine the equation of h.
Write your answer in its simplest form. (2)
[12]

MEMOS

6.5 **h(x) = sin x** ✓



Common Errors and Misconceptions

(f) As in Q6.4 candidates **again resorted to algebraic methods** to answer **Q6.5 instead of** translating **the graph** and observing the image obtained. It was disturbing that a number of candidates did not respond to this question.

QUESTION 6: Suggestions for Improvement

(a) Although these concepts are discussed in Grade 10, it is necessary for learners to be constantly reminded of the **meaning of** concepts like ***period***, ***domain***, ***amplitude*** and ***range***.

(b) Learners should be told that **the period** of a trigonometric function is the length of a function's cycle. Since this value is a length, it **is a single number** and not an interval of values.

(c) Learners should be shown **how to write intervals**, using both **inequalities** and **interval notation**.

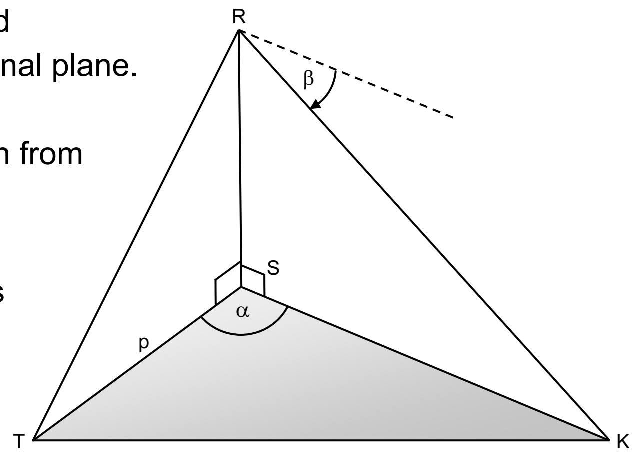
(d) Teachers should make learners aware of the **cyclic nature of trigonometric graphs**. This is useful in determining the coordinates of other points on the graph.

(e) Teachers should put more emphasis on teaching **graphical interpretation**, by **reading off values from graphs** and using the properties of the graphs and transformations rather than using long algebraic methods. This skill is particularly useful when the question is allocated only a few marks.

QUESTION 7 43%

In the diagram, S, T and K lie in the same horizonal plane. RS is a vertical tower. The angle of depression from R to K is β.

$T\hat{S}K = \alpha$, TS = p metres and the area of ΔSTK is q m^2.



7.1 Determine the length of SK in terms of p, q and α. (2)

46%

7.2 **Show that** $RS = \dfrac{2q \tan \beta}{p \sin \alpha}$ (2)

36%

MEMOS

7.1 $\frac{1}{2}TS.SK \sin \alpha = \text{Area of } \Delta STK$

$\therefore \frac{1}{2}p.SK \sin \alpha = q$

$\therefore p.SK.\sin \alpha = 2q$

$\therefore \mathbf{SK = \dfrac{2q}{p.\sin \alpha}}$ ◀

7.2 $R\hat{K}S = \beta$ ... *alt $\angle^s$; || lines*

In ΔRSK: $\dfrac{RS}{SK} = \tan \beta$

$\therefore RS = SK.\tan \beta$

Substitute from Q7.1 ...

$\therefore \mathbf{RS = \dfrac{2q.\tan \beta}{p.\sin \alpha}}$ ◀

Common Errors and Misconceptions

(a) In **Q7.1 many** candidates **attempted to use the sine formula instead of the area formula**. This resulted in them not being able to express the side of the triangle in terms of the required variables.

(b) **Q7.2** required candidates to analyse the diagram and create a trigonometric expression for RS and substitute into this expression the result obtained in Q7.1. Instead, some candidates tried to manipulate the expression given for RS algebraically. These **candidates assumed that the given expression was true**. They were not awarded any marks for their efforts.

QUESTION 7 (cont.)

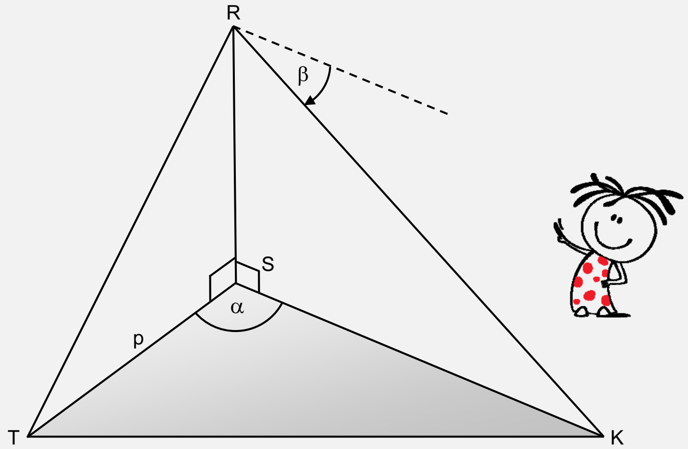
7.3 Calculate the size of α if $\alpha < 90°$ and RS = 70 m,

45% $p = 80$ m, $q = 2\,500$ m^2 and $\beta = 42°$. (3)

[7]

MEMOS

7.3



From 7.2: $RS = \dfrac{2q.\tan\beta}{p.\sin\alpha}$

$\therefore\ p.\sin\alpha.RS = 2q.\tan\beta$

$\therefore\ \sin\alpha = \dfrac{2q.\tan\beta}{p.RS}$

$= \dfrac{2 \times 2\,500 \times \tan 42°}{80 \times 70}$

$= \mathbf{0{,}803...}$ **. . . Do not round off here**

$\therefore\ \alpha = \mathbf{53{,}51°}$ ✓

Common Errors and Misconceptions

(c) Some candidates did not realise that they could use the expression given in Q7.2 to calculate the size of α. Instead, they tried to formulate other expressions involving α, most common of which was to use the sine formula in ΔSTK. This proved to be fruitless.

Some candidates rounded off the answer for $\sin\alpha$. Consequently, they arrived at $\alpha = 53{,}13°$ instead of $\alpha = 53{,}51°$.

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 7: Suggestions for Improvement

(a) Teachers should devote the appropriate amount of time to this section. This should allow learners to score the accessible marks in this section of work.

(b) Teachers need to **develop strategies** to be used when solving right-angled triangles and triangles that are not right-angled. Teach learners the conditions that determine which rule should be used to solve the question.

(c) It might be a good idea to give learners an exercise in which they identify which rule is to be used to solve the question. The learners must also substantiate why they think that the rule that they have selected applies to the question.

(d) Learners should be encouraged to highlight the different triangles using different colours.

(e) Initially, expose learners to numeric questions on solving 3D problems. This makes it easier for them to develop strategies on how to solve such questions. Once learners have gained confidence with numeric type questions, they should then be exposed to non-numeric and higher-order questions.

(f) Learners must be reminded that they should not round off intermediate values in their calculations. Early rounding off creates an error in the final answer. **Only the final answer should be rounded off** to the required number of places.