KZN 2024

Maths Subject Advisors Workshop

EUCLIDEAN GEOMETRY
Problem Solving

 # BREAK THE 70% CEILING

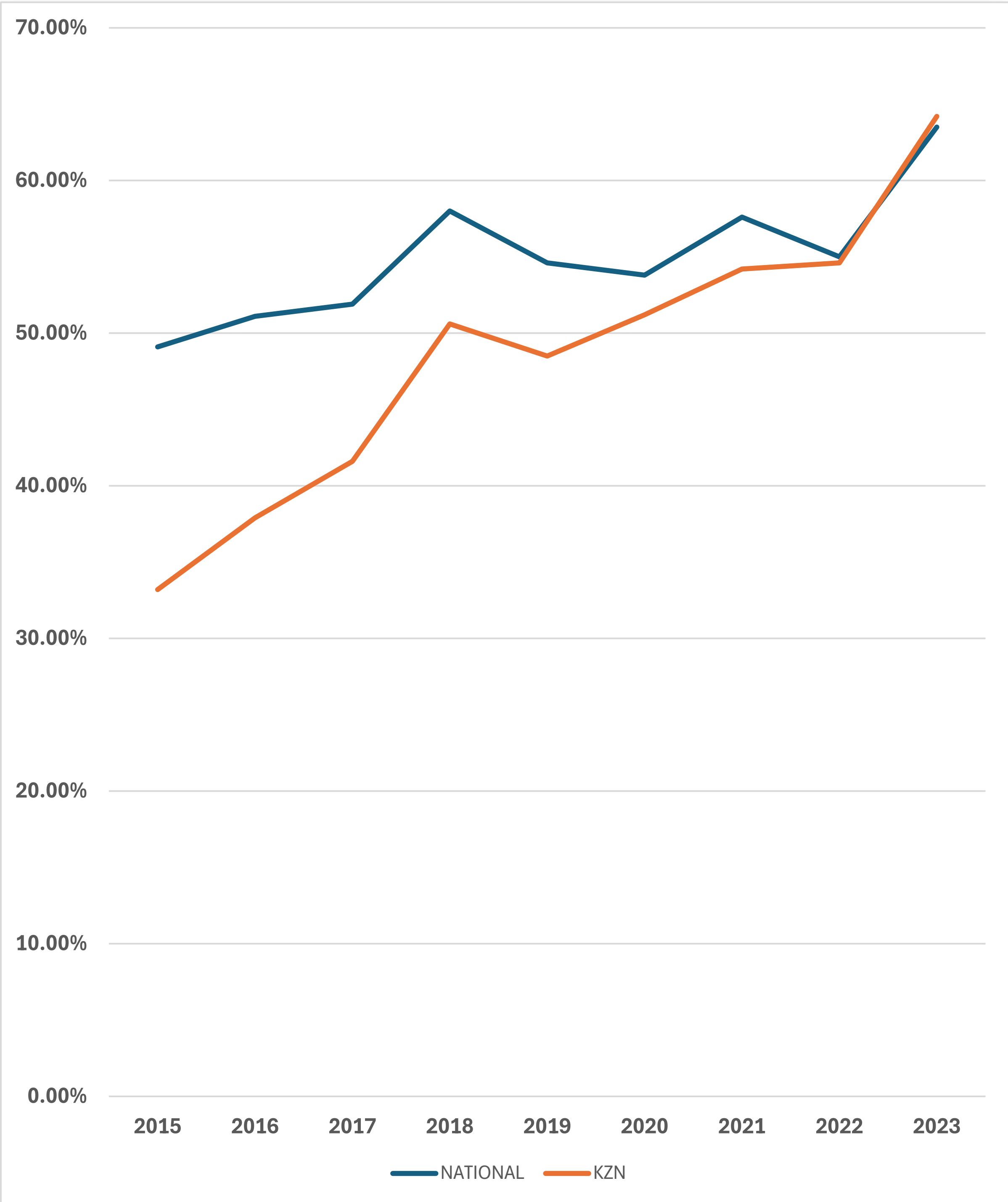
LEARN HOW – Remember for a moment

LEARN WHY – Remember for a life time

Presented by
Anne Eadie

THE ANSWER SERIES
Your Key to Exam Success

MATHS PASS RATE KZN vs NATIONAL



MATHS PER PROVINCE 2021 to 2023

CURRICULUM STRENGTHENING

OUR COMPASS TO IMPROVING LEARNING OUTCOMES

CURRENT CONTEXT: Where are we now?

VISION: Where do we want to be?

WAY FORWARD: How do we get to the vision?

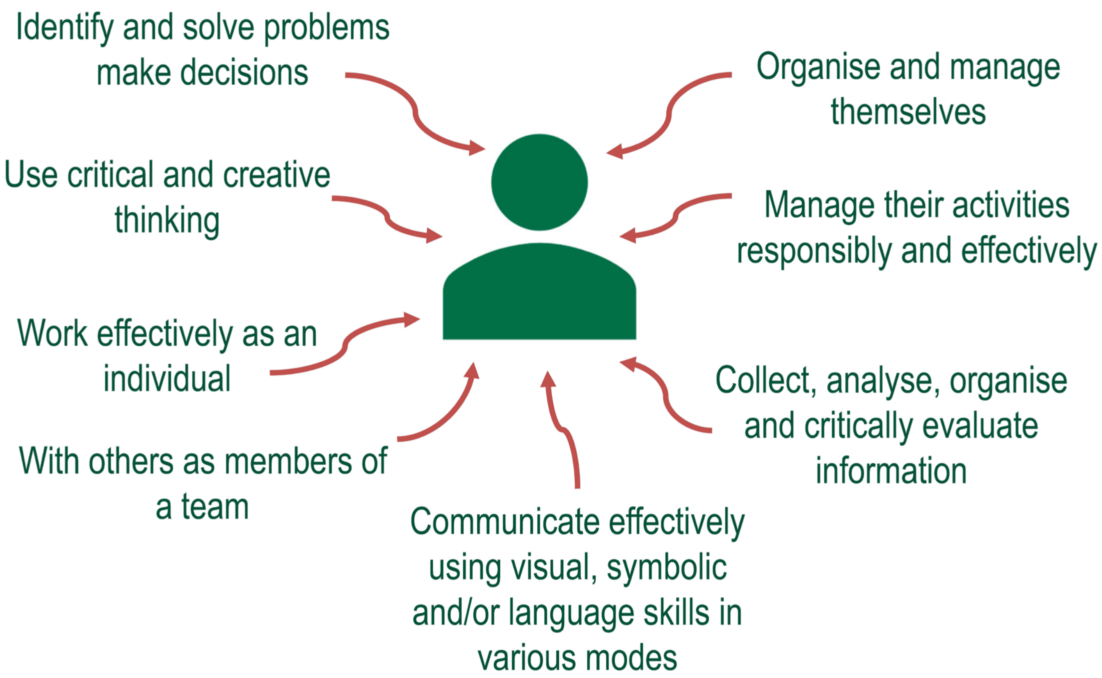
NATIONAL DEVELOPMENT PLAN 2030

Read to Lead
A Reading Nation is a Leading Nation

CURRICULUM STRENGTHENING

THE POLICY LANDSCAPE: CAPS

Section 1: General Aims



CURRICULUM STRENGTHENING

PERSISTENT CHALLENGES

Despite improvements, **learning outcomes remain low**er than many other middle-income countries. Targets remain elusive.

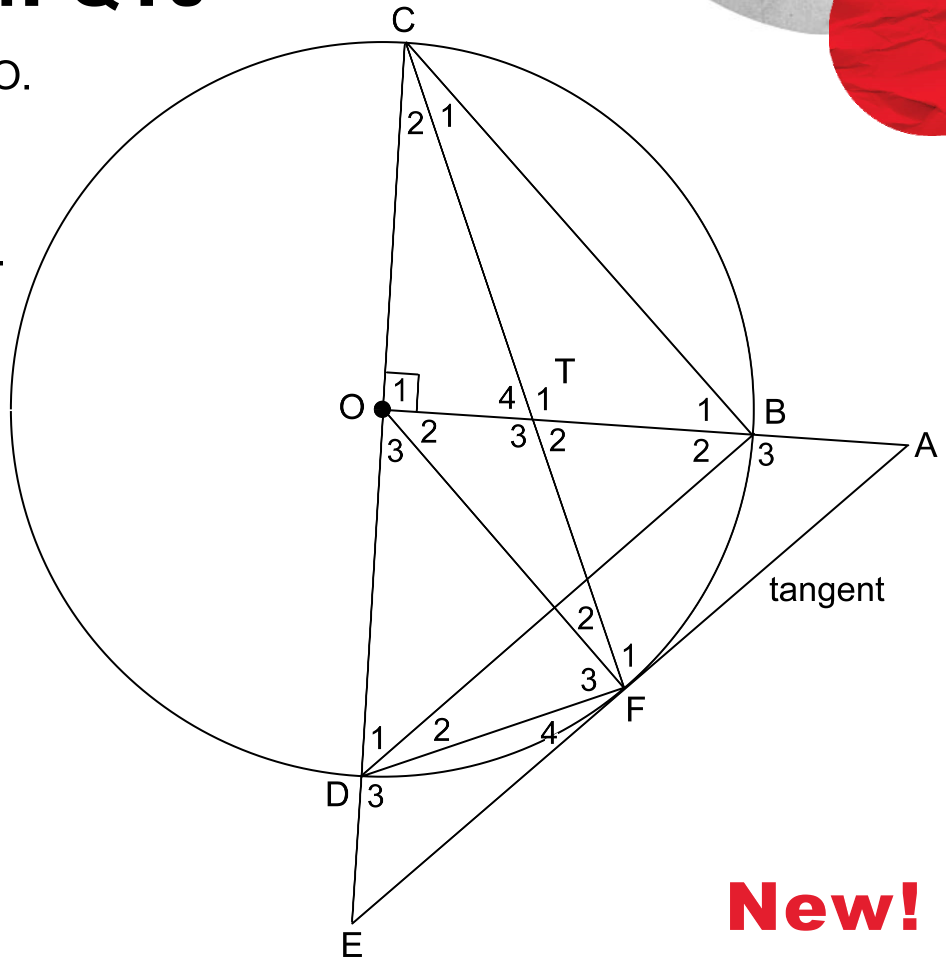
Low learning outcomes in early years contribute to many **learners exiting the system without adequate knowledge** and skills to succeed in life after school.

Together with other structural factors, this contributes to the **youth unemployment crisis** in the country.

A Warm-up Example

DBE May 2024: Q10

- In the diagram, COD is the diameter of the circle with centre O.
- EA is a tangent to the circle at F.
- AO ⊥ CE.
- Diameter COD produced intersects the tangent to the circle at E.
- OB produced intersects the tangent to the circle at A.
- CF intersects OB in T.
- CB, BD, OF and FD are drawn.



Prove, with reasons, that:

10.1 TODF is a cyclic quadrilateral (4)

10.2 $\hat{D}_3 = \hat{T}_1$ (3)

10.3 $\triangle TFO \,|||\, \triangle DFE$ (5)

10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)

10.5 Prove that $DO = \dfrac{TO \cdot FE}{AB}$ (5) **[19]**

New!

10.1 TODF is a cyclic quadrilateral (4)

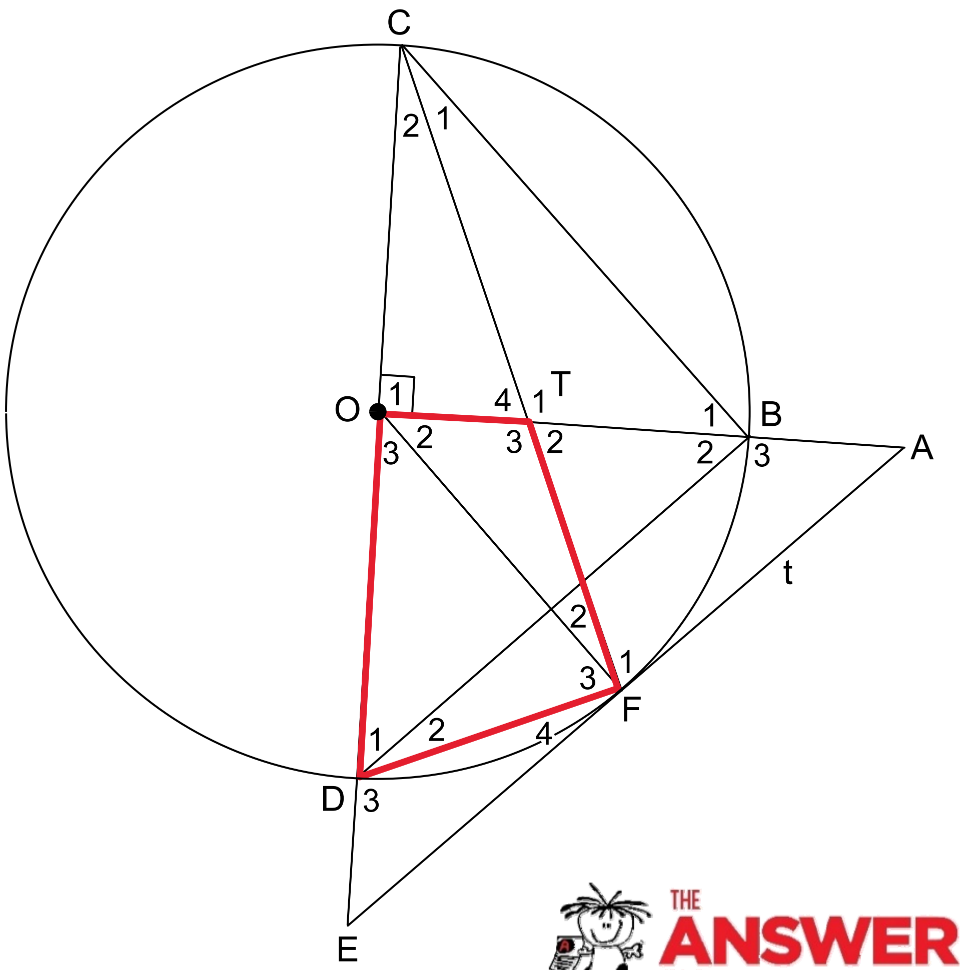
Solutions

10.1 $T\hat{O}D = 90^\circ$ ... *AO ⊥ CE*

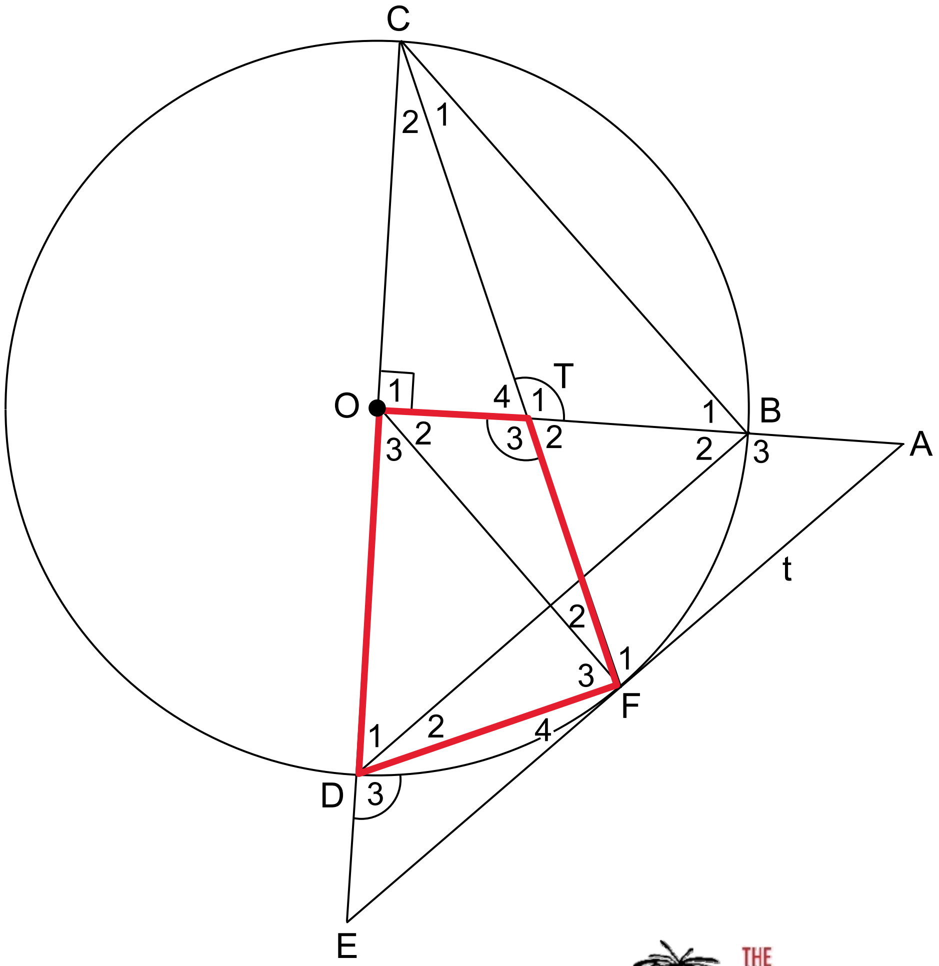
& $D\hat{F}T = 90^\circ$ ... *∠ in semi-⊙*

∴ TODF is a cyclic quad ... *converse ext ∠ of cyclic quad*

(or: converse opp ∠s of cyclic quad)



10.2 $\hat{D}_3 = \hat{T}_1$ (3)



Solutions

10.2 Let $\hat{D}_3 = x$

$\therefore \hat{T}_3 = x$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{T}_1 = x$ . . . *vert opp* $\angle^s =$

$\therefore \hat{D}_3 = \hat{T}_1$ ➤

10.3 ΔTFO ||| ΔDFE (5)

Solutions

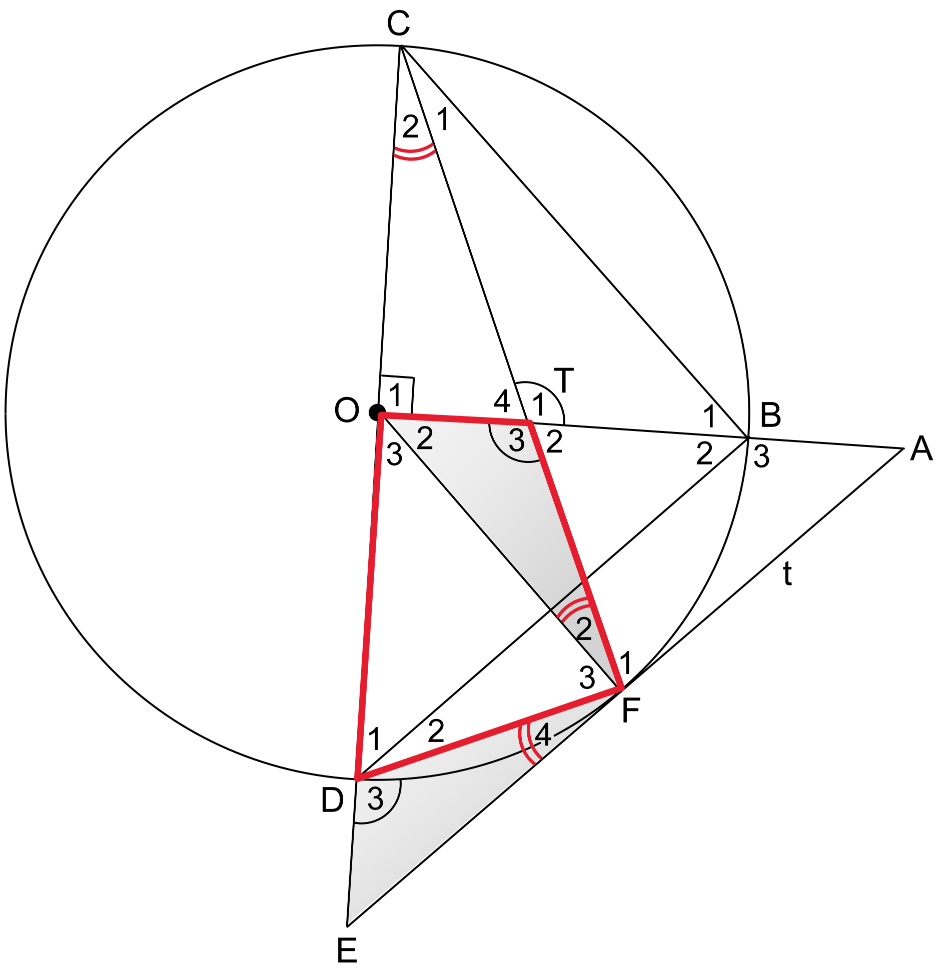
10.3 Δ^s TFO and ΔDFE

(1) $\hat{T}_3 = \hat{D}_3$ . . . *proved in 10.2*

(2) $\hat{F}_4 = \hat{C}_2$ . . . *tan chord theorem*

$= \hat{F}_2$ . . . *$\angle^s$ opp equal sides (radii)*

$\therefore$ ΔTFO ||| ΔDFE ➤ . . . *equiangular Δ^s*



10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)

Solutions

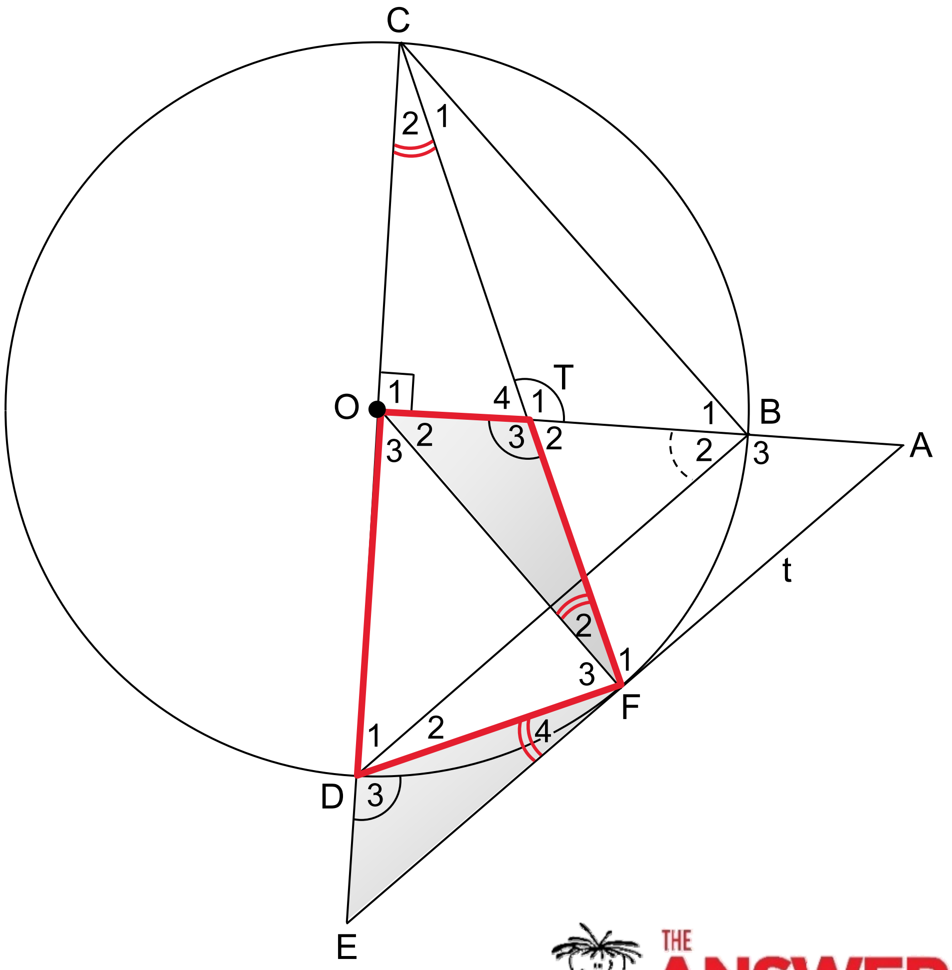
10.4 Let $\hat{E} = y$

$\therefore\ \hat{B}_2 = y$ . . . *given*

$\therefore\ \hat{D}_1 = y$ . . . *$\angle^s$ opp equal sides (radii)*

$\therefore\ \hat{D}_1 = \hat{E}$

$\therefore$ DB || EA ◀ . . . *corresp $\angle^s$ =*



10.5 Prove that $DO = \frac{TO.FE}{AB}$

Solutions

10.5 In $\triangle OEA$: $\frac{DO}{ED} = \frac{BO}{AB}$ ... *DB || EA (in 10,4); prop theorem*

$\therefore DO = \frac{BO.ED}{AB}$ ... ❶

Now, in $\triangle^s$ TFO & DFE: $\frac{TO}{ED} = \frac{FO}{FE}\left(= \frac{TF}{DF}\right)$ ... $|||\ \Delta^s$ *in 10.3*

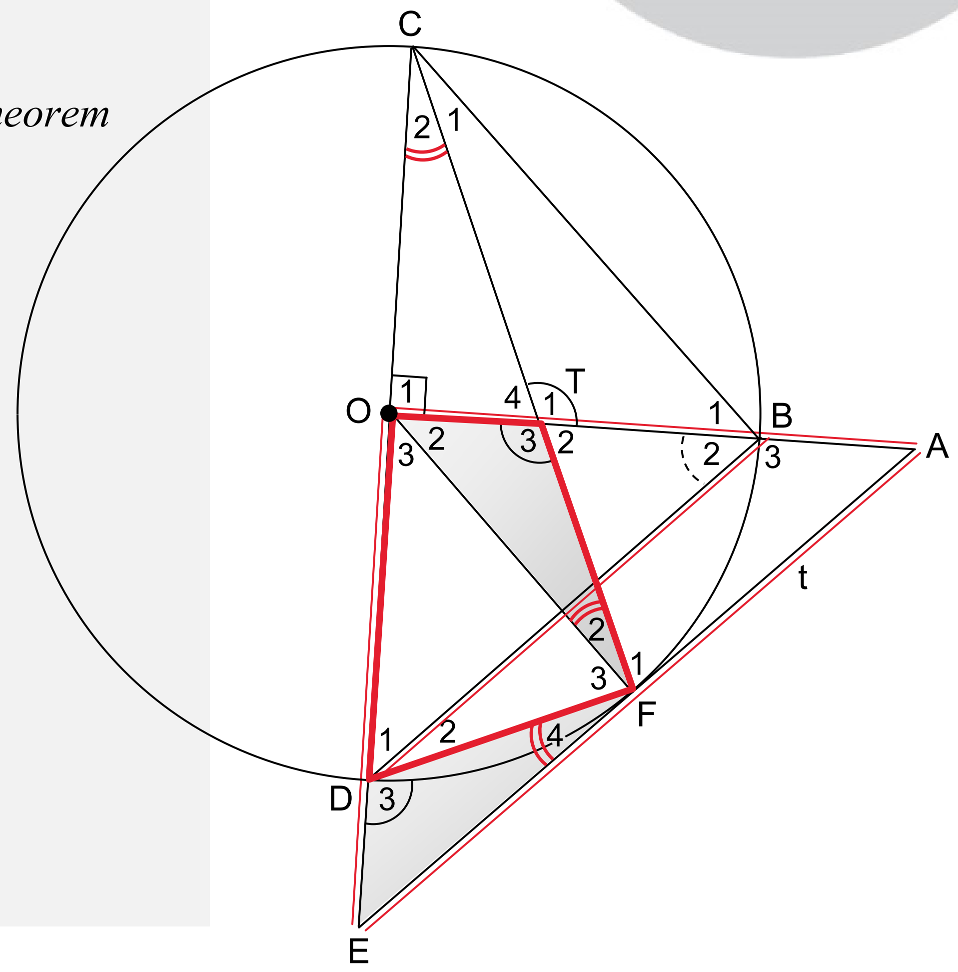
$\therefore FO.ED = TO.FE$

But FO = BO ... *radii*

$\therefore BO.ED = TO.FE$ ... ❷

Substitute ❷ in ❶:

$\therefore DO = \frac{BO.ED}{AB}$ ◄



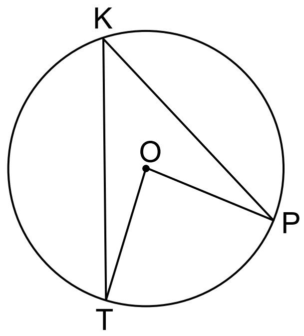
DBE NOV 2023 P2 EUCLIDEAN GEOMETRY (41%): QUESTIONS & PERFORMANCE

QUESTION 8 60%

8.1 In the diagram, O is the centre of the circle.

55%

Use the diagram alongside to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$.

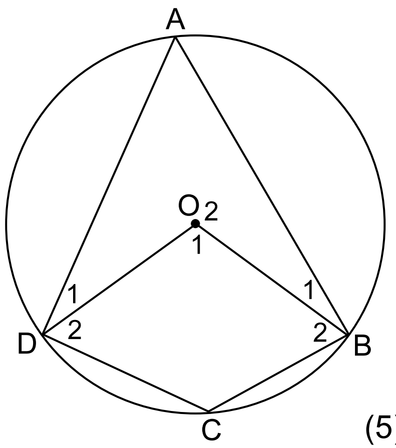


(5)

8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.

59%

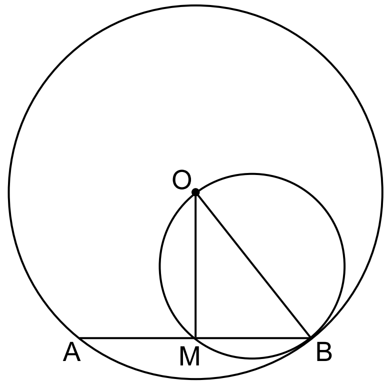
If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x.



(5)

8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.

65%



8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

8.3.2 If $AB = \sqrt{300}$ units and OM = 5 units, calculate, giving reasons, the length of OB. (4)

[16]

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 43
- TAS Website: www.theanswer.co.za
  - Diagnostic Report: Questions/Memos/Comments
  - 2023 Exam Reviews

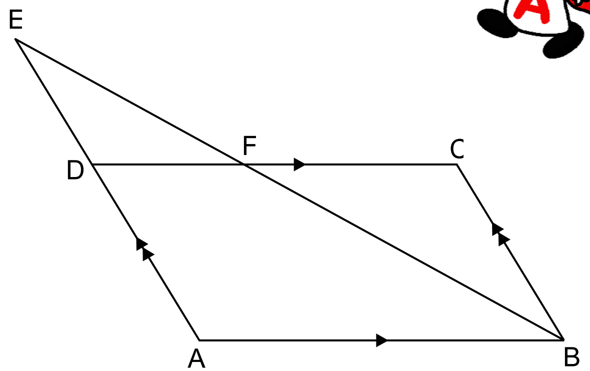
QUESTION 9 **44%**

In the diagram, ABCD is a parallelogram with AB = 14 units.

AD is produced to E such that AD : DE = 4 : 3.

EB intersects DC in F.

EB = 21 units.



9.1 Calculate, with reasons, the length of FB. (3)

51%

9.2 Prove, with reasons, that ΔEDF ||| ΔEAB. (3)

60%

9.3 Calculate, with reasons, the length of FC. (3)

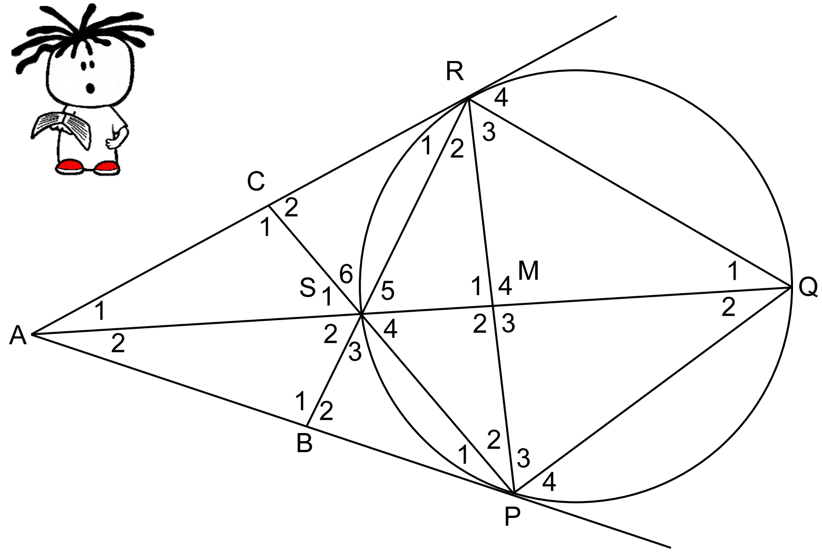
20%

[9]

THE ANSWER SERIES *Your Key to Exam Success*

QUESTION 10 **18%**

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.1 $\hat{S}_3 = \hat{S}_4$ (5)

29%

10.2 SMRC is a cyclic quadrilateral. (4)

16%

10.3 RP is a tangent to the circle passing through P, S and A at P. (6)

10%

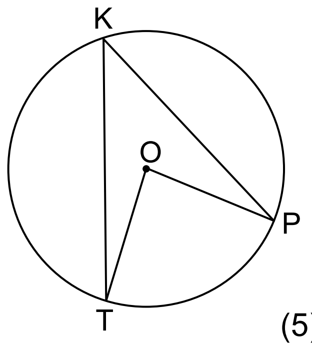
[15]

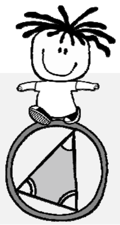
EUCLIDEAN GEOMETRY (41%): DBE NOVEMBER 2023

QUESTION 8   60%

8.1 In the diagram, O is the centre of the circle.
55%

Use the diagram alongside to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $T\hat{O}P = 2T\hat{K}P$. (5)



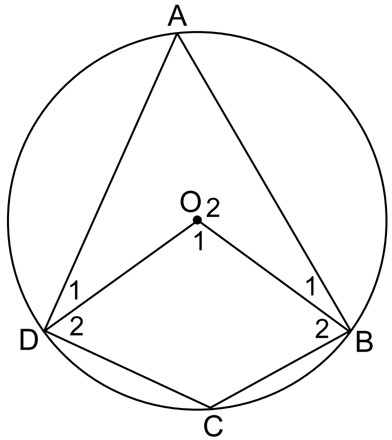


MEMOS

8.1 Theorem proof ➤

8.2 In the diagram, O is the centre of the circle and ABCD is a cyclic quadrilateral.
59%
OB and OD are drawn.

If $\hat{O}_1 = 4x + 100°$ and $\hat{C} = x + 34°$, calculate, giving reasons, the size of x. (5)



MEMOS

8.2 $\hat{A} = \frac{1}{2}(4x + 100°)$ . . . *∠ at centre = 2 × ∠ at circum*

$= 2x + 50°$

$\hat{A} + \hat{C} = 180°$ . . . *opp ∠^s of cyclic quad*

$\therefore\ 2x + 50° + x + 34° = 180°$

$\therefore\ 3x + 84° = 180°$

$\therefore\ 3x = 96°$

$\therefore\ \boldsymbol{x = 32°}$ ➤

Common Errors and Misconceptions

(a) **Q8.1** tested bookwork. Some candidates did not show or describe any construction. Some candidates labelled angles inappropriately, e.g. just $\hat{K}$, instead of $\hat{K}_1$ or $\hat{K}_2$.
Some candidates used as reason 'isosceles triangle', instead of 'angles opposite equal sides'.

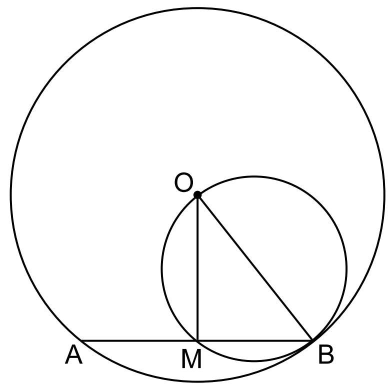
(b) Some candidates made the following **incorrect** statements when answering **Q8.2**:

- **DOBC is a cyclic quadrilateral.**
- $\boldsymbol{\hat{A} = 2\hat{O}_1}$
- $\boldsymbol{\hat{O}_2 = \frac{1}{2}\hat{C}}$
- $\boldsymbol{\hat{A} = \hat{C}}$

QUESTION 8 (cont.)

8.3 In the diagram, O is the centre of the larger circle. OB is a diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.

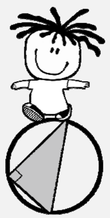
65%



8.3.1 Write down the size of $O\hat{M}B$. Provide a reason. (2)

8.3.2 If AB = $\sqrt{300}$ units and OM = 5 units, calculate, giving reasons, the length of OB. (4) **[16]**

MEMOS



8.3.1 $O\hat{M}B = 90^\circ$ . . . *∠ in semi-⊙*

8.3.2 $OB^2 = OM^2 + MB^2$ . . . *Pythag*

But $MB = \frac{1}{2}AB$ . . . *line from centre ⊥ to chord*

$= \frac{1}{2}\sqrt{300}$ . . . OR: $5\sqrt{3}$

$\therefore OB^2 = 5^2 + \left(\frac{1}{2}\sqrt{300}\right)^2$

$= 25 + \left(\frac{1}{4}\times 300\right)$

$= 25 + 75$

$= 100$

$\therefore$ **OB = 10 units** ◄

OR: $OB^2 = 5^2 + (\mathbf{5\sqrt{3}})^{\mathbf{2}}$

$= 25 + (25 \times 3)$

$= 25 + 75$

$= 100$, etc.

Common Errors and Misconceptions

(c) When answering **Q8.3.1** many candidates were able to state that $O\hat{M}B = 90^\circ$. However, they provided the following incorrect reasons for their statement:

- radius perpendicular to chord.
- line from centre perpendicular to chord.
- line from centre to midpoint of chord.

(d) In **Q8.3.2** some candidates were unable to provide the correct reason for AM being equal to MB. However, they were able to calculate the length of OB correctly. Some candidates **did not use brackets** when substituting into the expression for the *Theorem of Pythagoras*. They wrote $5\sqrt{3}^{\mathbf{2}}$ instead of $(5\sqrt{3})^{\mathbf{2}}$. Consequently, they went on to enter the same into the calculator and obtained an incorrect final answer.

QUESTION 8: Suggestions for Improvement

(a) Learners should be taught that a construction is required in order to prove a theorem. If the construction is not shown, then the proof is regarded as a breakdown and they get no marks.

(b) Teachers must cover the basic work thoroughly. An explanation of the **theorem** should be accompanied by showing the relationship in a diagram. In addition, learners should be made to prove theorems as part of their informal tasks. A good strategy is to expect learners to write the proof of a theorem as a task the day after the theorem was explained in class.

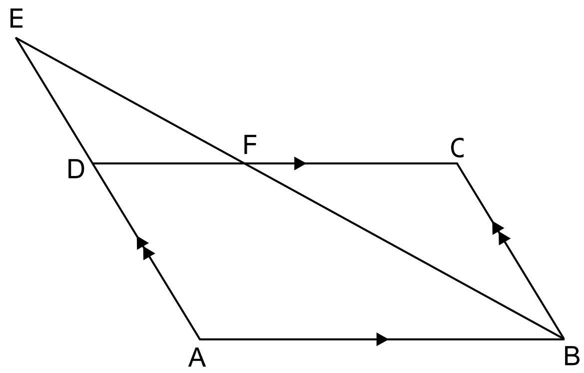
(c) Teachers are encouraged to use the **'Acceptable Reasons'** in the *Examination Guidelines* when teaching. This should start from as early as Grade 8. Learners should be issued with a copy of the 'Acceptable Reasons'.

(d) Learners should be encouraged to scrutinise the given information and the diagram for **clues** about which theorems could be used when answering the question.

(e) Learners should be taught that **all statements must be accompanied by reasons**. It is essential that the parallel lines be mentioned when stating that corresponding angles are equal, alternate angles are equal, the sum of the co-interior angles is 180° or when stating the proportional intercept theorem.

QUESTION 9 44%

In the diagram, ABCD is a parallelogram with AB = 14 units.
AD is produced to E such that AD : DE = 4 : 3.
EB intersects DC in F.
EB = 21 units.



9.1 Calculate, with reasons, the length of FB. (3)

51%

9.2 Prove, with reasons, that ΔEDF ||| ΔEAB. (3)

60%

MEMOS

9.1 $\frac{FB}{FE} = \frac{AD}{DE} \left(= \frac{4}{3} \right)$ . . . *prop thm; DF || AB*

$\therefore$ Let FB = $4x$ & FE = $3x$

$\therefore$ $7x$ = 21 units

$\therefore$ x = 3 units

$\therefore$ **FB = 12 units** ◄

9.2 In Δ^s EDF & EAB

(1) $\hat{E}$ is common

(2) $E\hat{F}D = E\hat{B}A$ . . . *corresp* $\angle^s$*; DF || AB*

$\therefore$ **ΔEDF ||| ΔEAB** ◄ . . . ∠∠∠

Common Errors and Misconceptions

(a) In **Q9.1** many candidates did not get a mark for the reason, because of only stating: 'proportionality theorem', instead of also stating which lines were parallel in the reason. Some candidates equated ratios between sides which were not actually equal, because they did not choose the sides appropriately, e.g. $\frac{FB}{EB} = \frac{DE}{EA}$.

(b) In **Q9.2** many candidates did not label the angles correctly, e.g. $\hat{F}$ and $\hat{B}$ instead of $E\hat{F}D$ and $E\hat{B}A$. Some candidates incorrectly gave the reason as 'alternate angles' or 'co-interior angles'. Other candidates correctly gave the reason as 'corresponding angles'. However, they did not state 'the lines parallel' and were not awarded a mark as the reason was incomplete.

QUESTION 9 (cont.)

9.3 Calculate, with reasons, the length of FC. (3) **[9]**

20%

MEMOS

9.3 $\frac{DF}{AB} = \frac{EF}{EB}$ . . . *similar* Δ^s

$\therefore \frac{DF}{14} = \frac{9}{21}$

$\therefore DF = \frac{{}^{3}\cancel{9} \times \cancel{14}^{2}}{\cancel{21}}$

$= 6$ units

But DC = AB = 14 units . . . *opp sides of* $||^m$

$\therefore$ **FC = 14 – 6 = 8 units**

Common Errors and Misconceptions

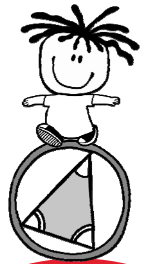
(c) Many candidates **incorrectly** used the *midpoint theorem* to answer **Q9.3**. They should have used the fact that the corresponding sides are in proportion when two triangles are similar.

A few candidates **incorrectly applied the *Theorem of Pythagoras*** even though there was **no right-angled triangle**.

They were not aware of the minimum conditions in which the *Theorem of Pythagoras* could be used.

QUESTION 9: Suggestions for Improvement

(a) Teachers should focus on developing learners' **skills to analyse the question and the diagram for clues** on which **theorems** are required to answer the questions correctly.



(b) Clearly explain to learners the difference between the *midpoint theorem*, the *proportionality theorem* and *similarity* so that they will know which of these concepts can be used in a specific situation.

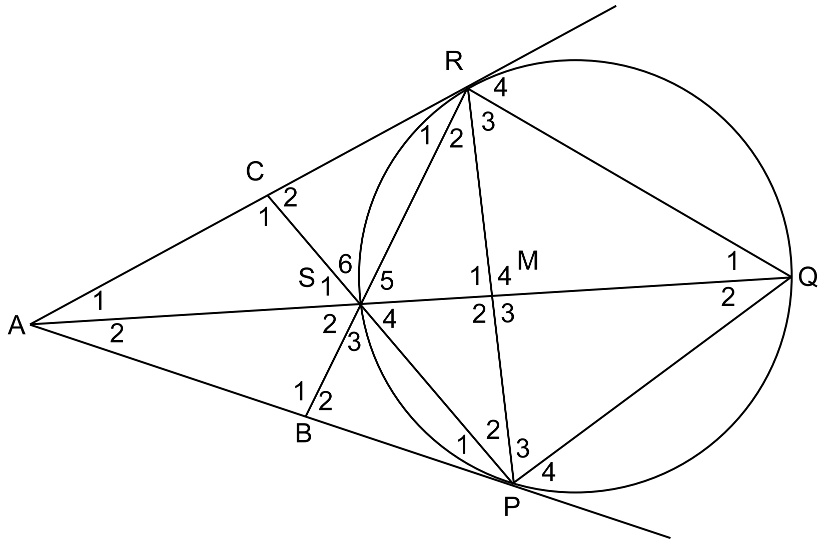
(c) When answering Euclidean Geometry, learners should be discouraged from writing correct statements that are not related to the solution. No marks are awarded for statements that do not lead to solving the problem.

(d) Learners need to be told that success in answering Euclidean Geometry comes from regular practice, starting off with the easy and progressing to the difficult.

(e) Teachers should take some time to discuss the naming of angles, for example, the acceptable methods are $\hat{T}$ or $\hat{T}_1$ or $O\hat{T}S$. Teachers should also clarify when it is acceptable to refer to an angle as $\hat{T}$ and when to refer to it as $\hat{T}_1$.

QUESTION 10 18%

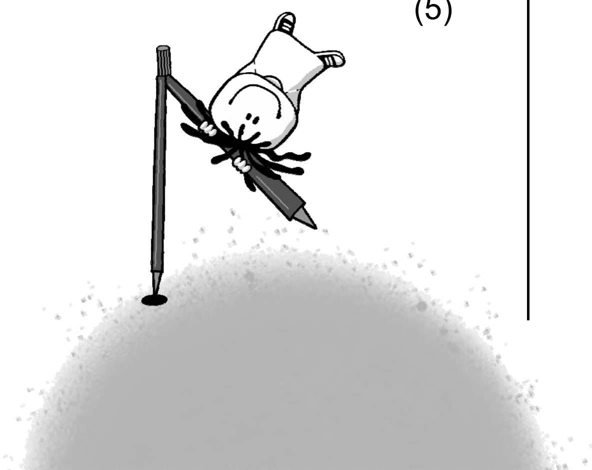
In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

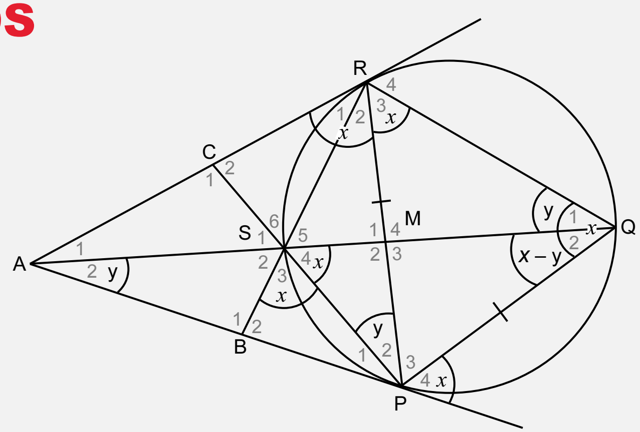
10.1 $\hat{S}_3 = \hat{S}_4$ (5)

29%



MEMOS

10.



10.1 Let $\hat{S}_3 = x$

$\therefore R\hat{Q}P = x$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{R}_3 = x$ . . . *∠ˢ opp = sides*

$\therefore \hat{S}_4 = x$ . . . *∠ˢ in the same seg*

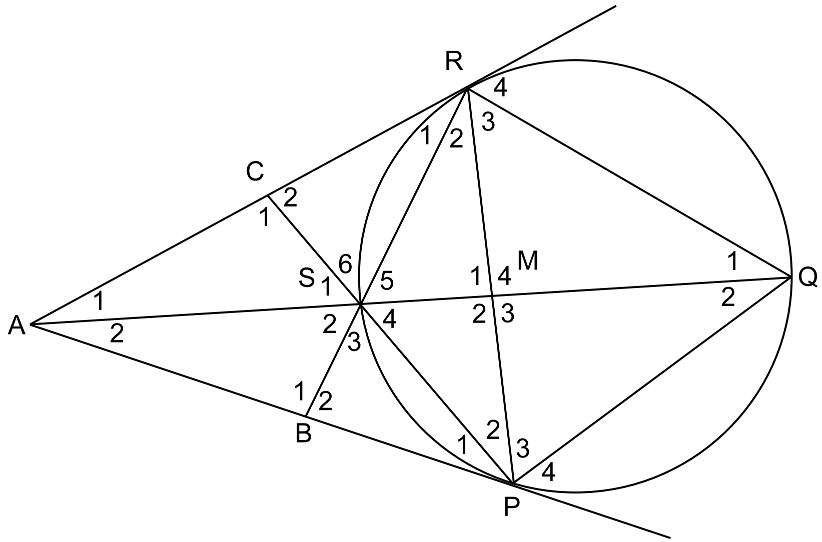
$\therefore \boldsymbol{\hat{S}_3 = \hat{S}_4}$

Common Errors and Misconceptions

(a) A fair number of candidates made **incorrect assumptions** when answering **Q10.1**. Among them were that: an exterior angle of the cyclic quadrilateral ($\hat{S}_3$) = the interior opposite angle ($\hat{R}_2$), PQ = RQ and therefore APQR is a kite, RQ || AP and $\hat{M}_1 = 90°$.

QUESTION 10 (cont.)

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



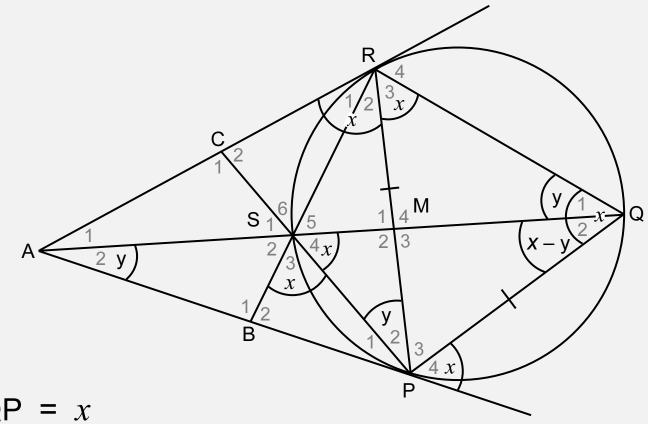
Prove, giving reasons, that:

10.2 SMRC is a cyclic quadrilateral. (4)

16%

MEMOS

10.



10.2 $\quad R\hat{Q}P = x$

$\therefore\ A\hat{R}P = x$ . . . *tan chord theorem*

But $\hat{S}_4 = x$

$\therefore\ \hat{S}_4 = A\hat{R}P$

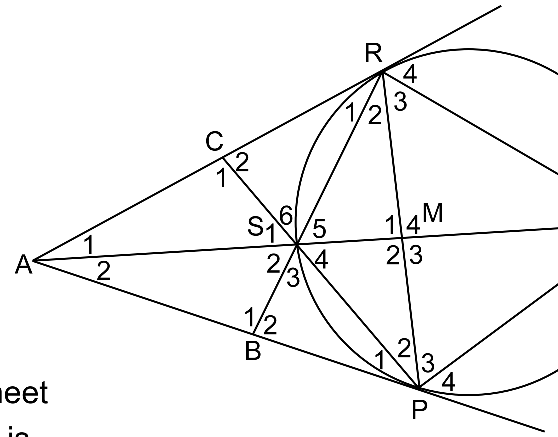
$\therefore$ **SMRC is a cyclic quad** ◂ . . . *converse ext ∠ of c.q.*

Common Errors and Misconceptions

(b) Candidates who could not answer **Q10.1** correctly could not understand how to start to answer **Q10.2**. Some candidates used the properties of a cyclic quadrilateral in their attempt to prove that the same quadrilateral is cyclic. Some candidates **did not know the difference** between ***a theorem*** and ***its converse***. They omitted the word 'converse' in the reason: 'exterior angle of cyclic quad'.

QUESTION 10 (cont.)

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



Prove, giving reasons, that:

10.3 RP is a tangent to the circle passing through P, S and A at P. (6)

10%

[15]

MEMOS

10.3 $\hat{P}_4 = x$ . . . *tan chord theorem*

Let $\hat{P}_2 = y$

$\therefore \hat{Q}_1 = y$ . . . $\angle^s$ *in the same seg*

$\therefore \hat{Q}_2 = x - y$

$\therefore \hat{A}_2 = y$ . . . *ext* $\angle$ *of* ΔQAP

$\therefore \hat{P}_2 = \hat{A}_2$

$\therefore$ RS is a tangent to the circle through P, S and A

. . . *conv tan chord thm*

OR:

$\hat{P}_4 = x$ . . . *tan chord thm*

$\therefore \hat{P}_4 = R\hat{Q}P$

$\therefore$ RQ || AP . . . *alt* $\angle^s =$

Let $\hat{A}_2 = y$

$\therefore \hat{Q}_1 = y$ . . . *alt* $\angle^s$*; RQ || AP*

$\therefore \hat{P}_2 = y$ . . . $\angle^s$ *in the same seg*

$\therefore \hat{P}_2 = \hat{A}_2$

$\therefore$ RS is a tangent to the circle through P, S and A

. . . *converse tan chord thm*

Common Errors and Misconceptions

(c) Very few candidates obtained full marks for **Q10.3**. The main reason for this was that candidates were unable to answer **Q10.1** and **Q10.2** correctly. Poor naming of angles in the answers often led to candidates themselves getting confused about which angle they were referring to.

(d) **Q10.3** required candidates to obtain a proportion from the similar triangles in **Q10.2**, using the *proportional intercept theorem* in ΔRAC to establish a second proportion and then to combine the two. Many candidates failed to establish one or the other proportion and therefore could not make the conclusion.

QUESTION 10: Suggestions for Improvement

(a) **More time needs to be spent on** the teaching of **Euclidean Geometry** in **all grades**. More practice on Grade 11 and 12 Euclidean Geometry will help learners to understand theorems and diagram analysis. They should **read** the given information carefully **without making any assumptions**. The work covered in class must include different activities and all levels of the taxonomy.

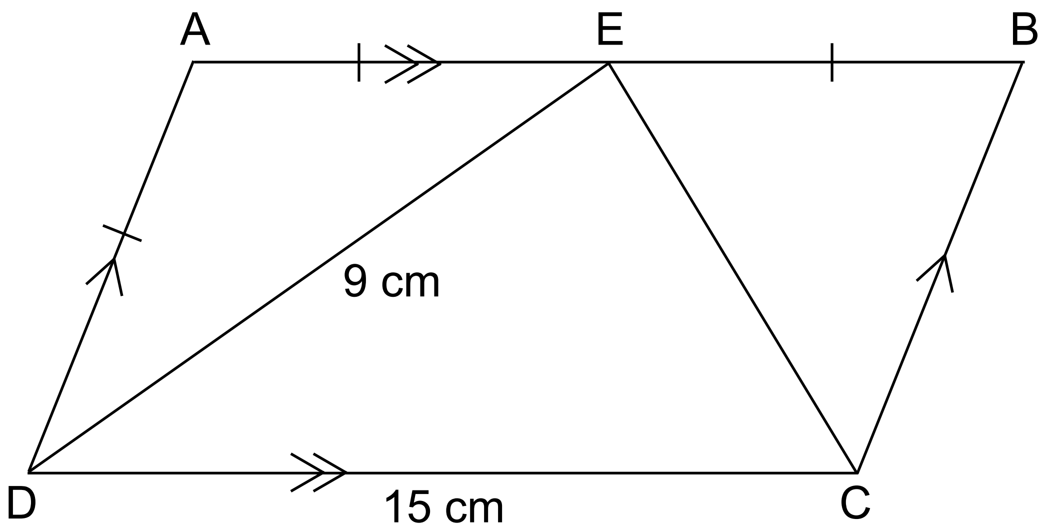
(b) Teach learners **not to assume any facts** in a geometry sketch but to **only use what was given** and that which was **proven already** in earlier questions.

(c) Learners need to be made aware that writing correct statements that are irrelevant to the answer in Euclidean Geometry will not earn them any marks in an examination.

(d) Consider teaching the approach of **'angle chasing'** where you label one angle as x and then relate other angles to x. In this way, learners should find it easy to identify angles that are equal but moreover, they should find it easier to establish the reasons for the relationships between the angles.

Problem Solving Questions

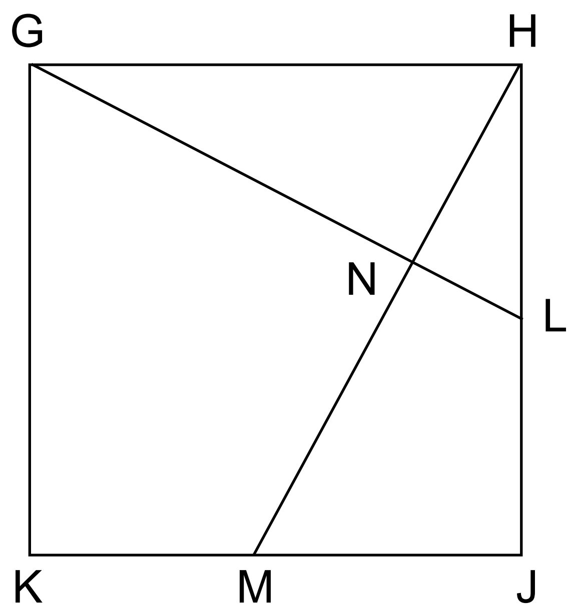
1.



ABCD is a parallelogram with AD = AE = EB. DE = 9 cm and DC = 15 cm.

Determine the length of EC.

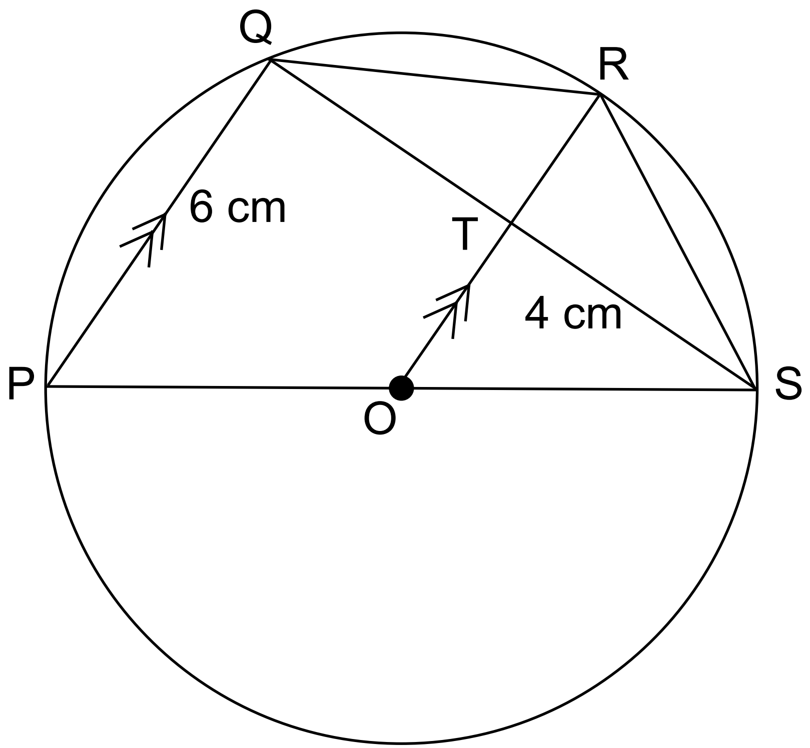
2.



GHJK is a square with L and M the midpoints of HJ and JK respectively.

Prove that GL $\perp$ HM.

3.



P, Q, R and S lie on the circumference of circle O.

PQ = 6 cm, ST = 4 cm, and the radius of the circle is 5 cm.

Determine the area of quadrilateral PQRS.

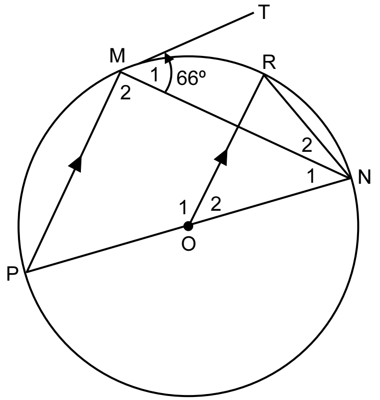
2018
EUCLIDEAN GEOMETRY

Give reasons for your statements in QUESTIONS 8, 9 and 10

QUESTION 8

8.1 PON is a diameter of the circle centred at O.
TM is a tangent to the circle at M, a point on the circle.
R is another point on the circle such that OR || PM.

NR and MN are drawn. Let $\hat{M}_1 = 66°$.



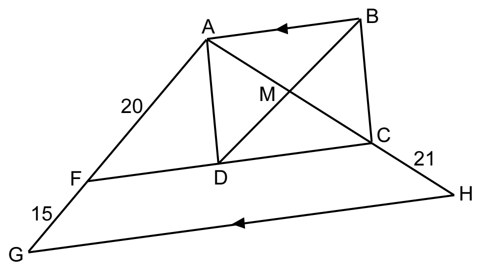
Calculate, with reasons, the size of EACH of the following angles:

8.1.1 $\hat{P}$ 8.1.2 $\hat{M}_2$ (2)(2)

8.1.3 $\hat{N}_1$ 8.1.4 $\hat{O}_2$ (1)(2)

8.1.5 $\hat{N}_2$ (3)

8.2 In the diagram, ΔAGH is drawn. F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units. D is a point on FC such that ABCD is a rectangle with AB also parallel to GH. The diagonals of ABCD intersect at M, a point on AH.



8.2.1 Explain why FC || GH. (1)

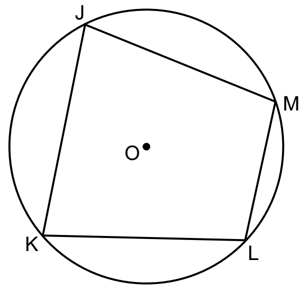
8.2.2 Calculate, with reasons, the length of DM. (5)

[16]

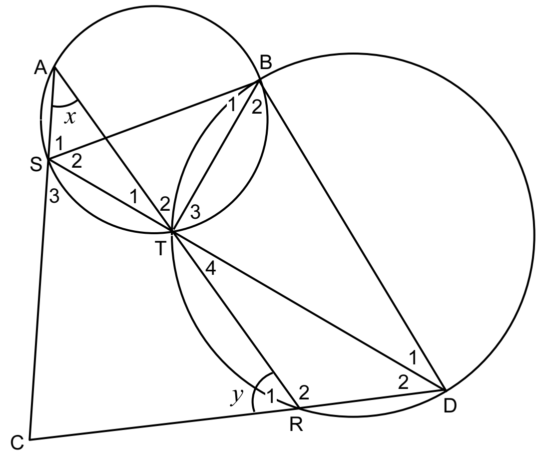
QUESTION 9

9.1 In the diagram, JKLM is a cyclic quadrilateral and the circle has centre O.

Prove the theorem which states that $\hat{J} + \hat{L} = 180°$. (5)



9.2 In the diagram, a smaller circle ABTS and a bigger circle BDRT are given. BT is a common chord.
Straight lines STD and ATR are drawn.
Chords AS and DR are produced to meet in C, a point outside the two circles.
BS and BD are drawn.
$\hat{A} = x$ and $\hat{R}_1 = y$.



9.2.1 Name, giving a reason, another angle equal to:

(a) x (b) y (2)(2)

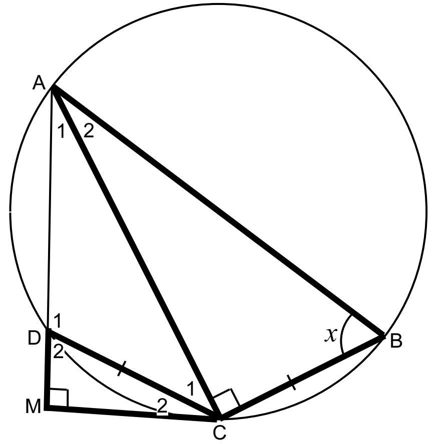
9.2.2 Prove that SCDB is a cyclic quadrilateral. (3)

9.2.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.
Prove that SD is not a diameter of circle BDS. (4)

[16]

QUESTION 10

In the diagram, ABCD is a cyclic quadrilateral such that AC ⊥ CB and DC = CB. AD is produced to M such that AM ⊥ MC.
Let $\hat{B} = x$.



10.1 Prove that:

10.1.1 MC is a tangent to the circle at C. (5)

10.1.2 ΔACB ||| ΔCMD (3)

10.2 Hence, or otherwise, prove that:

10.2.1 $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

10.2.2 $\frac{AM}{AB} = \sin^2 x$ (2)

[16]

TOTAL: 150

THE ANSWER SERIES Your Key to Exam Success

EUCLIDEAN GEOMETRY

THEOREM STATEMENTS & ACCEPTABLE REASONS

LINES

The adjacent angles on a straight line are supplementary.	$\angle^s$ on a str linep
If the adjacent angles are supplementary, the outer arms of these angles form a straight line.	adj $\angle^s$ supp
The adjacent angles in a revolution add up to 360º.	$\angle^s$ around a pt **OR** $\angle^s$ in a rev
Vertically opposite angles are equal.	vert opp $\angle^s$
If AB || CD, then the alternate angles are equal.	alt $\angle^s$; AB || CD
If AB || CD, then the corresponding angles are equal.	corresp $\angle^s$; AB || CD
If AB || CD, then the co-interior angles are supplementary.	co-int $\angle^s$; AB || CD
If the alternate angles between two lines are equal, then the lines are parallel.	alt $\angle^s$ =
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp $\angle^s$ =
If the co-interior angles between two lines are supplementary, then the lines are parallel.	co-int $\angle^s$ supp

TRIANGLES

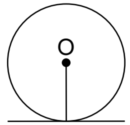
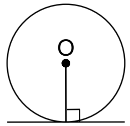
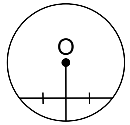
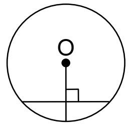
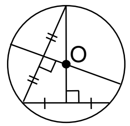
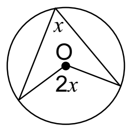
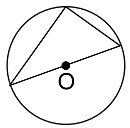
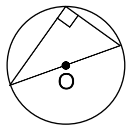
The interior angles of a triangle are supplementary.	$\angle$ sum in Δ **OR** sum of $\angle^s$ in Δ **OR** int $\angle^s$ in Δ
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	ext $\angle$ of Δ
The angles opposite the equal sides in an isosceles triangle are equal.	$\angle^s$ opp equal sides
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal $\angle^s$
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras **OR** Theorem of Pythagoras
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	**Converse** Pythagoras **OR Converse** Theorem of Pythagoras
If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.	SSS
If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS **OR** S$\angle$S
If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.	AAS **OR** $\angle\angle$S
If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent.	RHS **OR** 90ºHS
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the length of the third side.	Midpt Theorem
The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt || to 2nd side
A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line || one side of Δ **OR** prop theorem; name || lines
If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in prop
If two triangles are equiangular, then the corresponding sides are in proportion (and consequently the triangles are similar).	||| Δ^s **OR** equiangular Δ^s
If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and consequently the triangles are similar).	sides of Δ in prop
If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height **OR** equal bases; equal height

QUADRILATERALS

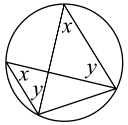
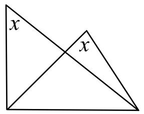
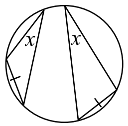
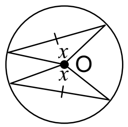
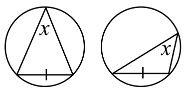
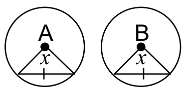
The interior angles of a quadrilateral add up to 360º.	sum of $\angle^s$ in quad
The opposite sides of a parallelogram are parallel.	opp sides of ||m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are || **OR** **converse** opp sides of ||m
The opposite sides of a parallelogram are equal in length.	opp sides of ||m
If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.	opp sides of quad are = **OR** **converse** opp sides of a parm
The opposite angles of a parallelogram are equal.	opp $\angle^s$ of ||m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp $\angle^s$ of quad are = **OR** **converse** opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of ||m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other **OR** **converse** diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and ||
The diagonals of a parallelogram bisect its area.	diag bisect area of ||m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles.	diag of kite

CIRCLES

GROUP I

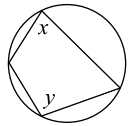
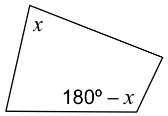
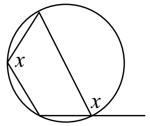
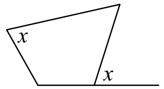
	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan $\perp$ radius tan $\perp$ diameter
	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line $\perp$ radius **OR** **converse** tan $\perp$ radius **OR** **converse** tan $\perp$ diameter
	The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
	The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre $\perp$ to chord
	The perpendicular bisector of a chord passes through the centre of the circle.	perp bisector of chord
	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre) No converse	$\angle$ at centre = 2 × $\angle$ at circumference
	The angle subtended by the diameter at the circumference of the circle is 90º.	$\angle^s$ in semi circle **OR** diameter subtends right angle **OR** $\angle$ in ½ $\odot$
	If the angle subtended by a chord at the circumference of the circle is 90º, then the chord is a diameter.	chord subtends 90º **OR** **converse** $\angle^s$ in semi circle

GROUP II

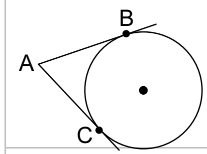
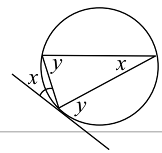
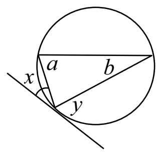
	Angles subtended by a chord of the circle, on the same side of the chord, are equal	$\angle^s$ in the same seg
	If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic. (This can be used to prove that the four points are concyclic).	line subtends equal $\angle^s$ **OR** **converse** $\angle^s$ in the same seg
	Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal $\angle^s$
	Equal chords subtend equal angles at the centre of the circle.	equal chords; equal $\angle^s$
	Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal $\angle^s$
	Equal chords in equal circles subtend equal angles at the centre of the circles. (A and B indicate the centres of the circles)	equal circles; equal chords; equal $\angle^s$

The highlighted statements are CONVERSE theorem statements.

GROUP III

	The opposite angles of a cyclic quadrilateral are supplementary (i.e. x and y are supplementary)	opp $\angle^s$ of cyclic quad
	If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic.	opp $\angle^s$ quad sup **OR** **converse** opp $\angle^s$ of cyclic quad
	The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.	ext $\angle$ of cyclic quad
	If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the quadrilateral is cyclic.	ext $\angle$ = int opp $\angle$ **OR** **converse** ext $\angle$ of cyclic quad

GROUP IV

	Two tangents drawn to a circle from the same point outside the circle are equal in length (AB = AC)	Tans from common pt **OR** Tans from same pt
	The angle between the tangent to a circle and the chord drawn from the point of contact is equal to the angle in the alternate segment.	tan chord theorem
	If a line is drawn through the end-point of a chord, making with the chord an angle equal to an angle in the alternate segment, then the line is a tangent to the circle. (If $x = b$ or if $y = a$ then the line is a tangent to the circle)	**converse** tan chord theorem **OR** $\angle$ between line and chord

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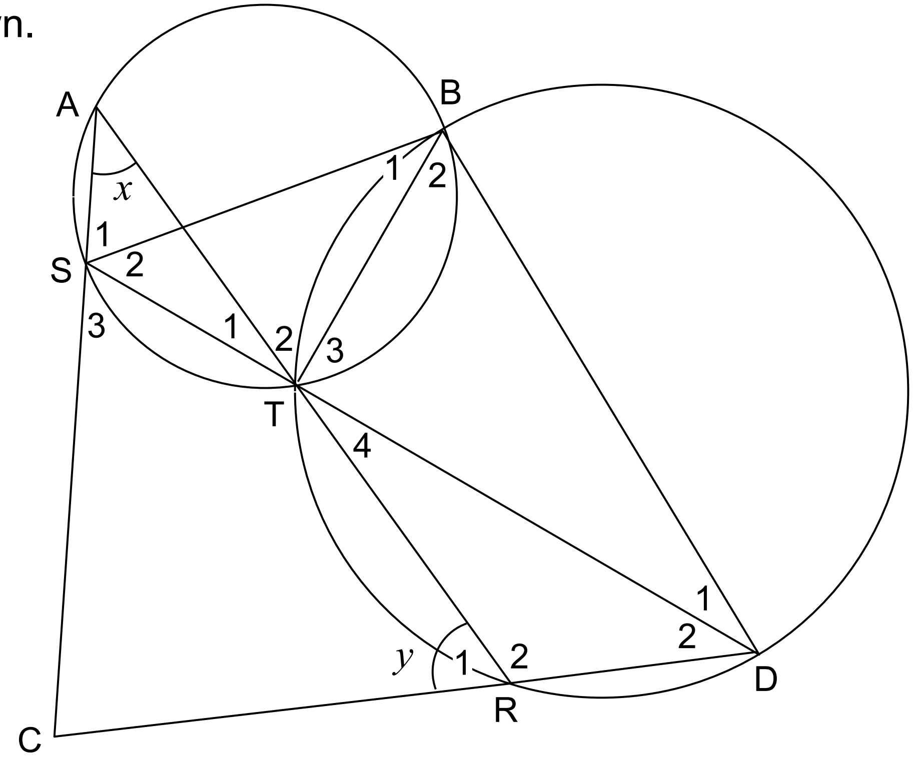
⊙ Geom, no tangents

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 17
- TAS Gr 12 Euclidean Geometry Video 2

Example 1 (DBE Nov 2018 Q9.2) 44%

- In the diagram, a smaller circle ABTS and a bigger circle BDRT are given.
- BT is a common chord. Straight lines STD and ATR are drawn.
- Chords AS and DR are produced to meet in C, a point outside the two circles.
- BS and BD are drawn.

$\hat{A} = x$ and $\hat{R}_1 = y$.



1.1 Name, giving a reason, another angle equal to:

(a) x (b) y (2)(2)

1.2 Prove that SCDB is a cyclic quadrilateral. (3)

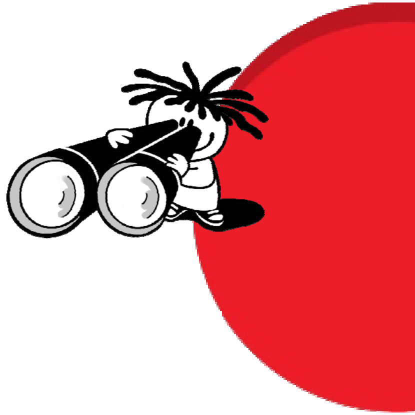
1.3 It is further given that $\hat{D}_2 = 30^\circ$ and $A\hat{S}T = 100^\circ$.
Prove that SD is not a diameter of circle BDS. (4)

1.1 Name, giving a reason, another angle equal to:

(a) x (2)

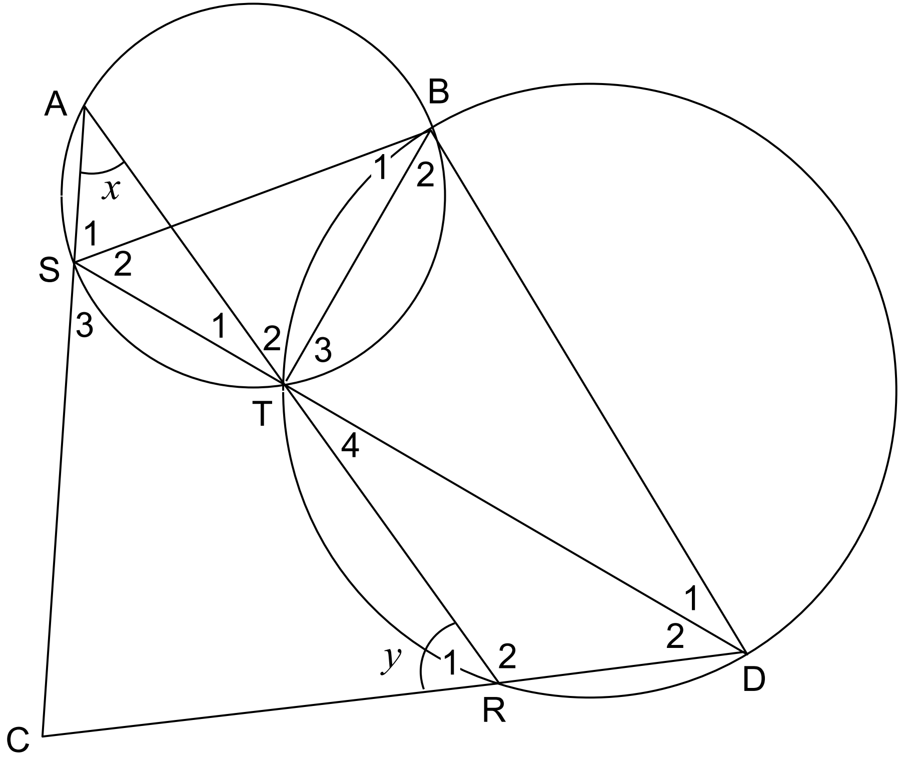
(b) y (2)

no circle centres
no tangents
no || lines



Circle without Centre

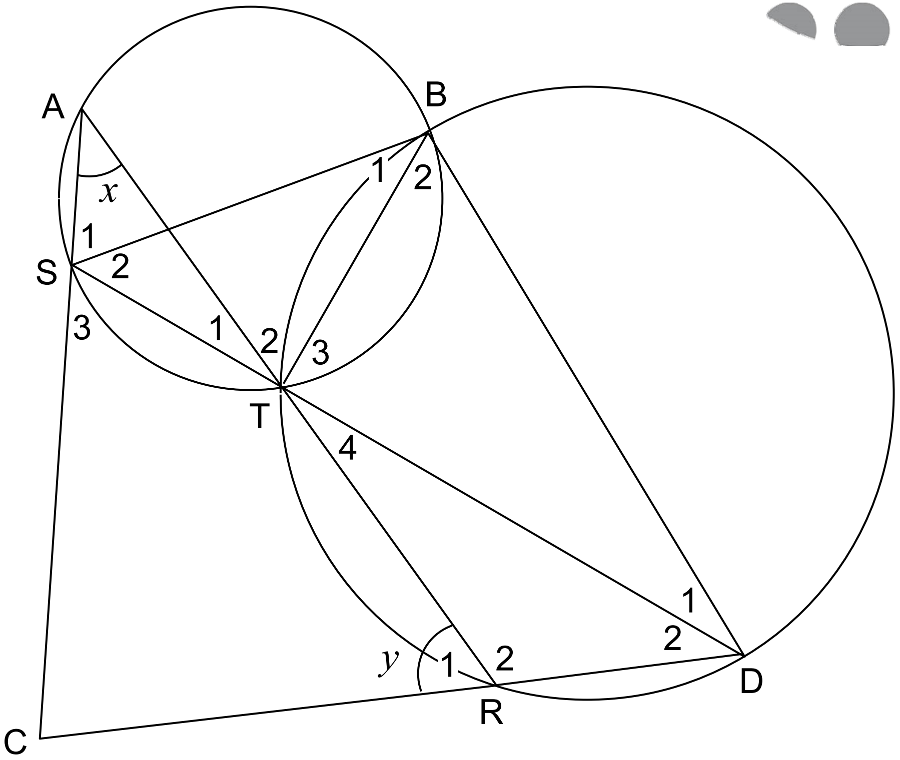
- angles in same segment
- exterior angle of a cyclic quad
- opposite angles of a cyclic quad
- equal chords, equal angles



1.1 Name, giving a reason, another angle equal to:

(a) x (2)

(b) y (2)



Solutions

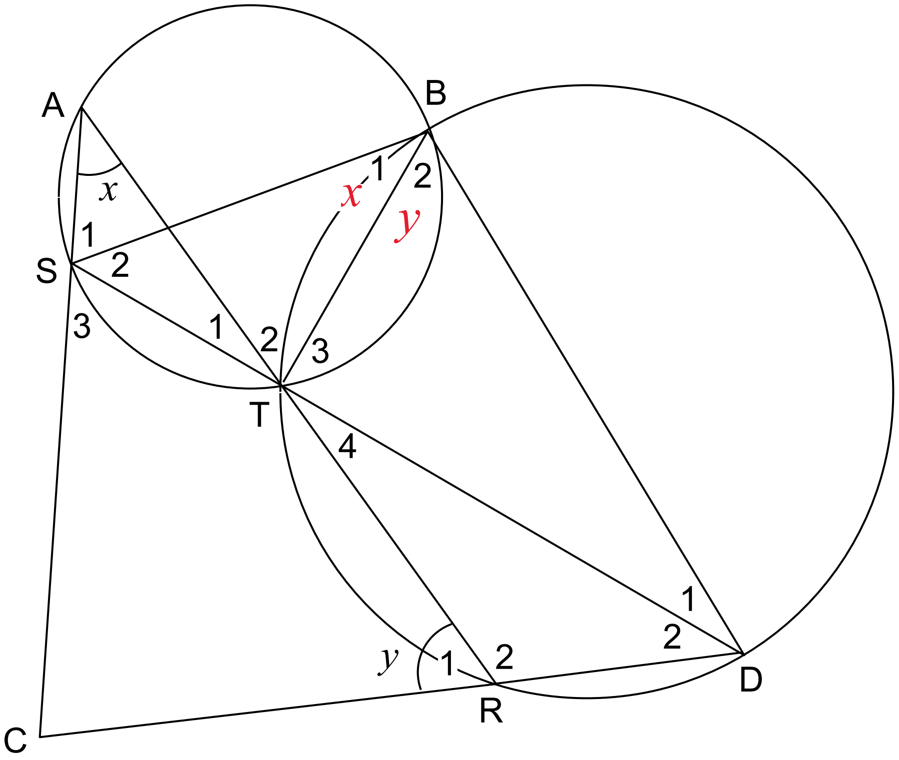
(a) $\hat{B}_1 = x$ ... $\angle^s$ *in the same segment*

(b) $\hat{B}_2 = y$ ... *exterior* $\angle$ *of cyclic quad. BTRD*

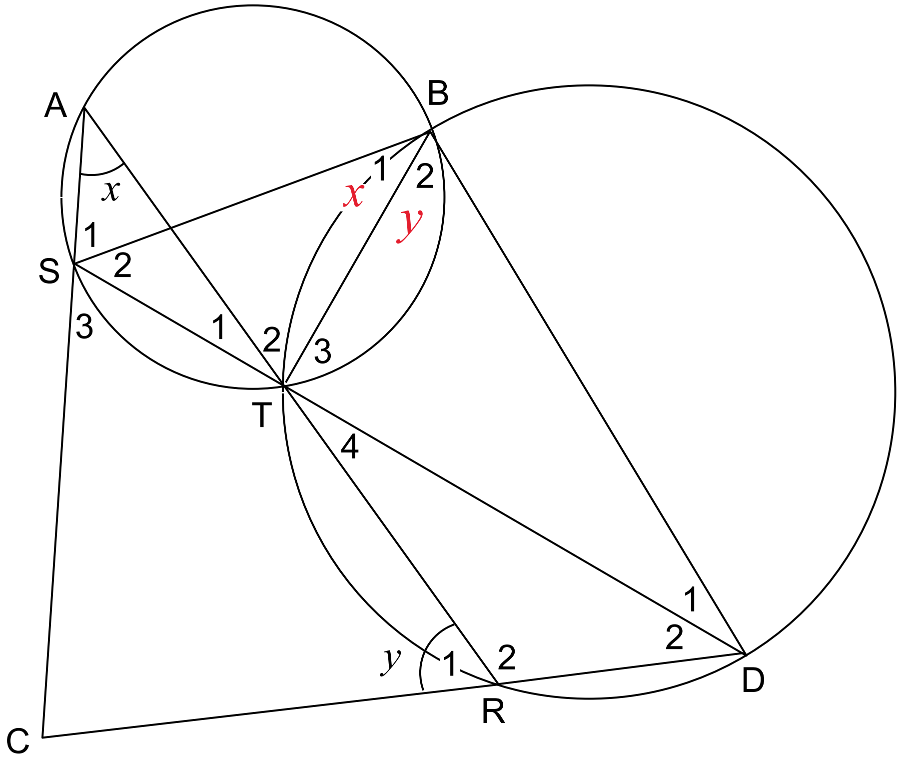
1.2 Prove that SCDB is a cyclic quadrilateral. (3)

Can I use my answers to 1.1?

If this is a cyclic quad, what must be true?



1.2 Prove that SCDB is a cyclic quadrilateral. (3)



Solutions

$\hat{C} = 180° - (x + y)$ ... *sum of $\angle^s$ in ΔACR*

$D\hat{B}S = x + y$ ... *from 1.1*

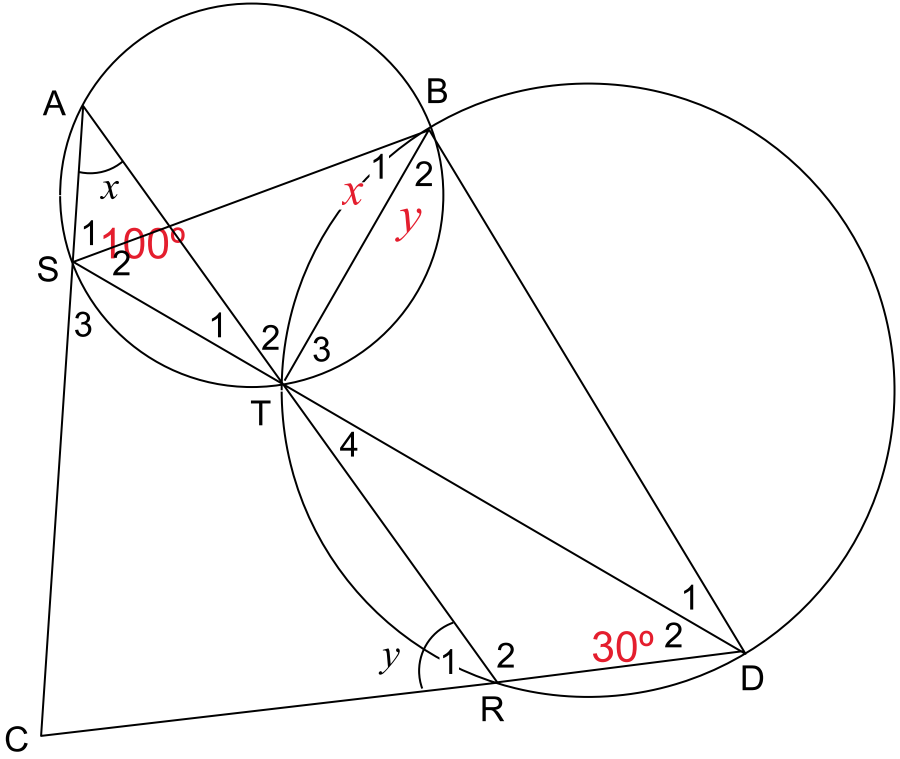
$\therefore \hat{C} + D\hat{B}S = 180°$

$\therefore$ SCDB is a cyclic quad

... *CONVERSE opposite $\angle^s$ of cyclic quad.*

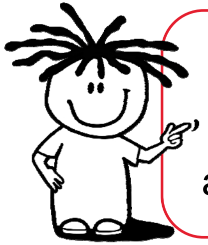
1.3 It is further given that $\hat{D}_2 = 30°$ and $A\hat{S}T = 100°$.

Prove that SD is not a diameter of circle BDS. (4)



If SD is a diameter, then $S\hat{B}D = 90°$

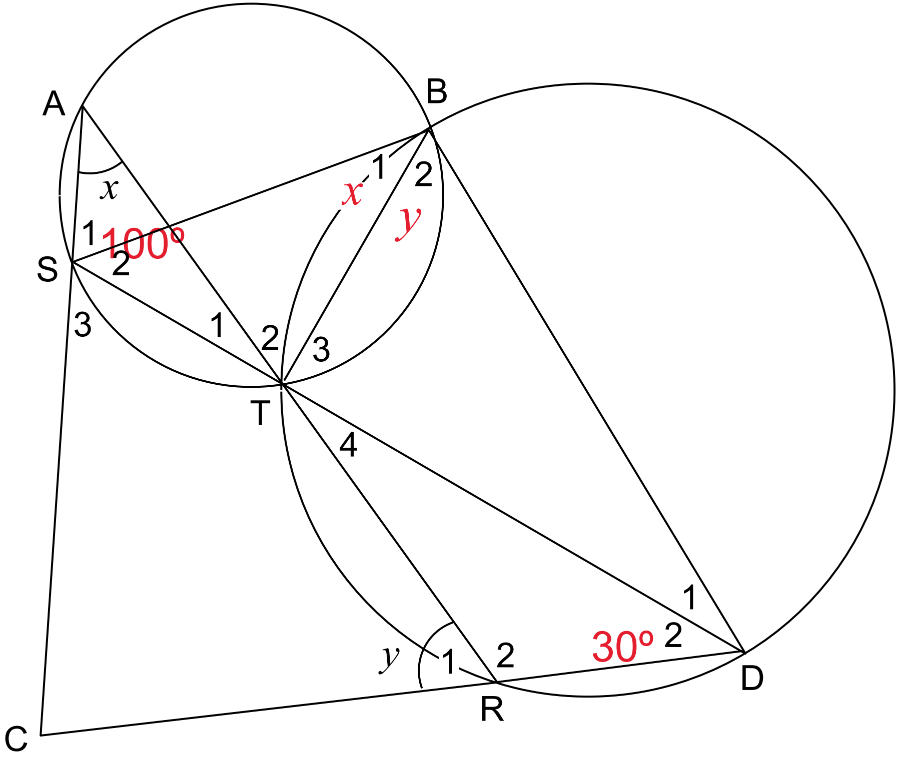
If SD is NOT a diameter, then $S\hat{B}D \neq 90°$



Update your diagram with new information, as well as angles found in 1.1 and/or 1.2.

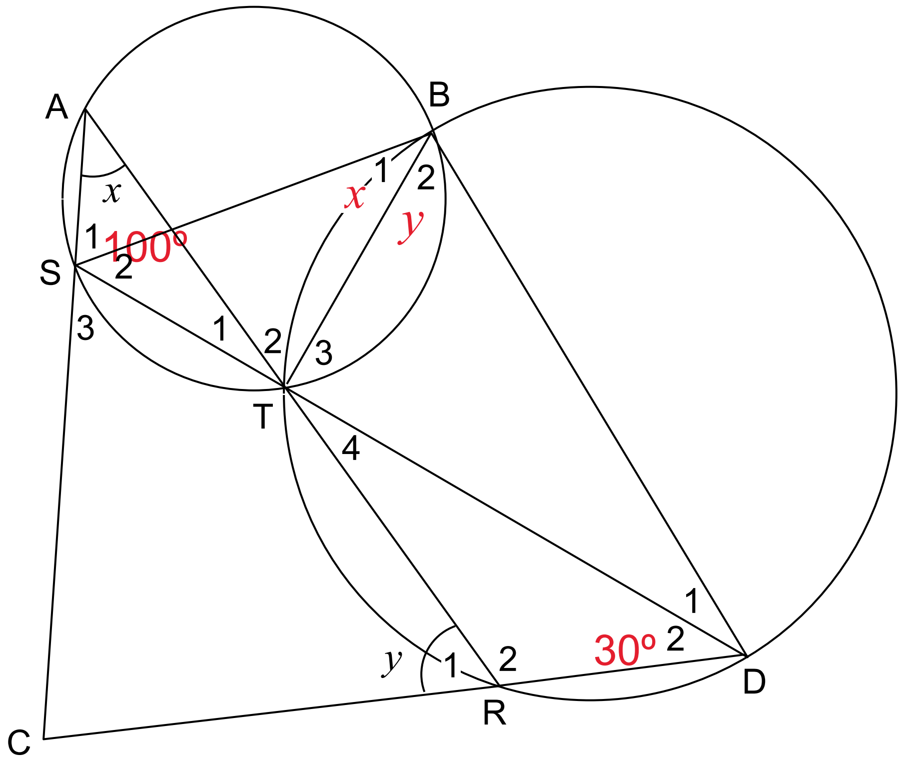
1.3 It is further given that $\hat{D}_2 = 30^\circ$ and $A\hat{S}T = 100^\circ$.

Prove that SD is not a diameter of circle BDS. (4)



1.3 It is further given that $\hat{D}_2 = 30^\circ$ and $A\hat{S}T = 100^\circ$.

Prove that SD is not a diameter of circle BDS. (4)



Solution

$S\hat{C}D = 70^\circ$ ... *exterior ∠ of ΔSCD*

$S\hat{B}D = 110^\circ$ ... *opposite ∠^{s} cyclic quadrilateral*

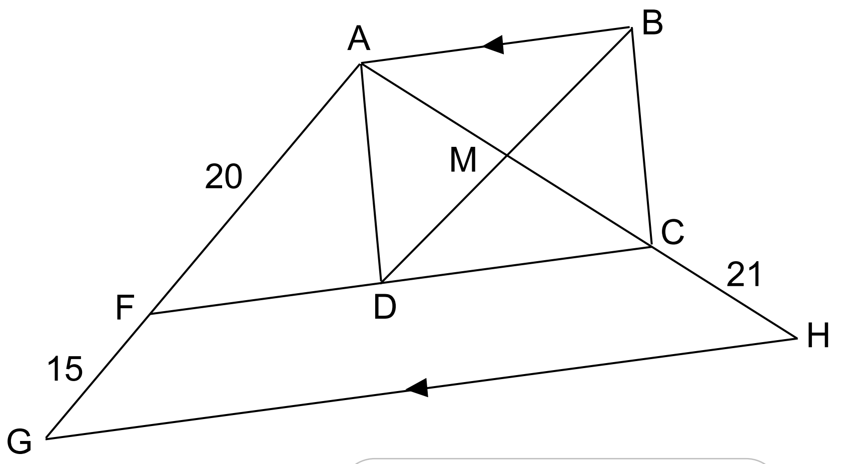
DS is not a diameter.

It does not subtend a right angle.

Proportionality

Example 2 (DBE Nov 2018 Q8.2) 59%

- In the diagram, ΔAGH is drawn.
- F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units.
- D is a point on FC such that ABCD is a rectangle with AB also parallel to GH.
- The diagonals of ABCD intersect at M, a point on AH.



2.1 Explain why FC || GH. (1)

2.2 Calculate, with reasons, the length of DM. (5)

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 17
- TAS Gr 12 Euclidean Geometry Video 5

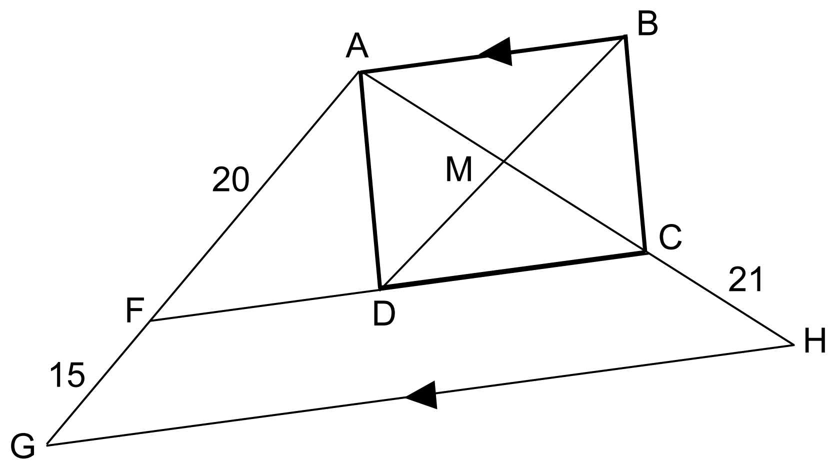
2.1 Explain why FC || GH. (1)

Solution

2.1 FC || AB . . . *opposite sides of a rectangle*

& AB || GH . . . *given*

∴ **FC || GH** ✓



2.2 Calculate, with reasons, the length of DM. (5)

Solution

2.2 **In ΔAGH:**

$$\frac{AC}{CH} = \frac{AF}{FG} \quad \ldots \textit{ proportion theorem; } FC \parallel GH$$

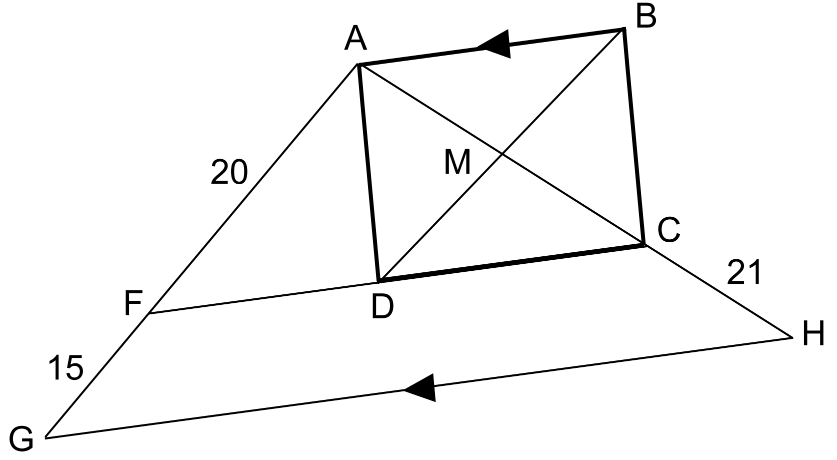
$$\therefore \frac{AC}{21} = \frac{20}{15}$$

$$\therefore AC = \frac{21 \times 20}{15} = 28 \text{ units}$$

$\therefore$ DB (= AC) = 28 units ... *diagonals of a rectangle are equal*

$\therefore$ DM $= \frac{1}{2}(28)$ ... *diagonals of a $||^m$ (and $\therefore$ a rectangle) bisect one another*

= **14 units** ➤

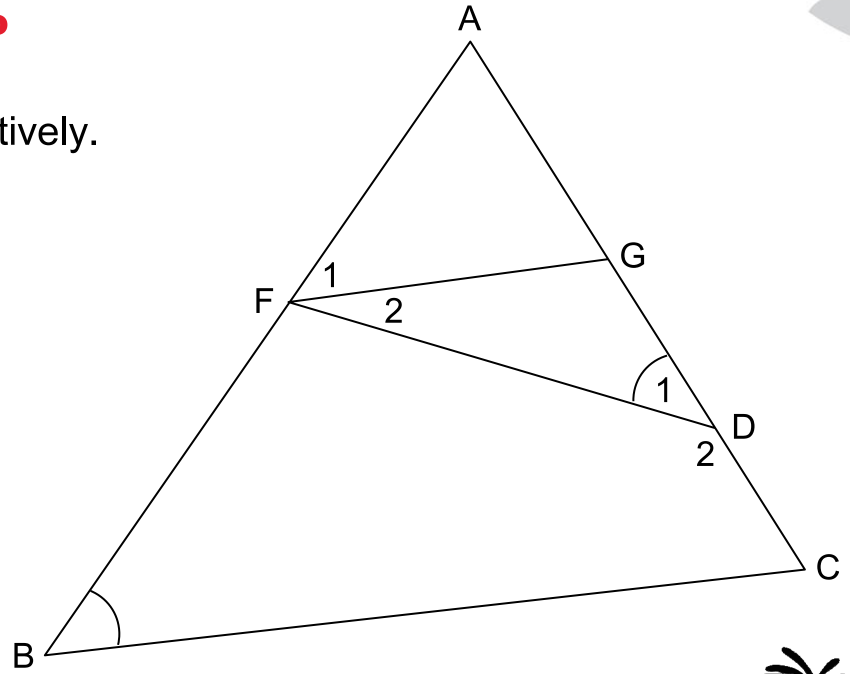


Proportionality

Example 3 (DBE Nov 2020 Q8.2) 52%

3. In ΔABC, F and G are points on sides AB and AC respectively.

D is a point on GC such that $\hat{D}_1 = \hat{B}$.

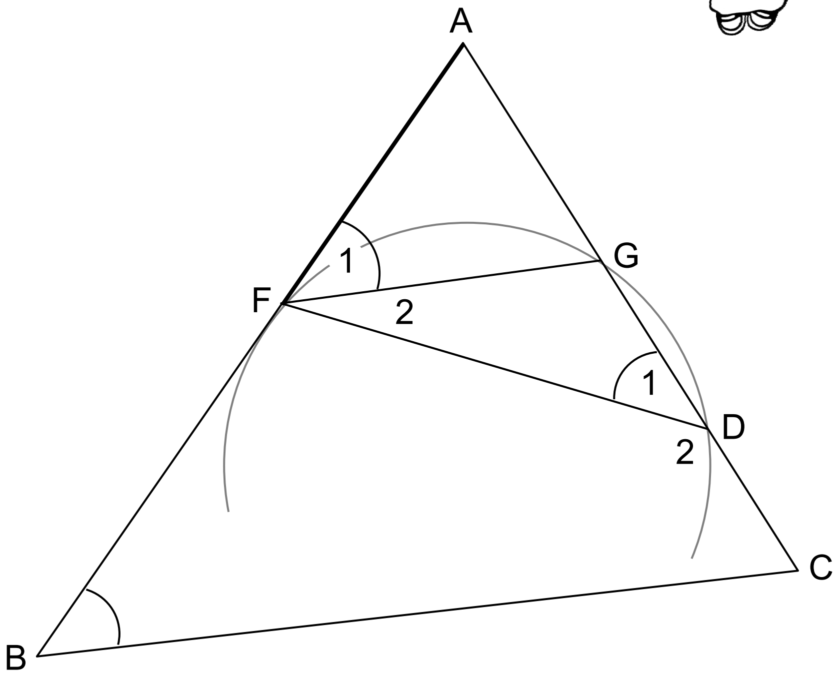


(a) If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)

(b) If it is further given that $\frac{AF}{FB} = \frac{2}{5}$, $AC = 2x - 6$ and $GC = x + 9$, then calculate the value of x. (4)

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 27
- TAS Gr 12 Euclidean Geometry Video 6

3. (a) If AF is a tangent to the circle passing through points F, G and D, then prove, giving reasons, that FG || BC. (4)



Solution

3. (a) $\hat{F}_1 = \hat{D}_1$ . . . *tan chord theorem*

But $\hat{D}_1 = \hat{B}$ . . . *given*

$\therefore\ \hat{F}_1 = \hat{B}$

$\therefore$ FG || BC . . . *corresponding* $\angle^s$ *equal*

a converse theorem

3. (b) If it is further given that $\frac{AF}{FB} = \frac{2}{5}$,

AC = $2x - 6$ and GC = $x + 9$,

then calculate the value of x. (4)

Solution

3. (b) $AG = (2x - 6) - (x + 9)$

$= x - 15$ ➡

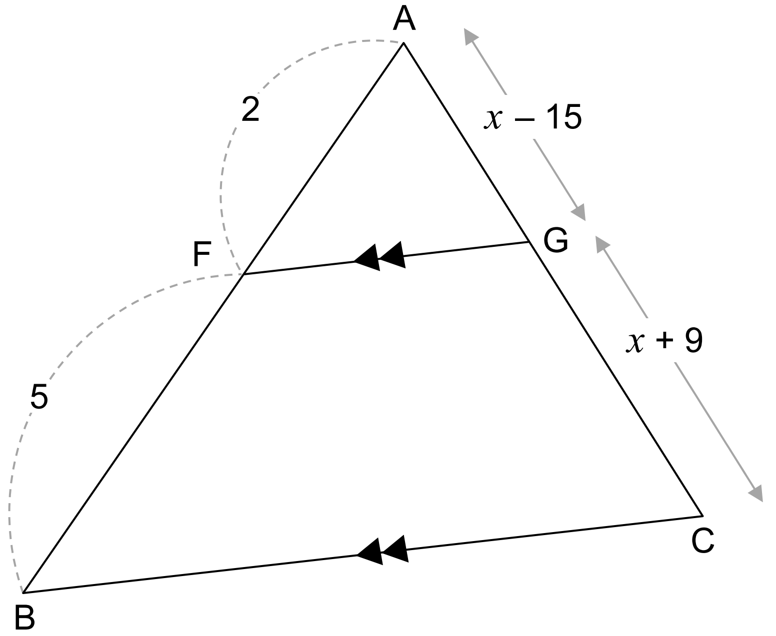
$\frac{AG}{GC} = \frac{AF}{FB}$ . . . *proportion theorem; FG || BC*

$\therefore \frac{x - 15}{x + 9} = \frac{2}{5}$

$\therefore 5x - 75 = 2x + 18$

$\therefore 3x = 93$

$\therefore \boldsymbol{x = 31}$ ◀



Solution

3. (b) cont.

OR:

AB : FB = 7 : 5

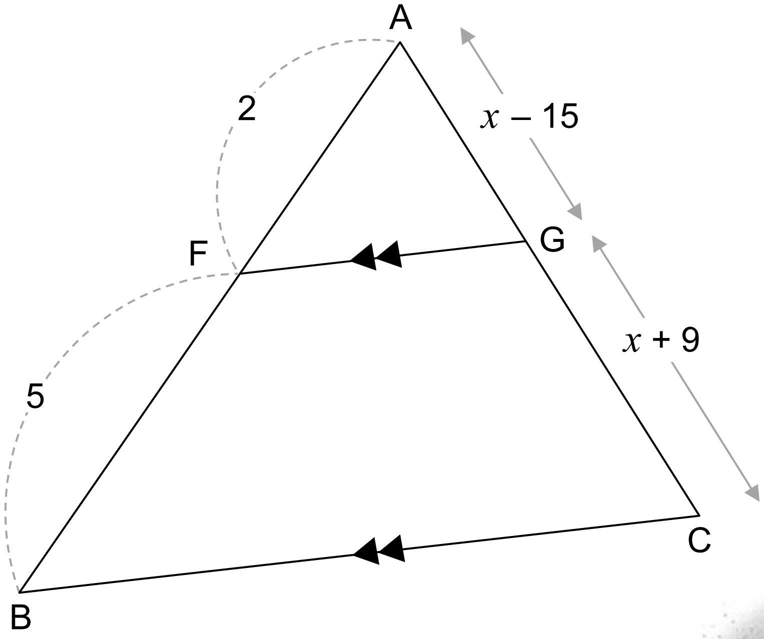
$$\frac{AC}{GC} = \frac{AB}{FB} \quad \ldots \textit{proportion theorem; } FG \parallel BC$$

$$\therefore \frac{2x - 6}{x + 9} = \frac{7}{5}$$

$$\therefore 10x - 30 = 7x + 63$$

$$\therefore 3x = 93$$

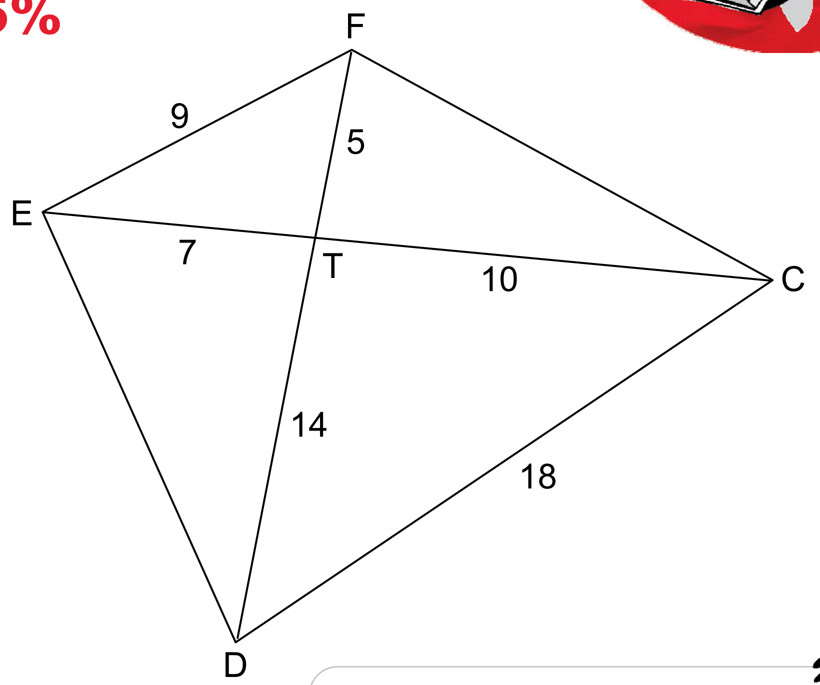
$$\therefore \mathbf{x = 31}$$



Similarity

Example 4 (DBE Nov 2019 Q8.2) 25%

- In the diagram, the diagonals of quadrilateral CDEF intersect at T.
- EF = 9 units, DC = 18 units, ET = 7 units, TC = 10 units, FT = 5 units and TD = 14 units.



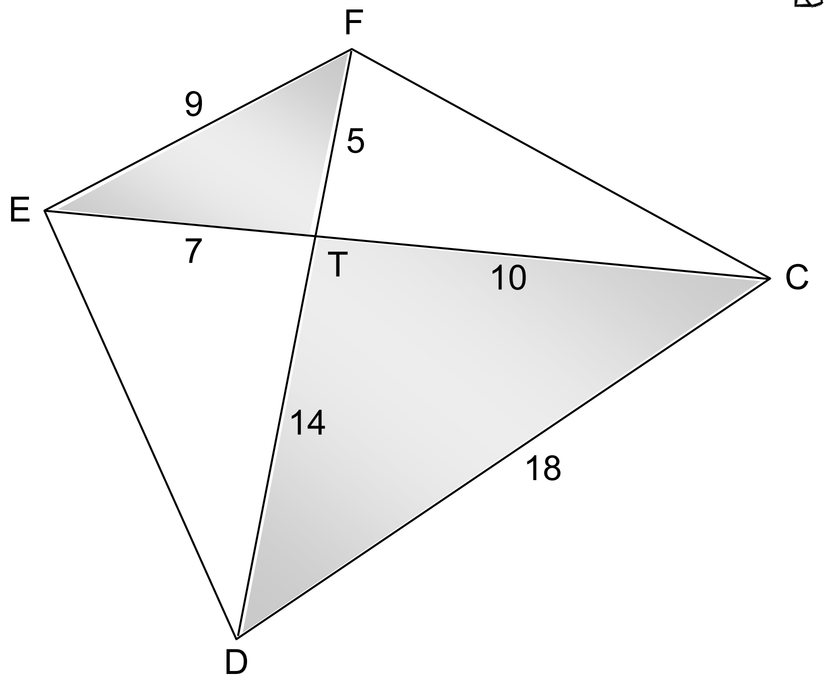
Prove, with reasons, that:

4.1 $E\hat{F}D = E\hat{C}D$ (4)

4.2 $D\hat{F}C = D\hat{E}C$ (3)

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 22
- TAS Gr 12 Euclidean Geometry Video 7

4.1 Prove, with reasons, that $E\hat{F}D = E\hat{C}D$ (4)



Solution

4.1 In Δ^s FTE and CTD:

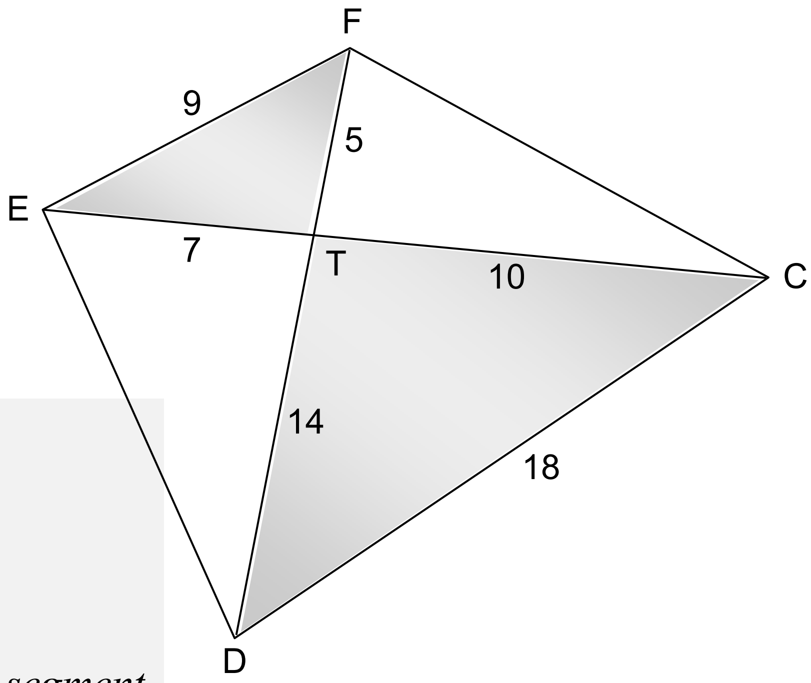
$$\frac{FT}{CT} = \frac{TE}{TD} = \frac{FE}{CD} = \frac{1}{2} \quad \ldots \frac{5}{10} = \frac{7}{14} = \frac{9}{18}$$

$\therefore$ $\Delta FTE \;|||\; \Delta CTD$ ... *proportional sides*

$\therefore$ $T\hat{F}E = T\hat{C}D$ ... *Δ^s are equiangular*

i.e. $\mathbf{E\hat{F}D = E\hat{C}D}$ ◄

4.2 Prove, with reasons, that $D\hat{F}C = D\hat{E}C$ (3)



Solution

4.2 $E\hat{F}D = E\hat{C}D$ . . . *proved in 4.1*

$\therefore$ CDEF is a cyclic quadrilateral . . . *converse* $\angle^s$ *in the same segment*

$\therefore$ **$D\hat{F}C = D\hat{E}C$** ✓ . . . $\angle^s$ *in the same segment*

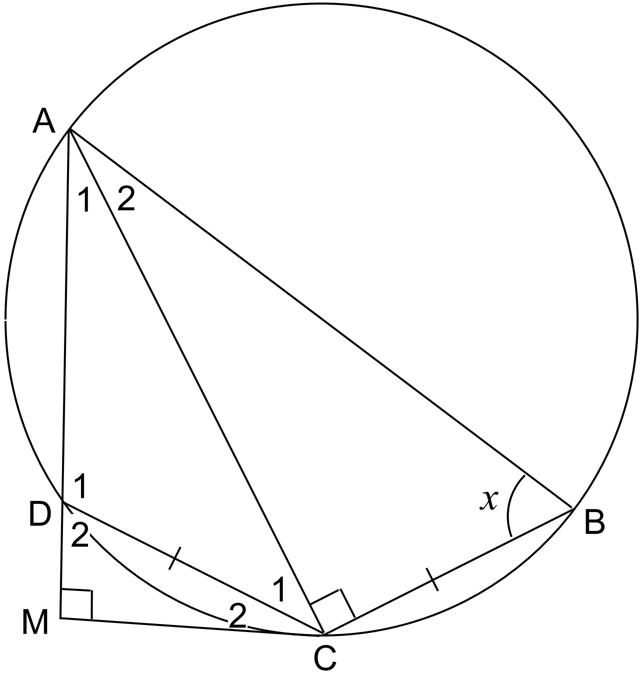
THE ANSWER SERIES *Your Key to Exam Success*

Mixed

Example 5 (DBE Nov 2018 Q10) 31%

- In the diagram, ABCD is a cyclic quadrilateral such that AC ⊥ CB and DC = CB.
- AD is produced to M such that AM ⊥ MC.

Let $\hat{B} = x$.



5.1 Prove that:

48%

(a) MC is a tangent to the circle at C. (5)

(b) ΔACB ||| ΔCMD (3)

5.2 Hence, or otherwise, prove that:

18%

(a) $\dfrac{CM^2}{DC^2} = \dfrac{AM}{AB}$ (6)

(b) $\dfrac{AM}{AB} = \sin^2 x$ (2)

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 17
- TAS Gr 12 Euclidean Geometry Video 7

5.1 (a) Prove that MC is a tangent to the circle at C. (5)

Solution

We will use the CONVERSE of the tan chord theorem.

(a) We need to prove that $\hat{C}_2 = \hat{A}_1$ . . .

$\hat{D}_2 = \hat{B}$ . . . *exterior ∠ of cyclic quad.*

$= \boldsymbol{x}$

$\therefore$ In ΔDMC:

$\hat{C}_2 = 90^\circ - x$ . . . *sum of* $\angle^s$ *in* Δ

In ΔACB: $\hat{A}_2 = 90^\circ - x$ . . . *sum of* $\angle^s$ *in* Δ

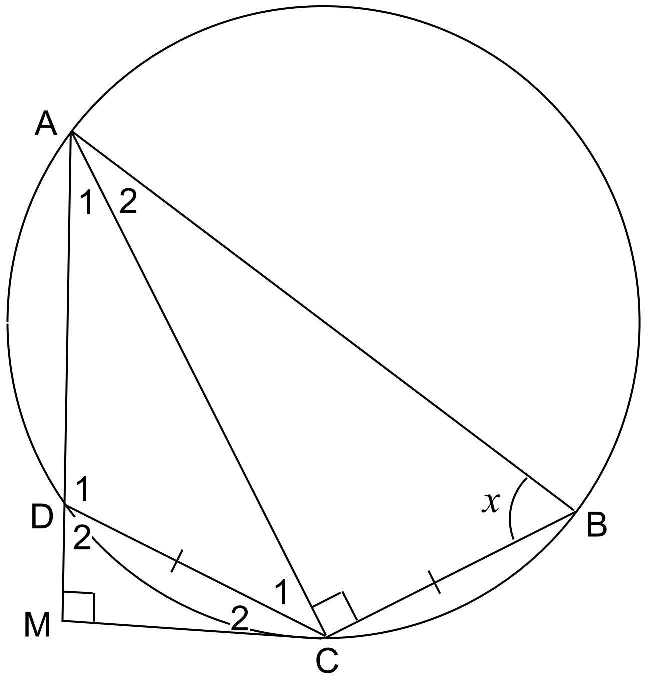
But $\hat{A}_1 = \hat{A}_2$ . . . *equal chords subtend equal angles*

$\therefore \hat{A}_1 = 90^\circ - x$

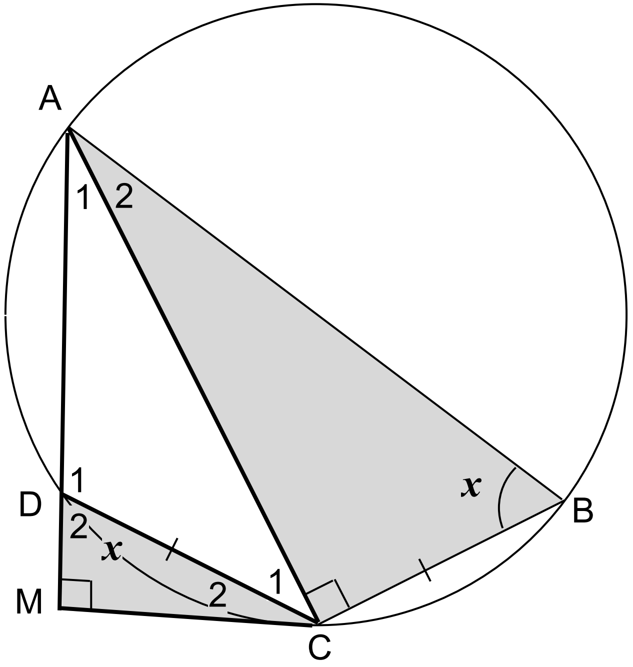
$\therefore \hat{C}_2 = \hat{A}_1$

$\therefore$ **MC is a tangent to the circle at C** ➤

. . . *CONVERSE of tan chord theorem*



5.1 (b) Prove that ΔACB ||| ΔCMD (3)



Solution

(b) In Δ^s ACB and CMD

(1) $A\hat{C}B = \hat{M}$ (= 90º) . . . *given*

(2) $\hat{B} = \hat{D}_2$ . . . *exterior ∠ of cylic quad.*

∴ **ΔACB ||| ΔCMD** ➤ . . . ∠∠∠

5.2 (a) Prove that $\frac{CM^2}{DC^2} = \frac{AM}{AB}$ (6)

Solution

(a) $\frac{CM}{DC} = \frac{AC}{AB}$ . . . ❶ . . . *ΔACB ||| ΔCMD*

But, in Δ^s CMD and AMC

(1) $\hat{M}$ (= 90°) is common

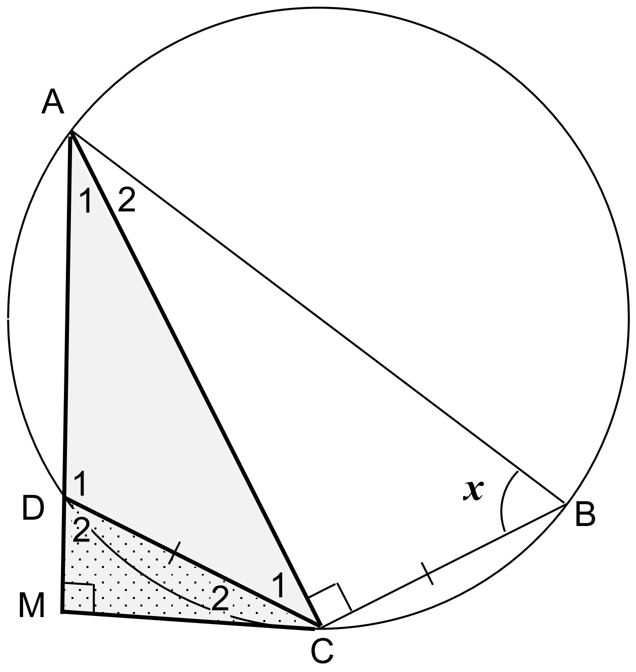
(2) $\hat{C}_2 = \hat{A}_1$ . . . *proved in 5.1(a)*

∴ ΔCMD ||| ΔAMC . . . *equiangular Δ^s*

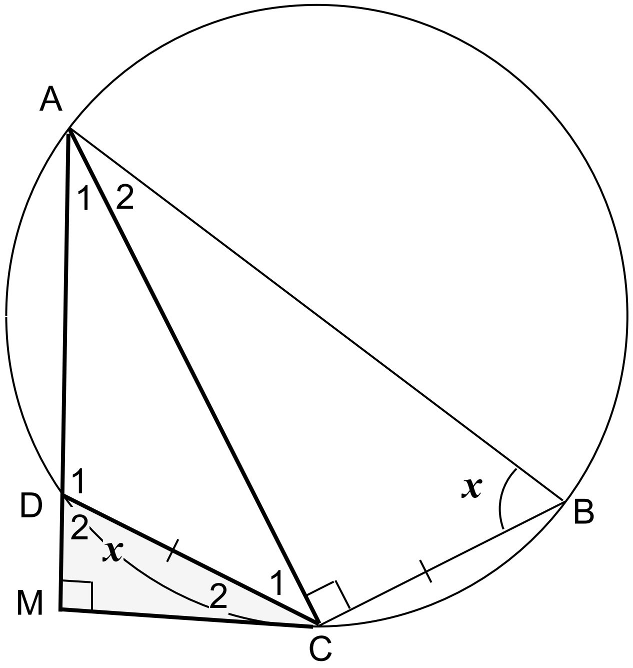
$\therefore \frac{CM}{DC} = \frac{AM}{AC}$ . . . ❷ . . . *ΔCMD ||| ΔAMC*

$\therefore \frac{CM}{DC} \times \frac{CM}{DC} = \frac{\cancel{AC}}{AB} \times \frac{AM}{\cancel{AC}}$ . . . *see* ❶ *and* ❷

$\therefore \mathbf{\frac{CM^2}{DC^2} = \frac{AM}{AB}}$ ➤



5.2 (b) Prove that $\frac{AM}{AB} = \sin^2 x$ (2)



Solution

(b) $\frac{AM}{AB} = \frac{CM^2}{DC^2}$ . . . *proved in 5.2(a)*

But, in ΔDMC: $\frac{CM}{DC} = \sin x$

$\therefore$ $\mathbf{\frac{AM}{AB} = \sin^2 x}$ ✓

Example 6 (DBE Nov 2020 Q10) **43%**

- Gr 12 Maths **Toolkit**: DBE Past Papers, p. 27
- TAS Gr 12 Euclidean Geometry Video 8

- In the diagram, a circle passes through D, B and E.
- Diameter ED of the circle is produced to C and AC is a tangent to the circle at B.
- M is a point on DE such that AM ⊥ DE.
- AM and chord BE intersect at F.

6.1 Prove, giving reasons, that: **55%**

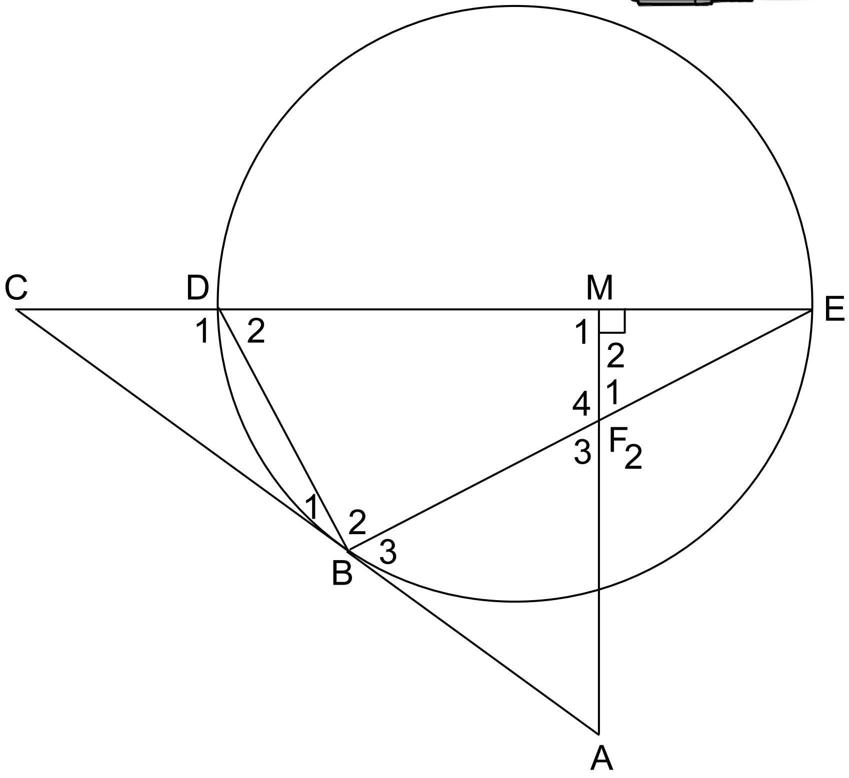
(a) FBDM is a cyclic quadrilateral (3)

(b) $\hat{B}_3 = \hat{F}_1$ (4)

(c) $\Delta CDB \parallel\!| \Delta CBE$ (3)

6.2 If it is further given that CD = 2 units and DE = 6 units, calculate the length of: **26%**

(a) BC (b) DB (3)(4)



6.1 (a) Prove, giving reasons, that FBDM is a cyclic quadrilateral (3)

Solution

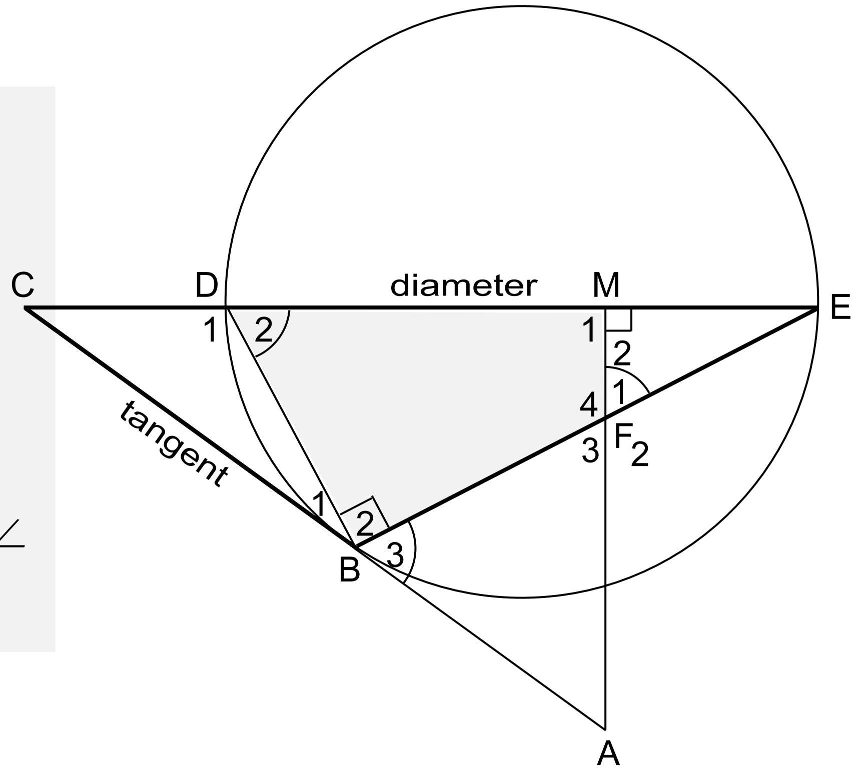
Tip: shade the quadrilateral

6.1 (a) $\hat{M}_2 = 90^\circ$

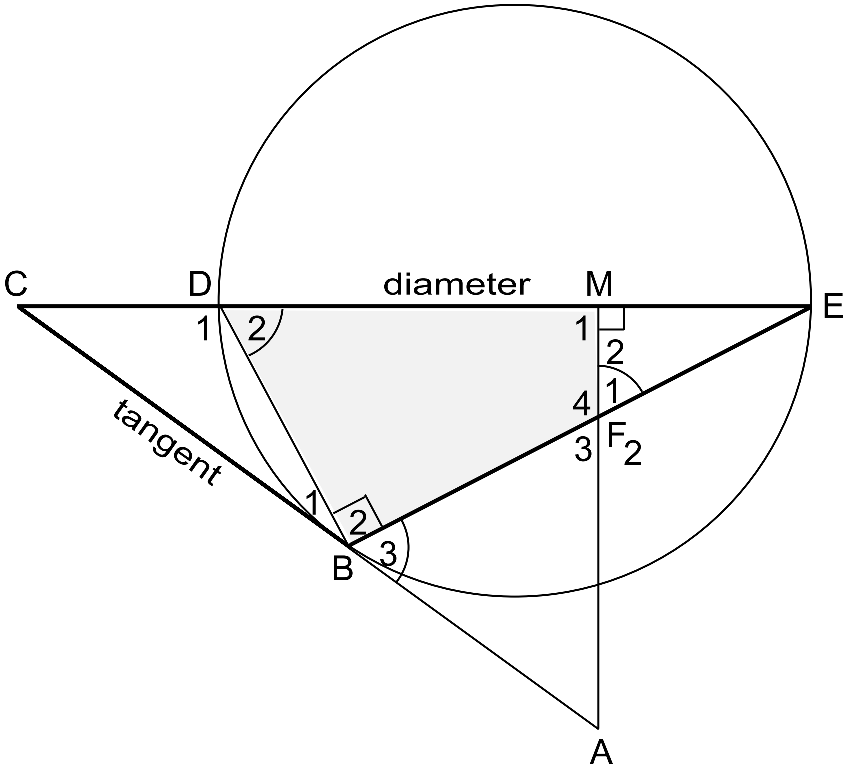
& $\hat{B}_2 = 90^\circ$ ... *∠ in semi-⊙*

$\therefore \hat{M}_2 = \hat{B}_2$

$\therefore$ **FBDM is a cyclic quad.** ◄ ... *CONVERSE of ext. ∠ of cyclic quad.*



6.1 (b) Prove, giving reasons, that $\hat{B}_3 = \hat{F}_1$ (4)



Solution

6.1 (b) $\hat{B}_3 = \hat{D}_2$ . . . *tan chord theorem*

$= \hat{F}_1$ ◄ . . . *ext.∠ of c.q. FBDM (6.1(a))*

6.1 (c) Prove, giving reasons, that $\Delta CDB \,|||\, \Delta CBE$ (3)

Solution

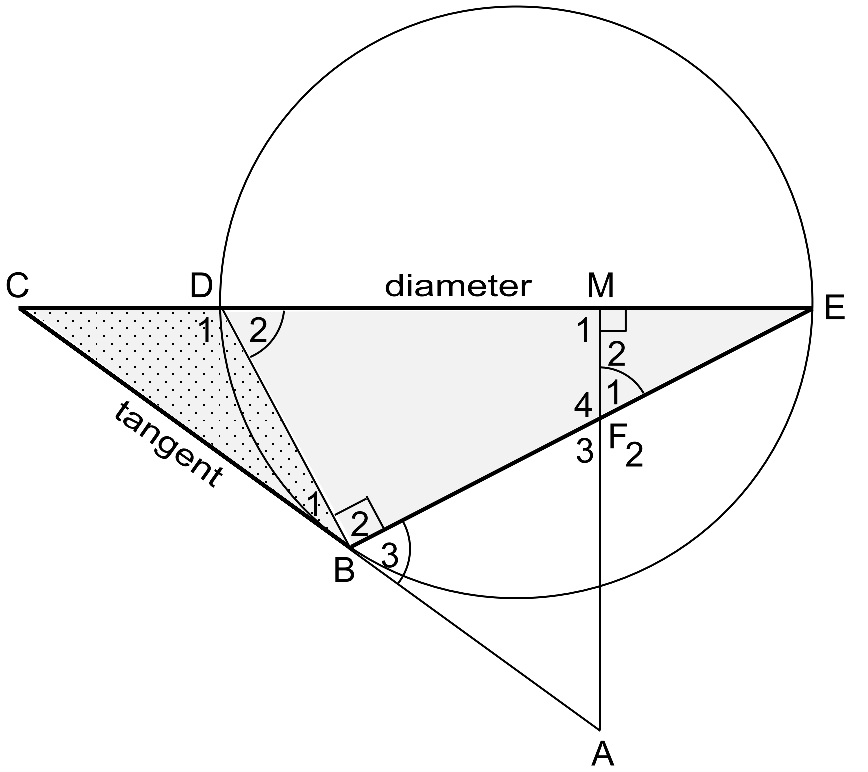
Tip: mark the Δ^s clearly

6.1 (c) In Δ^s CDB & CBE

(1) $\hat{C}$ is common

(2) $\hat{B}_1 = \hat{E}$ . . . *tan chord theorem*

$\therefore$ $\Delta CDB \,|||\, \Delta CBE$ . . . $\angle\angle\angle$



6.2 (a) If it is further given that CD = 2 units and DE = 6 units, calculate the length of BC (3)

Solution

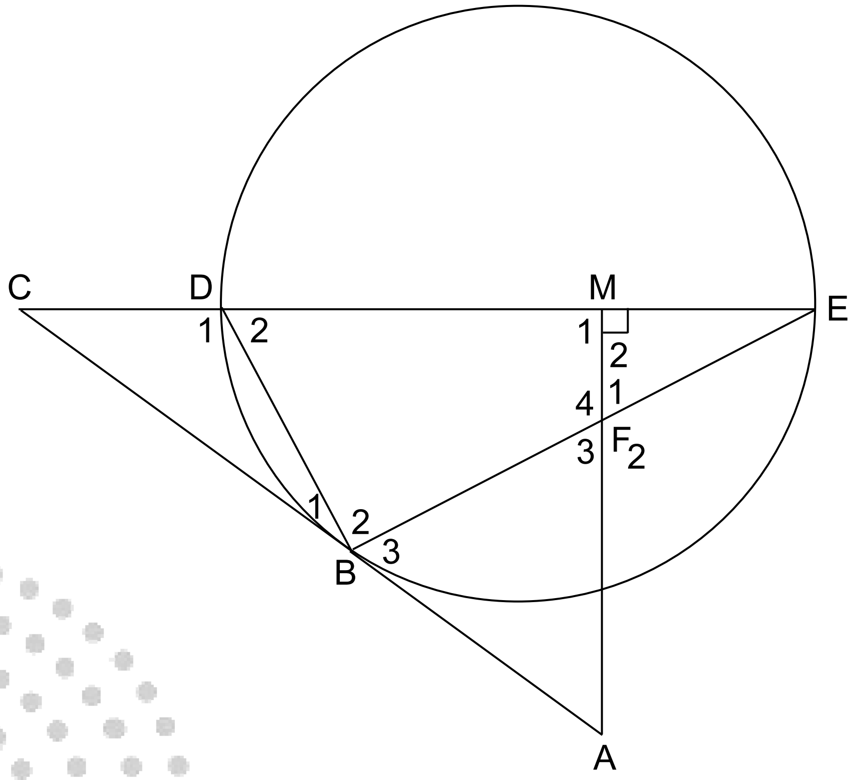
6.2 (a) $\therefore$ $\Delta CDB \;|||\; \Delta CBE$ . . . *from 6.1(c)*

$\therefore \dfrac{CD}{BC} = \dfrac{BC}{CE} \left(= \dfrac{DB}{BE} \right)$ . . . *equiangular* Δ^s

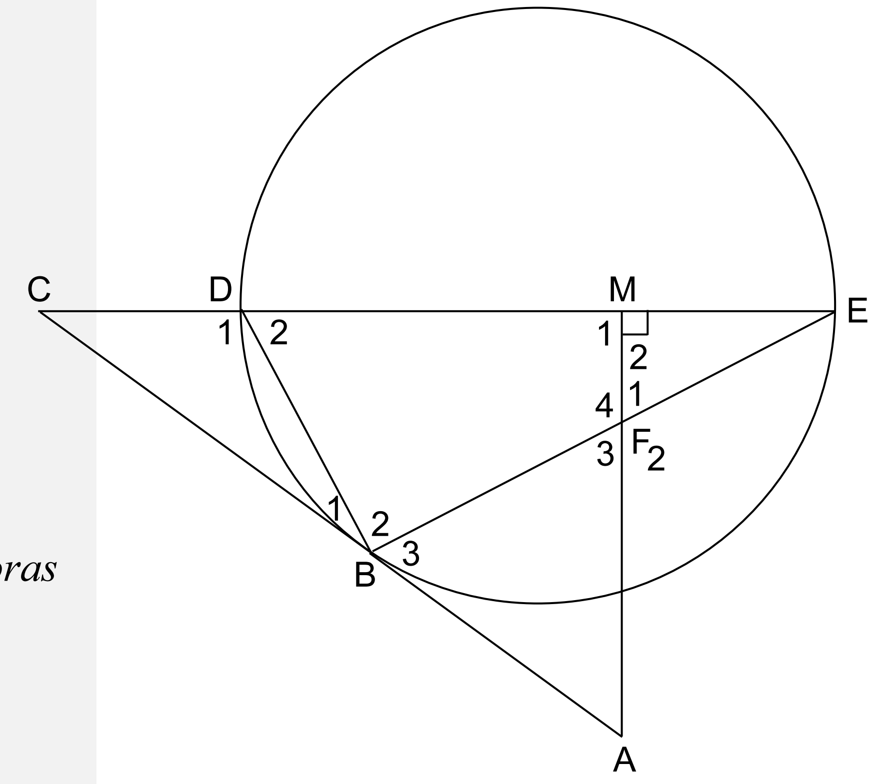
$\therefore \dfrac{2}{BC} = \dfrac{BC}{2 + 6}$

$\therefore BC^2 = 16$

$\therefore$ **BC = 4 units**



6.2 (b) If it is further given that CD = 2 units and DE = 6 units, calculate the length of DB (4)



Solution

6.2 (b) $\frac{CD}{BC}\left(=\frac{BC}{CE}\right) = \frac{DB}{BE}$

$\frac{DB}{BE} = \frac{CD}{BC} = \frac{2}{4} = \frac{1}{2}$

$\therefore$ BE = 2DB

Let DB = x, then BE = $2x$

In ΔDBE: $\hat{B}_2 = 90°$ ... *$\angle$ in semi-$\odot$*

$\therefore DB^2 + BE^2 = DE^2$ ... *Theorem of Pythagoras*

$\therefore x^2 + (2x)^2 = 6^2$

$\therefore x^2 + 4x^2 = 36$

$\therefore 5x^2 = 36$

$\therefore x^2 = \frac{36}{5}$

$\therefore$ **x ≈ 2,68 units** ◄

The Major Issues

Language

Knowledge

Logic

and then

Strategies

THE ANSWER SERIES Your Key to Exam Success

Theory without practice is empty

Practice without theory is blind

Philosopher, Immanuel Kant (18th century philosopher)

The Answer Series

CONTENT FRAMEWORK: Gr 8 – 12

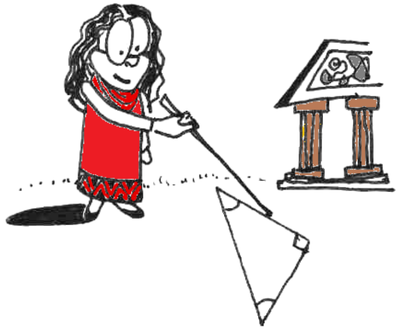
- LINES
- TRIANGLES
- QUADRILATERALS
- CIRCLES (Gr 11)

(Gr 8 → 10) [LINES, TRIANGLES, QUADRILATERALS]

Gr 12?

Gr 12

- **THEOREM OF PYTHAGORAS (Gr 8)**
- **SIMILAR Δs (Gr 9)**
- **MIDPOINT THEOREM (Gr 10)**



- **THE PROPORTION THEOREM (Gr 12)**

Ratio Proportion Area

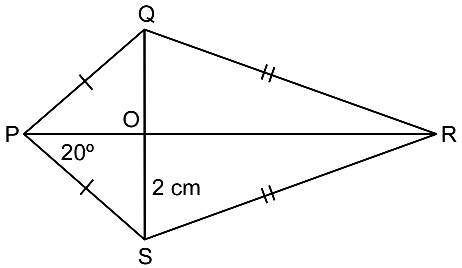
KZN Grade 12 2024 ATP

TERM 2				
NUMBER OF DAYS	**DATE STARTED**	**DATE COMPLETED**	**TOPIC**	**CURRICULUM STATEMENT**
03 – 10/04 (6 days)			EUCLIDEAN GEOMETRY	1. Revise earlier work on the necessary and sufficient conditions for polygons to be similar. 2. Prove (accepting results established in earlier grades) that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Midpoint Theorem as a special case of this theorem). 3. Solve proportionality problems and prove riders.
11 – 18/04 (6 days)			EUCLIDEAN GEOMETRY	4. Prove (accepting results established in earlier grades): 4.1 that equiangular triangles are similar; 4.2 that triangles with sides in proportion are similar; and 4.3 the Pythagorean Theorem by similar triangles. 5. Solve similarity problems and prove riders.

GR 10 – 12 EXEMPLAR GEOMETRY

GRADE 10: QUESTIONS

1. PQRS is a kite such that the diagonals intersect in O. OS = 2 cm and $\hat{OPS}$ = 20º.

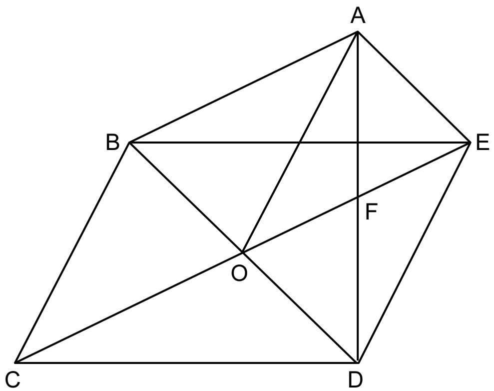


1.1 Write down the length of OQ. (2)

1.2 Write down the size of $\hat{POQ}$. (2)

1.3 Write down the size of $\hat{QPS}$. (2) [6]

2. In the diagram, BCDE and AODE are parallelograms.



2.1 Prove that OF || AB. (4)

2.2 Prove that ABOE is a parallelogram. (4)

2.3 Prove that ΔABO ≡ ΔEOD. (5) [13]

GRADE 10: MEMOS

1.1 OQ = 2 cm ✓ . . . *the longer diagonal of a kite bisects the shorter diagonal*

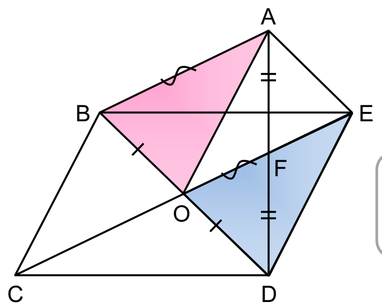
1.2 $\hat{POQ}$ = 90º ✓ . . . *the diagonals of a kite intersect at right angles*

1.3 $\hat{QPO}$ = 20º . . . *the longer diagonal of a kite bisects the (opposite) angles of a kite*

∴ $\hat{QPS}$ = 40º ✓

---

2. **Hint:** Use highlighters to mark the various $||^{ms}$ and Δ^s



The highlighted Δ^s (and their sides) refer to Question 2.3.

2.1 In ΔDBA:

O is the midpt of BD . . . *diagonals of $||^m$ BCDE bisect each other*

& F is the midpt of AD . . . *diagonals of $||^m$ AODE bisect each other*

∴ OF || AB ✓ . . . *the line joining the midpoints of two sides of a Δ is || to the 3rd side*

2.2 AE || OD . . . *opp. sides of $||^m$ AODE*

∴ AE || BO

and OF || AB . . . *proven above*

∴ OE || AB

∴ ABOE is a $||^m$ . . . *both pairs of opposite sides are parallel*

OR: In $||^m$ AODE: AE = and || OD . . . *opp. sides of $||^m$*

But OD = BO . . . *O proved midpt of BD of BD in 2.1*

∴ AE = and || BO

∴ ABOE is a $||^m$ ✓ . . . *1 pr of opp. sides = and ||*

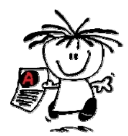
2.3 In Δ^s ABO and EOD

1) AB = EO . . . *opposite sides of $||^m$ ABOE*

2) BO = OD . . . *proved in 2.1*

3) AO = ED . . . *opposite sides of $||^m$ AODE*

∴ ΔABO ≡ ΔEOD ✓ . . . *SSS*



GRADE 11: QUESTIONS

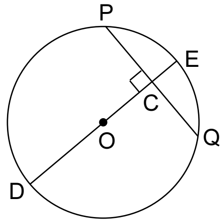
1.1 Complete the statement so that it is valid:

The line drawn from the centre of the circle perpendicular to the chord . . . (1)

1.2 In the diagram, O is the centre of the circle.

The diameter DE is perpendicular to the chord PQ at C.

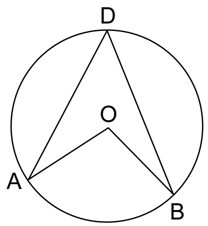
DE = 20 cm and CE = 2 cm.



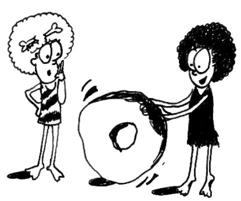
Calculate the length of the following with reasons:

1.2.1 OC 1.2.2 PQ (2)(4) [7]

2.1 In the diagram, O is the centre of the circle and A, B and D are points on the circle.



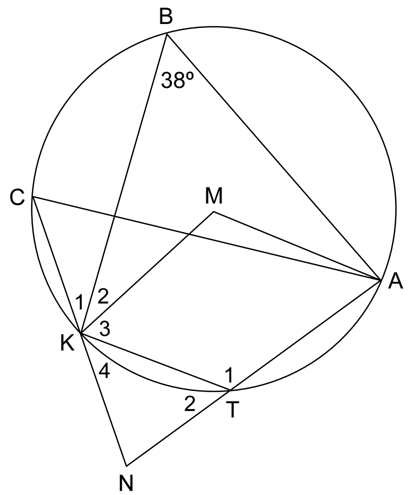
Use Euclidean geometry methods to prove the theorem which states that $A\hat{O}B = 2A\hat{D}B$. (5)



2.2 In the diagram, M is the centre of the circle. A, B, C, K and T lie on the circle.

AT produced and CK produced meet in N.

Also NA = NC and $\hat{B} = 38°$.



2.2.1 Calculate, with reasons, the size of the following angles:

(a) $K\hat{M}A$ (b) $\hat{T}_2$ (2)(2)

(c) $\hat{C}$ (d) $\hat{K}_4$ (2)(2)

2.2.2 Show that NK = NT. (2)

2.2.3 Prove that AMKN is a cyclic quadrilateral. (3) [18]

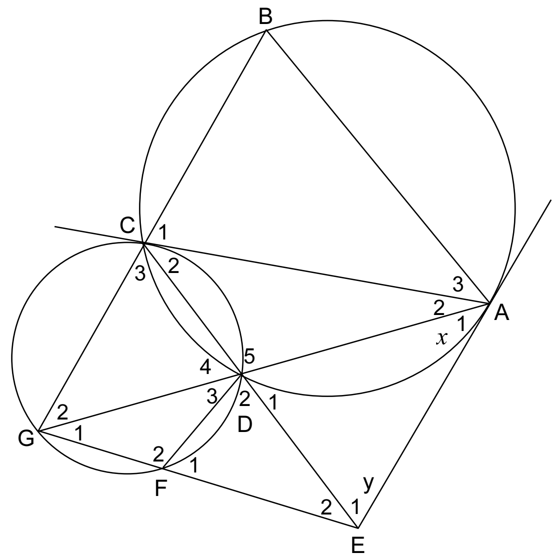
3.1 Complete the following statement so that it is valid:

The angle between a chord and a tangent at the point of contact is . . . (1)

3.2 In the diagram, EA is a tangent to circle ABCD at A.

AC is a tangent to circle CDFG at C.

CE and AG intersect at D.



If $\hat{A}_1 = x$ and $\hat{E}_1 = y$, prove the following with reasons:

3.2.1 BCG || AE (5)

3.2.2 AE is a tangent to circle FED (5)

3.2.3 AB = AC (4) [15]

GRADE 11: MEMOS

1.1 . . . **bisects the chord** ◄

1.2.1 OE = OD = $\frac{1}{2}$(20) = 10 cm . . . *radii* $= \frac{1}{2}$ *diameter*

∴ **OC = 8 cm** ◄ . . . *CE = 2 cm*

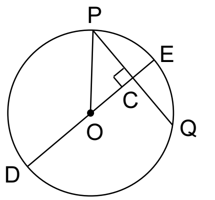
1.2.2 In ΔOPC:

$PC^2 = OP^2 - OC^2$ . . . *Pythagoras*

$= 10^2 - 8^2$

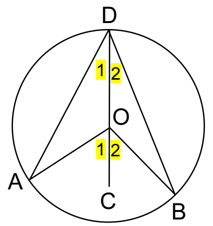
$= 36$

∴ PC = 6 cm



∴ **PQ = 12 cm** ◄ . . . *line from centre ⊥ chord*

2.1 Construction: Join DO and produce it to C



Proof:

Let $\hat{D}_1 = x$

then $\hat{A} = x$ . . . *radii; ∠s opp = sides*

∴ $\hat{O}_1 = 2x$ . . . *ext. ∠ of ΔDAO*

Similarly: Let $\hat{D}_2$ = y

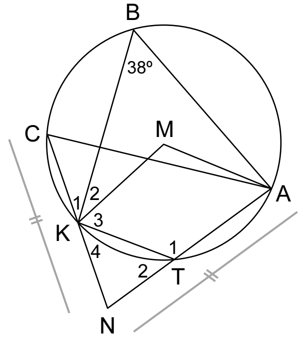
then, $\hat{O}_2$ = 2y

∴ $A\hat{O}B = 2x + 2y$

$= 2(x + y)$

$=$ **$2\,A\hat{D}B$** ◄

2.2



2.2.1 (a) $K\hat{M}A = 2(38°)$ . . . *∠ at centre = 2 × ∠ at circumference*

$=$ **76º** ◄

(b) **$\hat{T}_2$ = 38º** ◄ . . . *ext. ∠ of cyclic quad. BKTA*

(c) **$\hat{C}$ = 38º** ◄ . . . *∠s in the same segment or, ext. ∠ of cyclic quad. CKTA*

(d) $N\hat{A}C$ = 38º . . . *∠s opp = sides*

∴ **$\hat{K}_4$ = 38º** ◄ . . . *ext. ∠ of c.q. CKTA*

2.2.2 In ΔNKT: $\hat{K}_4 = \hat{T}_2$ . . . *both = 38º in 2.2.1*

∴ **NK = NT** ◄ . . . *sides opp equal ∠s*

2.2.3 $K\hat{M}A$ = 2(38º) . . . *see 2.2.1(a)*

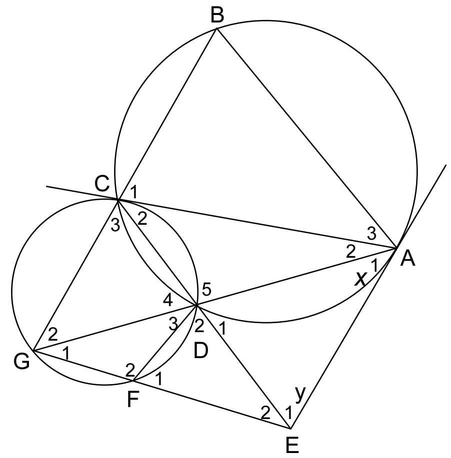
& $\hat{N}$ = 180º - 2(38º) . . . *sum of ∠s in ΔNKT (see 2.2.2)*

∴ $K\hat{M}A + \hat{N}$ = 180º

∴ **AMKN is a cyclic quadrilateral** ◄ . . . *opposite ∠s supplementary*

3.1 . . . **equal to the angle subtended by the chord in the alternate segment.** ◄

3.2



3.2.1 $\hat{A}_1 = x$ . . . *given*

∴ $\hat{C}_2 = x$ . . . *tan chord theorem*

∴ $\hat{G}_2 = x$ . . . *tan chord theorem*

∴ $\hat{A}_1$ = (alternate) $\hat{G}_2$

∴ **BCG || AE** ◄ . . . *(alternate ∠s equal)*

3.2.2 $\hat{F}_1 = \hat{C}_3$ . . . *ext. ∠ of cyclic quad. CGFD*

$= \hat{E}_1$ (= y) . . . *alternate ∠s ; BCG || AE*

∴ **AE is a tangent to ⊙FED** ◄ . . . *converse of tan chord theorem*

3.2.3 $\hat{C}_1 = C\hat{A}E$ . . . *alternate ∠s ; BCG || AE*

$= \hat{B}$ . . . *tan chord theorem*

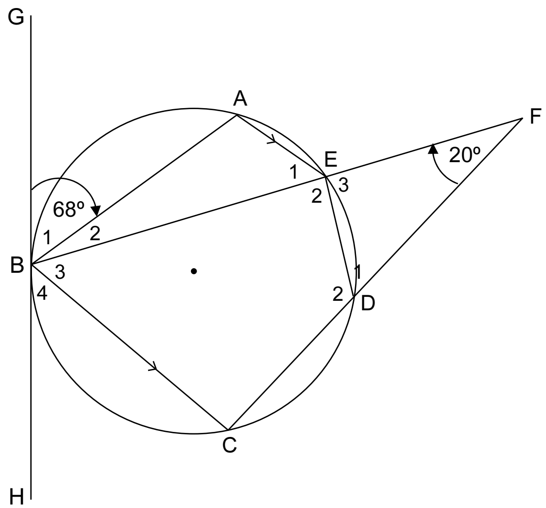
∴ **AB = AC** ◄ . . . *sides opposite equal ∠s*

GRADE 12: QUESTIONS

1.1 Complete the following statement:

The angle between the tangent and the chord at the point of contact is equal to . . . (1)

1.2 In the diagram, A, B, C, D and E are points on the circumference of the circle such that AE || BC. BE and CD produced meet in F. GBH is a tangent to the circle at B. $\hat{B}_1 = 68^\circ$ and $\hat{F} = 20^\circ$.



Determine the size of each of the following:

1.2.1 $\hat{E}_1$ (2)

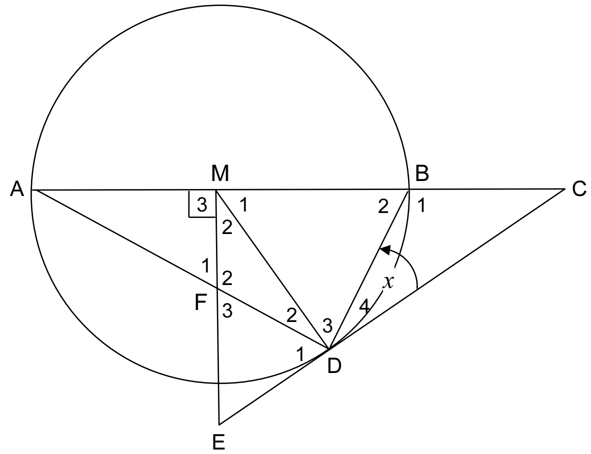
1.2.2 $\hat{B}_3$ (1)

1.2.3 $\hat{D}_1$ (2)

1.2.4 $\hat{E}_2$ (1)

1.2.5 $\hat{C}$ (2) [9]

2. In the diagram, M is the centre of the circle and diameter AB is produced to C. ME is drawn perpendicular to AC such that CDE is a tangent to the circle at D. ME and chord AD intersect at F. MB = 2BC.



2.1 If $\hat{D}_4 = x$, write down, with reasons, TWO other angles each equal to x. (3)

2.2 Prove that CM is a tangent at M to the circle passing through M, E and D. (4)

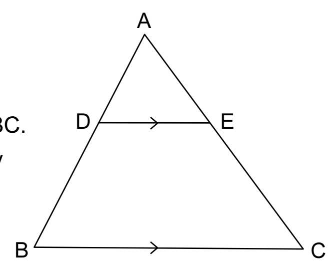
2.3 Prove that FMBD is a cyclic quadrilateral. (3)

2.4 Prove that $DC^2 = 5BC^2$. (3)

2.5 Prove that ΔDBC ||| ΔDFM. (4)

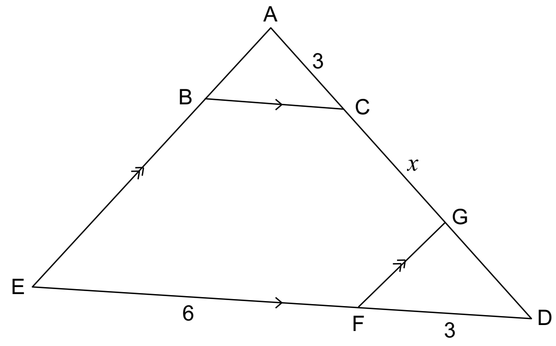
2.6 Hence, determine the value of $\frac{DM}{FM}$. (2) [19]

3.1 In the diagram, points D and E lie on sides AB and AC respectively of ΔABC such that DE || BC. Use Euclidean Geometry methods to prove the theorem which states that $\frac{AD}{DB} = \frac{AE}{EC}$.



(6)

3.2 In the diagram, ADE is a triangle having BC || ED and AE || GF. It is also given that AB : BE = 1 : 3, AC = 3 units, EF = 6 units, FD = 3 units and CG = x units.



Calculate, giving reasons:

3.2.1 the length of CD (3)

3.2.2 the value of x (4)

3.2.3 the length of BC (5)

3.2.4 the value of $\frac{\text{area } \Delta ABC}{\text{area } \Delta GFD}$ (5) [23]

GRADE 12: MEMOS

1.1 . . . the angle subtended by the chord in the alternate segment.

1.2.1 $\hat{E}_1 = \hat{B}_1$ . . . *tan chord theorem*
$= \mathbf{68^\circ}$ ◄

1.2.2 $\hat{B}_3 = \hat{E}_1$ . . . *alt.* $\angle^s$ *; AE || BC*
$= \mathbf{68^\circ}$ ◄

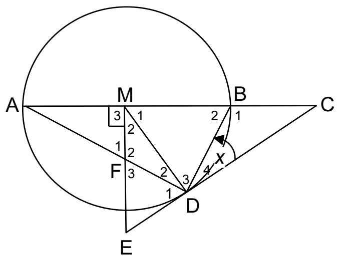
1.2.3 $\hat{D}_1 = \hat{B}_3$ . . . *ext.* $\angle$ *of cyclic quad.*
$= \mathbf{68^\circ}$ ◄

1.2.4 $\hat{E}_2 = \hat{D}_1 + 20^\circ$ . . . *ext.* $\angle$ *of* Δ
$= \mathbf{88^\circ}$ ◄

1.2.5 $\hat{C} = 180^\circ - \hat{E}_2$ . . . *opp.* $\angle^s$ *of cyclic quad.*
$= \mathbf{92^\circ}$ ◄

2.1 $\hat{A} = x$ . . . *tan chord theorem*
$\hat{D}_2 = x$ . . . $\angle^s$ *opp. equal sides*

2.2



$\hat{M}_1 = \hat{A} + \hat{D}_2$ . . . *ext.* $\angle$ *of* Δ
$= 2x$
$\therefore \hat{M}_2 = 90^\circ - 2x$ . . . $ME \perp AC$
& $M\hat{D}E = 90^\circ$ . . . *radius* $MD \perp$ *tangent CDE*
$\therefore \hat{E} = 2x$ . . . *sum of* $\angle^s$ *in* ΔMED
$\therefore \hat{M}_1 = \hat{E}$

$\therefore$ **CM is a tangent at M to ⊙MED** ◄ . . . *converse tan chord theorem*

2.3 $A\hat{D}B = 90^\circ$ . . . $\angle$ *in semi-⊙*
& $\hat{M}_3 = 90^\circ$ . . . $ME \perp AC$
$\therefore \hat{M}_3 = A\hat{D}B$

$\therefore$ **FMBD is a cyclic quad** ◄ . . . *converse ext.* $\angle$ *of cyclic quad*

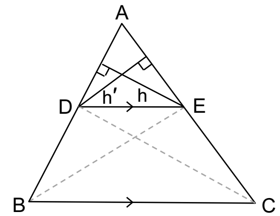
2.4 Let BC = a ; then MB = 2a
$\therefore$ MD = 2a . . . *radii*

In ΔMDC: $M\hat{D}C = 90^\circ$ . . . *radius* $\perp$ *tangent*
$\therefore DC^2 = MC^2 - MD^2$ . . . *theorem of Pythagoras*
$= (3a)^2 - (2a)^2$
$= 9a^2 - 4a^2$
$= 5a^2$
$= \mathbf{5BC^2}$ ◄

2.5 In Δ^s DBC and DFM
(1) $\hat{B}_1 = \hat{F}_2$ . . . *ext* $\angle$ *of c.q. FMBD*
(2) $\hat{D}_4 = \hat{D}_2$ . . . *both = x*
$\therefore$ **ΔDBC ||| ΔDFM** ◄ . . . *equiangular* Δ^s

2.6 $\therefore \dfrac{DM}{FM} = \dfrac{DC}{BC}$ . . . ||| Δ^s
$= \dfrac{\sqrt{5}\,BC}{BC}$ . . . *see 2.4*
$= \mathbf{\sqrt{5}}$ ◄

3.1 **Construction:**
Join DC and EB
and heights h and h′

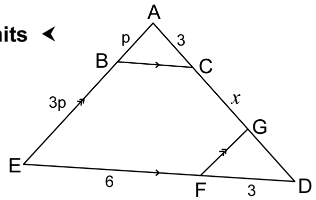


Proof:
$\dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta DBE} = \dfrac{\frac{1}{2} AD \cdot h}{\frac{1}{2} DB \cdot h}$
$= \dfrac{AD}{DB}$ . . . *equal heights*

& $\dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta EDC} = \dfrac{\frac{1}{2} AE \cdot h'}{\frac{1}{2} EC \cdot h'} = \dfrac{AE}{EC}$ . . . *equal heights*

But, area of ΔDBE = area of ΔEDC . . . *same base DE & betw. same || lines, i.e. same height*
$\therefore \dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta DBE} = \dfrac{\text{area of } \Delta ADE}{\text{area of } \Delta EDC}$
$\therefore \mathbf{\dfrac{AD}{DB} = \dfrac{AE}{EC}}$ ◄

3.2.1 Let AB = p ; then BE = 3p
In ΔAED: $\dfrac{CD}{3} = \dfrac{3p}{p}$ . . . *proportion thm; BC || ED*
$\times 3)$ $\therefore$ **CD = 9 units** ◄



3.2.2 CG = x ; so GD = 9 - x
In ΔDAE: $\dfrac{9 - x}{x + 3} = \dfrac{3}{6}$ . . . *prop. thm. ; AE || GF*
$\therefore 54 - 6x = 3x + 9$
$\therefore -9x = -45$
$\therefore \mathbf{x = 5}$ ◄

3.2.3 In Δ^s ABC and AED
(1) $\hat{A}$ is common
(2) $A\hat{B}C = \hat{E}$ . . . *corr.* $\angle^s$*; BC || ED*
$\therefore$ ΔABC ||| ΔAED . . . *equiangular* Δ^s
$\therefore \dfrac{BC}{ED} = \dfrac{AB}{AE}$ . . . ||| Δ^s
$\therefore \dfrac{BC}{9} = \dfrac{p}{4p}$
$\times 9)$ $\therefore$ **BC = $\mathbf{\frac{9}{4}}$ units** ◄

3.2.4 $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta GFD} = \dfrac{\frac{1}{2} AC \cdot BC \sin A\hat{C}B}{\frac{1}{2} DG \cdot DF \sin \hat{D}}$
$= \dfrac{\frac{1}{2} \cdot 3 \cdot \frac{9}{4} \cdot \sin \hat{D}}{\frac{1}{2} \cdot 4 \cdot 3 \cdot \sin \hat{D}}$ . . . *corr.* $\angle^s$*; BC || ED*
$= \dfrac{\frac{9}{4}}{4}$
$= \mathbf{\dfrac{9}{16}}$ ◄

OR: $\dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta AED} = \dfrac{\frac{1}{2} \cdot p \cdot 3 \cdot \sin \hat{A}}{\frac{1}{2} \cdot 4p \cdot 12 \cdot \sin \hat{A}} = \dfrac{1}{16}$

$\therefore$ area of ΔABC $= \dfrac{1}{16}$ area of ΔAED . . . ❶

& $\dfrac{\text{area of } \Delta GFD}{\text{area of } \Delta AED} = \dfrac{\frac{1}{2} \cdot 4 \cdot 3 \cdot \sin D}{\frac{1}{2} \cdot 12 \cdot 9 \cdot \sin D} = \dfrac{1}{9}$

$\therefore$ area of ΔGFD $= \dfrac{1}{9}$ area of ΔAED . . . ❷

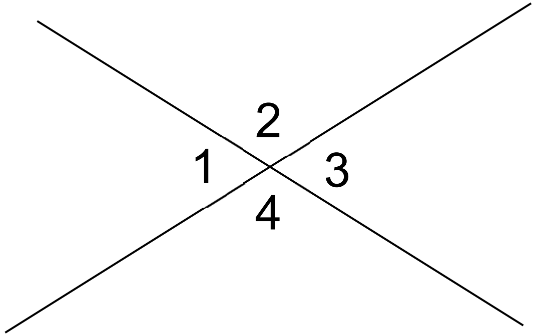
❶ ÷ ❷: $\therefore \dfrac{\text{area of } \Delta ABC}{\text{area of } \Delta GFD} = \dfrac{\frac{1}{16} \text{ area of } \Delta AED}{\frac{1}{9} \text{ area of } \Delta AED}$
$= \mathbf{\dfrac{9}{16}}$ ◄

LINES

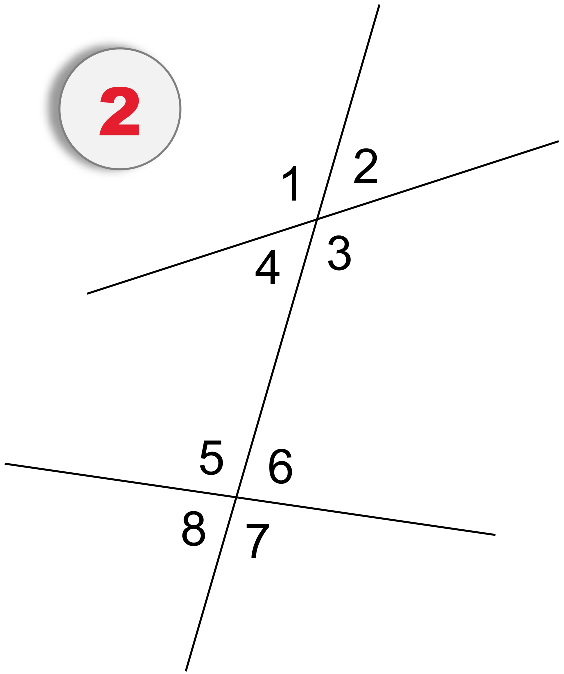
See Gr 11 Maths 3-in-1:
Module 9A, p. 9.1

2 Situations

1



2



Converse Statements

Logic !

NB:
Vocabulary first, then facts

TRIANGLES

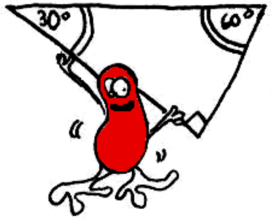
Sum of Interior $\angle^s$ Exterior $\angle$ of Δ

NB:
Vocabulary first, then facts

* Isosceles Δ Equilateral Δ Right-$\angle^d$ Δ

– * Theorem of Pythagoras

Area of a Δ and related facts



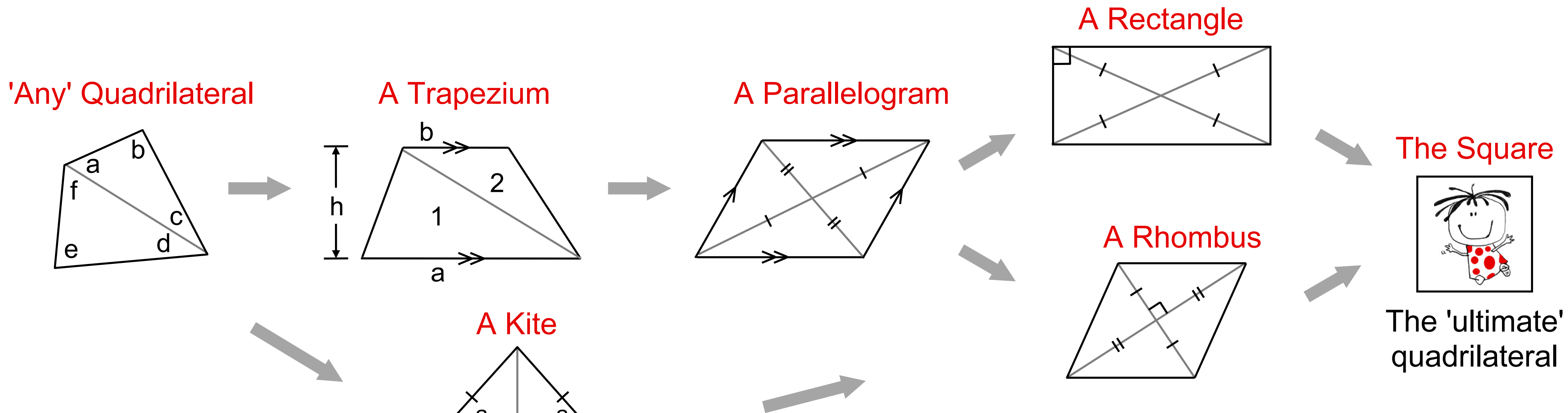
* Similar Δ^s Congruent Δ^s

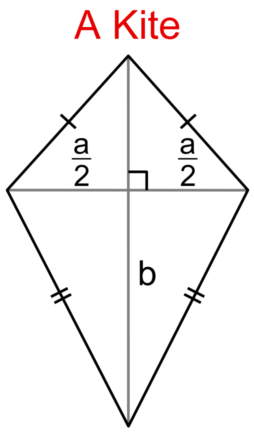
* Midpoint Theorem

* These involve converse theorems

QUADRILATERALS

An Assignment

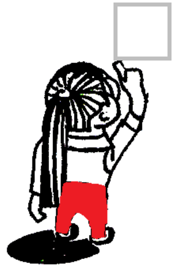




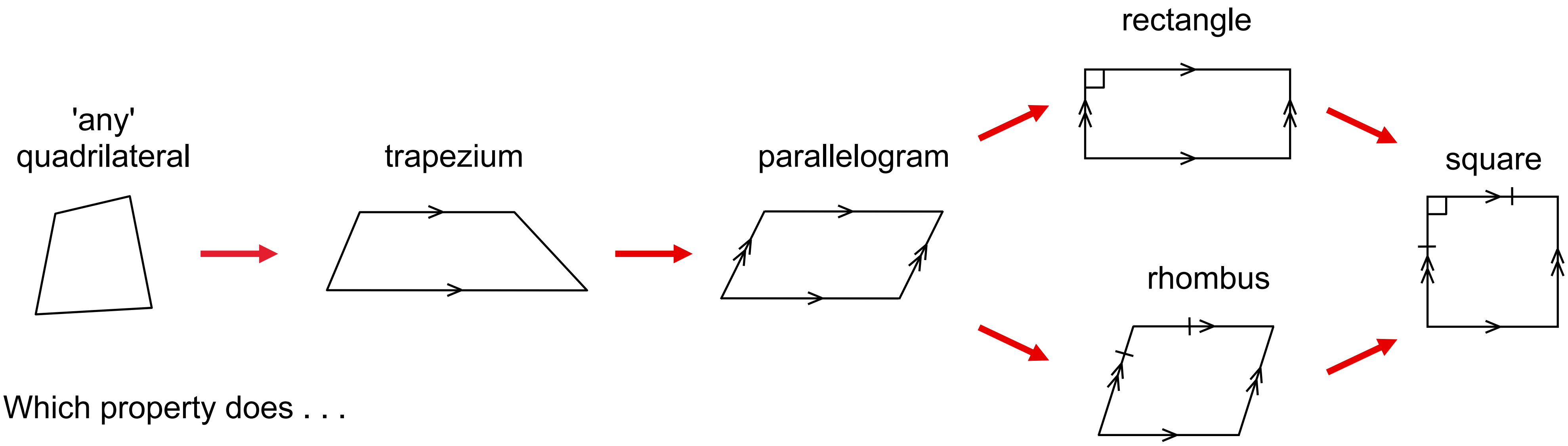
*The arrows indicate various '**pathways**' from 'any' quadrilateral to the square (the 'ultimate quadrilateral')*

Consider

- Sides: parallel? equal?
- Angles: equal? supplementary? right ∠?
- Diagonals . . . . ?

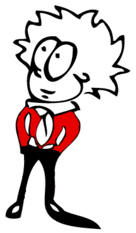


Observe the progression below as we discuss further definitions . . .



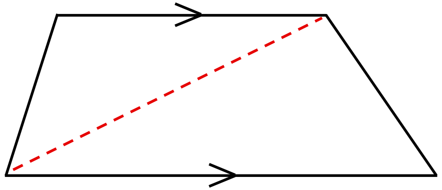
Which property does . . .

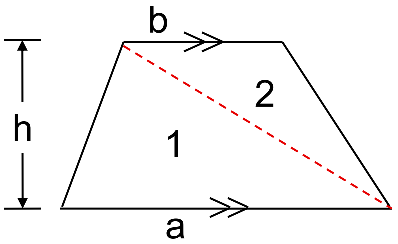
1. a **parallelogram** need to become a **rectangle**?
2. a **parallelogram** need to become a **rhombus**?
3. a **rectangle** need to become a **square**?
4. a **rhombus** need to become a **square**?
5. And, which property(s) does a **parallelogram** need to become a **square**?



A Trapezium

- Can you derive a formula for the area of a trapezium?





The **Area** $= \Delta 1 + \Delta 2$

$= \frac{1}{2}ah + \frac{1}{2}bh$

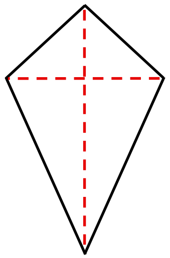
$= \mathbf{\frac{1}{2}(a + b).h}$

$\therefore$ The area of a trapezium:

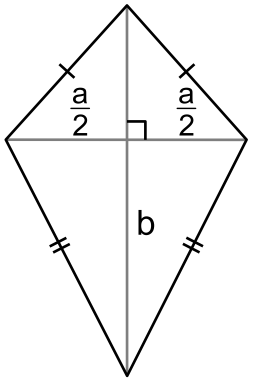
Half the sum of the || sides $\times$ the distance between them.

A Kite

Can you derive a formula for the area of a kite?



A Kite

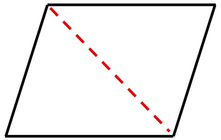


Given diagonals a and b . . .

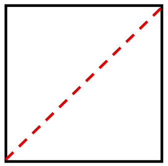
Area $= 2\Delta^s = 2\left(\frac{1}{2}b \cdot \frac{a}{2}\right) = \frac{ab}{2}$ . . . $\frac{\text{the product of the diagonals}}{2}$

$\therefore$ The area of a kite: **'Half the product of the diagonals'**

Could this formula apply to a **rhombus**?

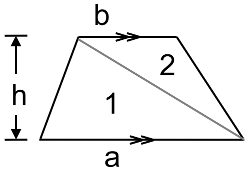


And to a **square**?



SUMMARY: AREAS

A Trapezium



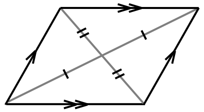
Area = $\Delta 1 + \Delta 2$

$= \frac{1}{2}ah + \frac{1}{2}bh$

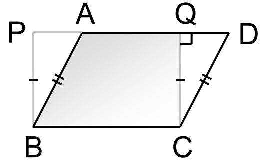
$= \mathbf{\frac{1}{2}(a+b).h}$

'Half the sum of the || sides × the distance between them.'

A Parallelogram



Area = base × height



$||^m$ ABCD = **ABCQ** + ΔQCD

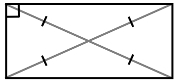
rect. PBCQ = **ABCQ** + ΔPBA

where ΔQCD $\equiv$ ΔPBA . . . *SS90°*

$\therefore$ $||^m$ ABCD = rect. PBCQ (in area)

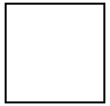
= BC × QC

A Rectangle



Area = ℓ × b

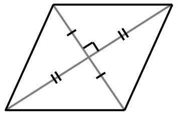
The Square



Area = s^2

Since a square is a rectangle, a rhombus, a parallelogram, a kite, . . . ALL the properties of these quadrilaterals apply.

A Rhombus



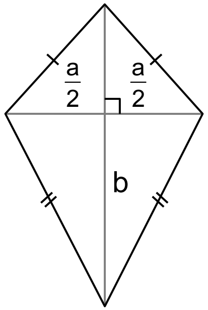
Area of a rhombus

= $\frac{1}{2}$ **product of diagonals** (as for a kite)

or

= **base × height** (as for a parallelogram)

A Kite



Given diagonals a and b

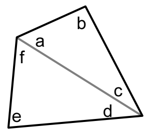
Area = $2\Delta^s = 2\left(\frac{1}{2}b \cdot \frac{a}{2}\right) = \mathbf{\frac{ab}{2}}$

'Half the product of the diagonals'

QUADRILATERALS - definitions, areas & properties

All you need to know

'Any' Quadrilateral



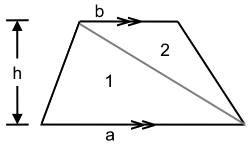
Sum of the $\angle^s$ of *any quadrilateral* = 360°

Sum of the interior angles
= (a + b + c) + (d + e + f)
= 2 × 180° . . . (2 Δ^s)
= 360°

*The arrows indicate various '**pathways**' from 'any' quadrilateral to the square (the 'ultimate quadrilateral'). These pathways, which combine logic and fact, are essential to use when proving specific types of quadrilaterals.*

See how the properties accumulate as we move from left to right, i.e. the first quad. has no special properties and each successive quadrilateral has all preceding properties.

A Trapezium

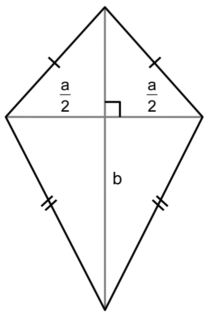


DEFINITION:
Quadrilateral with **1 pair of opposite sides ||**

Area = Δ 1 + Δ 2
$= \frac{1}{2}ah + \frac{1}{2}bh$
$= \frac{1}{2}(a + b).h$

'Half the sum of the || sides × the distance between them.'

A Kite



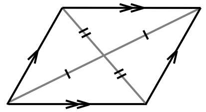
DEFINITION:
Quadrilateral with **2 pairs of adjacent sides equal**

Given diagonals a and b
Area $= 2\Delta^s = 2\left(\frac{1}{2}b.\frac{a}{2}\right) = \frac{ab}{2}$

'Half the product of the diagonals'

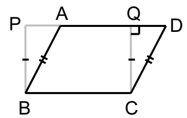
THE DIAGONALS
- *cut perpendicularly*
- ***ONE DIAGONAL** bisects the other diagonal, the opposite angles and the area of the kite*

A Parallelogram



DEFINITION:
Quadrilateral with **2 pairs opposite sides ||**

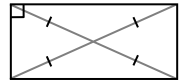
Area = base × height



$||^m$ ABCD = ABCQ + ΔQCD
rect. PBCQ = ABCQ + ΔPBA
where ΔQCD ≡ ΔPBA . . . *RHS/90°HS*
∴ $||^m$ ABCD = rect. PBCQ (in area)
= BC × QC

Properties:
2 pairs opposite sides equal
2 pairs opposite angles equal
*& **DIAGONALS BISECT ONE ANOTHER***

A Rectangle

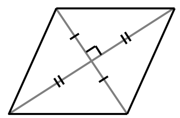


DEFINITION:
A $||^m$ with one right ∠

Area = ℓ × b

***DIAGONALS** are EQUAL*

A Rhombus

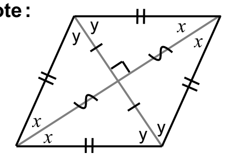


DEFINITION:
A $||^m$ with one pair of adjacent sides equal

Area
$= \frac{1}{2}$ **product of diagonals** (as for a kite)
or
= **base × height** (as for a parallelogram)

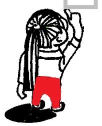
THE DIAGONALS
- *bisect one another PERPENDICULARLY*
- *bisect the angles of the rhombus*
- *bisect the area of the rhombus*

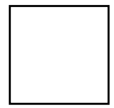
Note:



2x + 2y = 180° . . . *$\angle^s$ of Δ or co-int. $\angle^s$; || lines*
➜ x + y = 90°

The Square



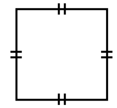


the 'ultimate' quadrilateral!

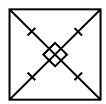
Area = s²

Properties:
It's all been said 'before'!
Since a square is a rectangle, a rhombus, a parallelogram, a kite, . . . ALL the properties of these quadrilaterals apply.

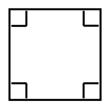
sides



diagonals



angles



Quadrilaterals play a prominent role in both Euclidean & Analytical Geometry right through to Grade 12!

THE ANSWER SERIES *Your Key to Exam Success*

An Assignment: Quadrilaterals
Theorems and Proofs

❑ Theorems and Proofs

The following section deals with the properties of a parallelogram. We firstly prove all the properties. Secondly, we prove that a quadrilateral with any of these properties has to be a parallelogram.

Geometry is an exercise in LOGIC. Initially, we observe, we measure, we record . . . But, finally . . . We decide on how to **define** something and then we prove various **properties** logically, using the **definition**.

THE DEFINITION OF A PARALLELOGRAM

A parallelogram is a quadrilateral with
2 PAIRS OF OPPOSITE SIDES PARALLEL.

Beyond the DEFINITION of a parallelogram, we noticed other facts/properties regarding the lines, angles and diagonals of a parallelogram. The statement and proofs of these properties make up our first three THEOREMS!

The PROPERTIES of a parallelogram

All the properties are to be deduced **from the definition!**

Theorem 1: The opposite angles of a parallelogram are equal.

Theorem 2: The opposite sides of a parallelogram are equal.

Theorem 3: The diagonals of a parallelogram bisect one another.

Gr 10 Maths 3-in-1
pp. 7.11 – 7.13

The CONVERSE theorems

Given a property, prove the quadrilateral is a parallelogram, i.e. prove both pairs of opposite sides are parallel.

There are four converse statements, each claiming that IF a quadrilateral has a particular property, it must be a parallelogram.

In these cases, we work **towards the definition!**

Theorem 4: If a QUADRILATERAL has 2 pairs of opposite angles equal, then the quadrilateral is a parallelogram.

Theorem 5: If a QUADRILATERAL has 2 pairs of opposite sides equal, then the quadrilateral is a parallelogram.

Theorem 6: If a QUADRILATERAL has 1 pair of opposite sides equal and parallel, then the quadrilateral is a parallelogram.

Theorem 7: If a QUADRILATERAL has diagonals which bisect one another, then the quadrilateral is a parallelogram.

❖ AN ASSIGNMENT ❖

TASK A: Theorems 1 → 3

Prove each of these properties yourself,

STARTING WITH THE DEFINITION as the 'given'.

TASK B: Theorems 4 → 7

Prove these four converse theorems,

Hint

Use your FACTS on II lines and congruent triangles.

WORKING TOWARDS THE DEFINITION,

i.e. you need to prove, given any one of these situations, that the quadrilateral would have 2 pairs of opposite sides parallel, i.e. that, *by definition*, the quadrilateral is a parallelogram.

❑ The Theorem Proofs

THE PROOFS OF THE PROPERTIES

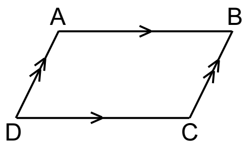
DON'T EVER MEMORISE THEOREM PROOFS!
Develop the proofs/logic for yourself *before* checking against the methods shown below.

Theorems: Definition ➡ Property
Converse theorems: Property ➡ Definition

Make sense of THE LOGIC!

➤ **Theorem 1: The opposite angles of a $||^m$ are equal.**

Given: $||^m$ ABCD
i.e. AB || DC and AD || BC



RTP: $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$

Proof: $\hat{A} + \hat{B} = 180°$ . . . *co-interior* $\angle^s$; *AD || BC*
But, $\hat{A} + \hat{D} = 180°$ . . . *co-interior* $\angle^s$; *AB || DC*
$\therefore$ $\hat{B} = \hat{D}$
Similarly, $\hat{A} = \hat{C}$

RTP: Required to prove

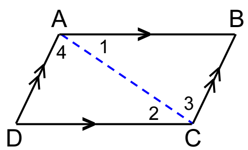
➤ **Theorem 2: The opposite sides of a $||^m$ are equal.**

Given: $||^m$ ABCD
i.e. AB || DC and AD || BC

RTP: AB = CD and AD = BC

Construction: Draw diagonal AC . . . *It doesn't matter which diagonal you draw*

Proof: In Δ^s ABC and ADC
1) $\hat{1} = \hat{2}$ . . . *alternate* $\angle^s$; *AB || DC*
2) $\hat{3} = \hat{4}$ . . . *alternate* $\angle^s$; *AD || BC*
3) AC is common
$\therefore$ $\Delta ABC \equiv \Delta CDA$ . . . $\angle\angle S$
$\therefore$ AB = CD and AD = BC

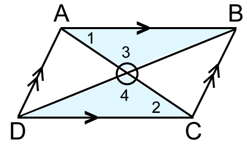


We could, of course, also have proved the first theorem this way!

➤ **Theorem 3: The diagonals of a parallelogram bisect one another.**

Given: $||^m$ ABCD with diagonals AC and BD intersecting at O.

RTP: AO = OC and BO = OD



Proof: In Δ^s AOB and DOC
1) $\hat{1} = \hat{2}$ . . . *alt* $\angle^s$; *AB || DC*
2) $\hat{3} = \hat{4}$ . . . *vert opp* $\angle^s$
3) AB = DC . . . *opposite sides of* $||^m$ – *see theorem 2 above*
$\therefore$ $\Delta AOB \equiv \Delta COD$ . . . $\angle\angle S$
$\therefore$ AO = OC and BO = OD

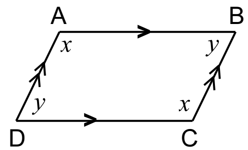
Note: We used the result in theorem 2 in the proof of theorem 3 – but, we **could've** started from the beginning, i.e. ***from the definition*** of a parallelogram. We would just have needed to prove an extra pair of Δ^s congruent *(as in theorem 2)*.

THE CONVERSE PROOFS

➤ **Theorem 4: If a QUADRILATERAL has 2 pairs of opposite angles equal, then the quadrilateral is a $||^m$**

Given: Quadrilateral ABCD with $\hat{A} = \hat{C}$ and $\hat{B} = \hat{D}$

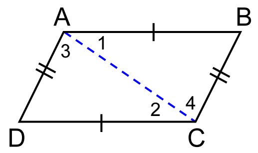
RTP: ABCD is a parallelogram,
i.e. AB || DC and AD || BC



Proof: Let $\hat{A} = \hat{C} = x$ and $\hat{D} = \hat{B} = y$
then $\hat{A} + \hat{B} + \hat{C} + \hat{D} = 360°$ . . . *sum of the* $\angle^s$ *of a quadrilateral*
$\therefore$ $2x + 2y = 360°$
$\div 2)$ $\therefore$ $x + y = 180°$
i.e. $\hat{A} + \hat{D} = 180°$ and $\hat{A} + \hat{B} = 180°$
$\therefore$ AB || DC and AD || BC . . . *co-interior* $\angle^s$ *are supplementary*
$\therefore$ ABCD is a parallelogram . . . *both pairs of opposite sides ||*

➤ Theorem 5: If a QUADRILATERAL has 2 pairs of opposite sides equal, then the quadrilateral is a $||^m$

Given: Quadrilateral ABCD with AB = CD and AD = BC



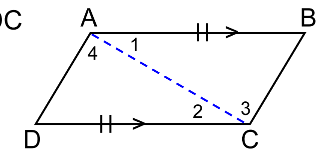
RTP: ABCD is a parallelogram,
i.e. AB || DC and AD || BC

Construction: Draw diagonal AC . . . *it doesn't matter which diag. you draw*

Proof: In Δ^s ACD and CAB
1) AC is common
2) AD = BC . . . *given*
3) CD = AB . . . *given*
∴ ΔACD ≡ ΔCAB . . . *SSS*
∴ $\hat{1} = \hat{2}$ and $\hat{3} = \hat{4}$
∴ AB || DC and AD || BC . . . *alternate* $\angle^s$ *are equal*
∴ ABCD is a parallelogram . . . *both pairs of opposite sides ||*

➤ Theorem 6: If a QUADRILATERAL has 1 pair of opposite sides equal and ||, then the quadrilateral is a $||^m$

Given: Quadrilateral ABCD with AB = and || DC



RTP: ABCD is a parallelogram,
i.e. AB || DC and AD || BC

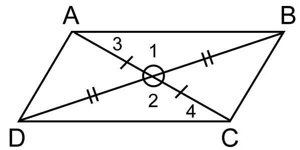
Construction: Draw diagonal AC . . . *It doesn't matter which diagonal you draw*

Proof: In Δ^s ABC and CDA
1) AB = DC . . . *given*
2) $\hat{1} = \hat{2}$ . . . *alternate* $\angle^s$*; AB || DC*
3) AC is common
∴ ΔABC ≡ ΔCDA . . . *S∠S*
∴ $\hat{3} = \hat{4}$
∴ AD || BC . . . *alternate* $\angle^s$ *equal*

But AB || DC . . . *given*
∴ ABCD is a parallelogram . . . *both pairs of opposite sides ||*

➤ Theorem 7: If a QUADRILATERAL has diagonals which bisect one another, then the quadrilaterals is a $||^m$.

Given: Quadrilateral ABCD with diagonals AC and BD intersecting at O and AO = OC and BO = OD.



RTP: ABCD is a parallelogram,
i.e. AB || DC and AD || BC

Proof: In Δ^s AOB and COD
1) AO = OC . . . *given*
2) $\hat{1} = \hat{2}$ . . . *vert opp* $\angle^s$
3) BO = O . . . *given*
∴ ΔAOB ≡ ΔCOD . . . *S∠S*
∴ $B\hat{A}D = O\hat{C}D$
∴ AB || DC . . . *alternate* $\angle^s$ *equal*

In Geometry, we never have to repeat a 'logic sequence' (as would've been required here) – we just say: Similarly, . . . !

Similarly, by proving ΔAOD ≡ ΔCOB it can be shown that AD || BC
∴ ABCD is a parallelogram . . . *2 pairs of opp. sides are ||*

---

*In our **sums**, we may use ALL properties and theorem statements . . .*

To prove that a quadrilateral is a $||^m$ we may choose one of 5 ways:

1) Prove both pairs of opposite sides || *(the definition)*.
2) Prove both pairs of opposite sides = *(a property)*.
3) Prove 1 pair of opposite sides = and || *(a property)*.

} . . . ***THE SIDES***

4) Prove both pairs of opposite angles = *(a property)*. . . . ***THE ANGLES***
5) Prove that the diagonals bisect one another *(a property)*. . . . ***THE DIAGONALS***

Using diagonals . . .

To prove a parallelogram is a rectangle: prove that the diagonals are equal.

To prove a parallelogram is a rhombus: prove that the diagonals intersect at right angles, or prove that the diagonals bisect the angles of the rhombus.

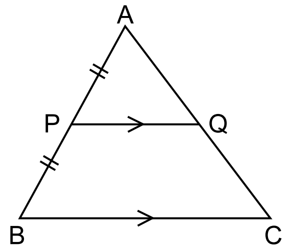
Gr 10: THE MIDPOINT THEOREM

FACT 1

The line segment through the midpoint of one side of a triangle, parallel to a second side, bisects the third side.

Given:
P midpoint AB & PQ || BC

Result:
Q midpoint AC & $PQ = \frac{1}{2}BC$

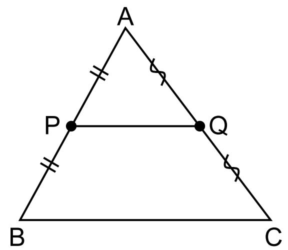


FACT 2

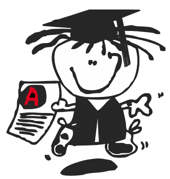
The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half of the third side.

Given:
P & Q midpoints of AB & AC

Result:
PQ || BC & $PQ = \frac{1}{2}BC$



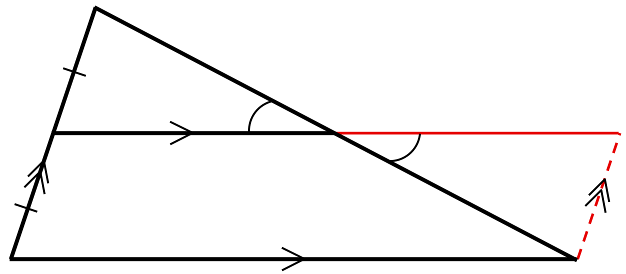
Regard these Facts 1 & 2 as **a special case** of the Proportion Theorem in Gr 12 Geometry.



AN ASSIGNMENT: PROOFS

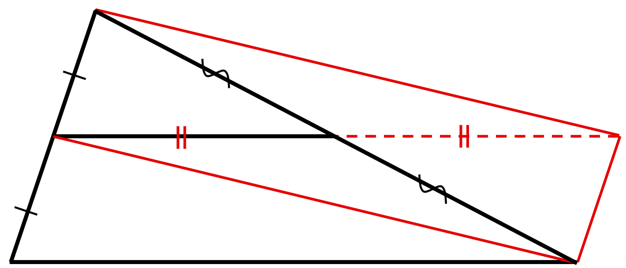
Use the diagrams below to prove facts 1 and 2:

1.



(See Exercise 4 Q3.2 for the proof)

2.

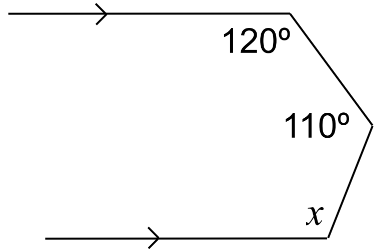


(See Exercise 4 Q3.1 for the proof)

∠s, Lines, ∆s

2.2 Calculate, with reasons, the values of x.

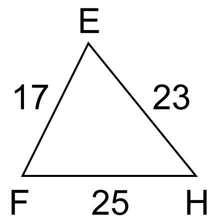
2.2.2

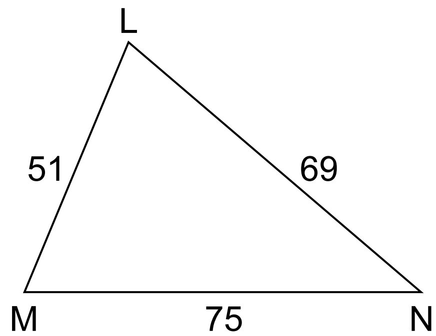


Gr 10 Maths 3-in-1 p. 7.6 Q2.2.2

5. State whether the following pairs of triangles are congruent or similar, giving reasons for your choice.

5.2

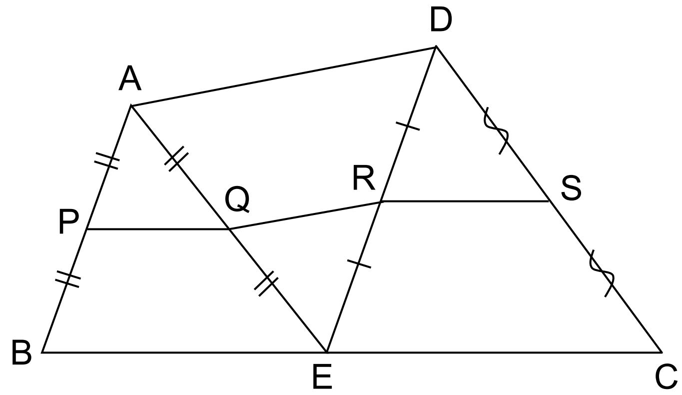




Gr 10 Maths 3-in-1 p. 7.6 Q5.2

9. ABCD is a quadrilateral.

E is a point on BC. P, Q, R and S are the midpoints of AB, AE, DE and DC respectively.



Prove that:

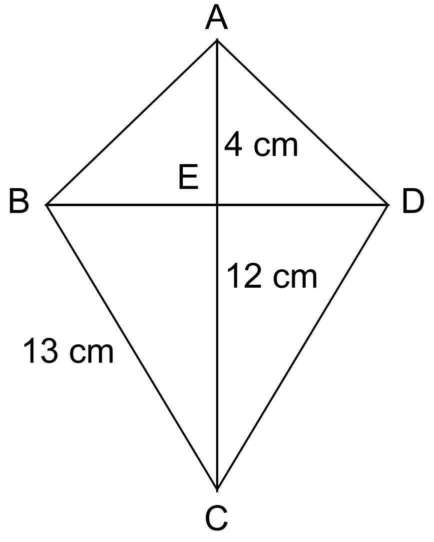
9.1 PQ || RS

9.2 $PQ + QR + RS = \frac{1}{2}(AD + BC)$

Gr 10 Maths 3-in-1 p. 7.17 Q9

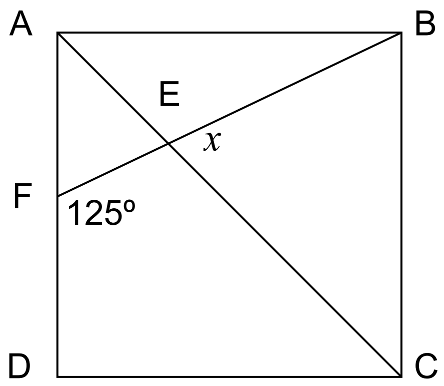
Quadrilaterals

7. Calculate the area of the kite alongside.



Gr 10 Maths 3-in-1
p. 7.6 Q7

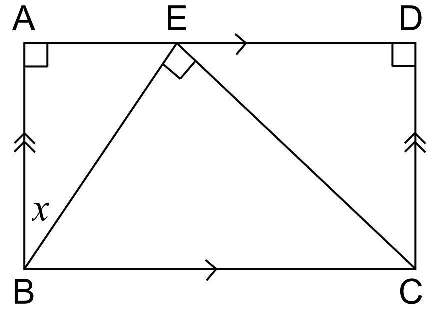
7. Calculate the value of x giving reasons, given that ABCD is a square and $B\hat{F}D = 125^\circ$.



Gr 10 Maths 3-in-1
p. 7.14 Q7.2

15.1 Make a neat copy of this sketch and fill in all the other angles in terms of x.

Reasons are not required.



15.2 Complete the following statement:

ΔABE ||| Δ . . . ||| Δ . . .

15.3 If BC = 18 cm and BE = 12 cm, calculate the length of

15.3.1 AE

15.3.2 AB correct to two decimals.

15.4 Hence calculate the area of rectangle ABCD to the nearest cm^2.

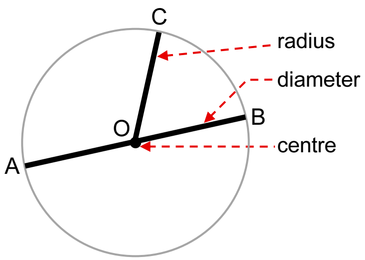
Gr 10 Maths 3-in-1
p. 7.7 Q15

CIRCLE GEOMETRY

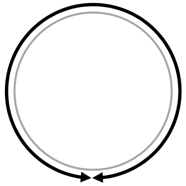
The Language (Vocabulary)

GROUP 1 AND 2

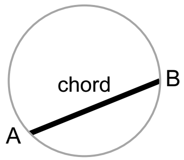
- **Centre**
- **Diameter**
- **Radius**



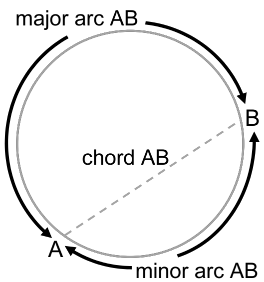
- **Circumference**



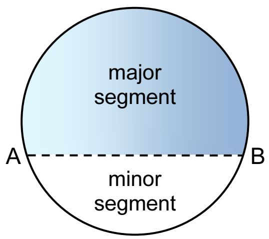
- **Chords**



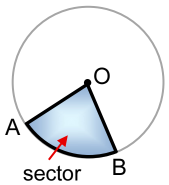
- **Arcs (major & minor)**



- **Segments (major & minor)**



- **Sectors**



- **"SUBTEND" . . .   Understand the word!**

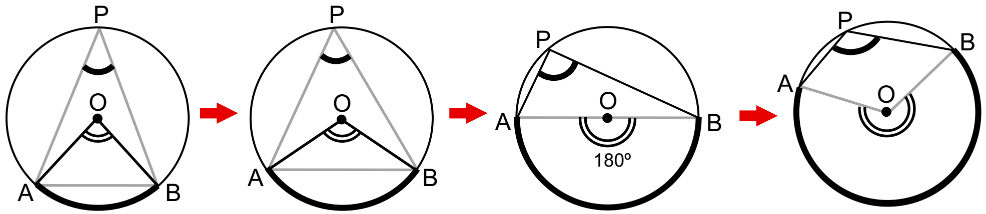


Figure 1   Figure 2   Figure 3   Figure 4

- **Central and Inscribed angles**

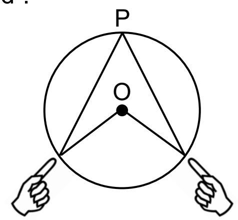
In all the figures, arc AB $(\overset{\frown}{AB})$, or chord AB, **subtends**:

- a **central A$\hat{O}$B** at the **centre** of the circle, and
- an **inscribed A$\hat{P}$B** at the **circumference** of the circle.

Consider that **subtend** means **support**.

To ensure that you grasp the meaning of the word 'subtend':

- Take **each** of the figures:
  - Place your index fingers on A & B;
  - move along the radii to meet at O and back; then,
  - move to meet at P on the circumference and back.



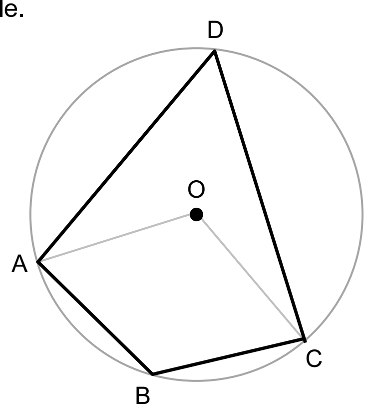
- Turn your book upside down and sideways.
  You need to recognise different views of these situations.
- Take note of whether the angles are acute, obtuse, right, straight or reflex.
- Redraw figures 1 to 4 leaving out the chord AB completely and **observe the arc** subtending the central and inscribed angles in each case.

See Gr 11 Maths 3-in-1: Module 9B, p. 9.6

GROUP 3

Cyclic Quadrilaterals

A cyclic quadrilateral is a quadrilateral which has all 4 vertices on the circumference of a circle.



Points A, B, C and D are **concyclic**, i.e. they lie on the same circle.

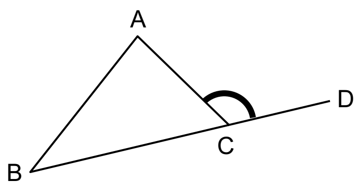
Note: Quadrilateral AOCB is ***not*** a cyclic quadrilateral because point O is **not** on the circumference! (A, O, C and B are **not** concyclic)

We name *quadrilaterals* by going around, either way, using consecutive vertices, i.e. ABCD or ADCB, **not** ADBC.

Exterior angles of polygons

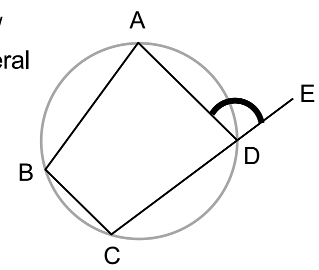
The **exterior angle** of any polygon is an angle which is formed between one side of the polygon and another side *produced*.

e.g. A triangle



$A\hat{C}D$ is an **exterior** ∠ of ΔABC.
[NB: BCD is a straight line!]

e.g. A quadrilateral/ cyclic quadrilateral



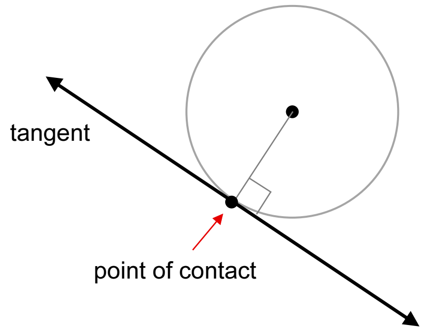
$A\hat{D}E$ is an **exterior** ∠ of c.q. ABCD.
[NB: CDE is a straight line!]

GROUP 4

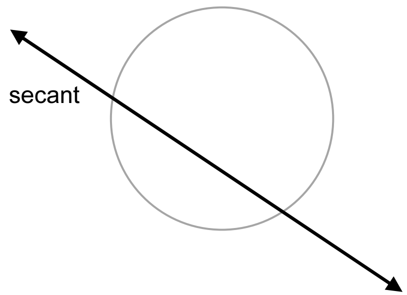
Tangents

Special lines

- A **tangent** is a line which *touches* a circle at a point.



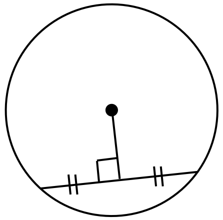
- A **secant** is a line which *cuts* a circle (in two points).

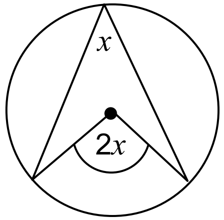


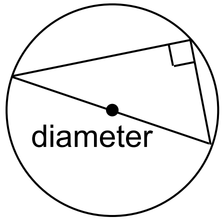
NB: It is assumed that the tangent is perpendicular to the radius (or diameter) at the point of contact.

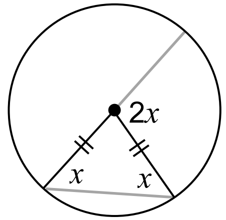
SUMMARY OF CIRCLE GEOMETRY THEOREMS

I The 'Centre' group



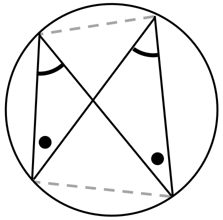


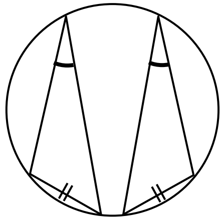




Equal radii!

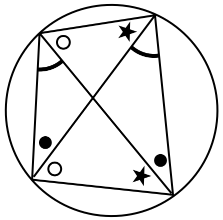
II The 'No Centre' group

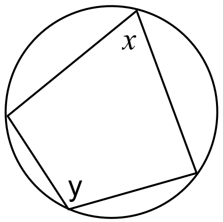




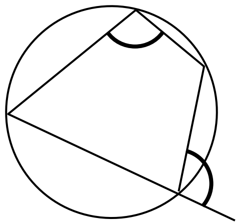
Equal chords!

III The 'Cyclic Quad.' group



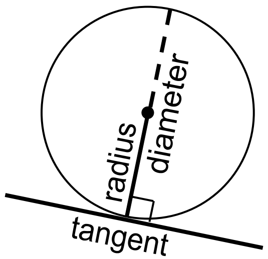


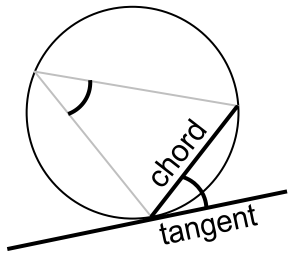
$x + y = 180^\circ$

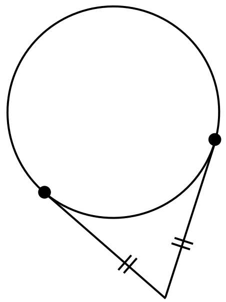


*There are '**3 ways** to prove that a quad. is a cyclic quad'.*

IV The 'Tangent' group





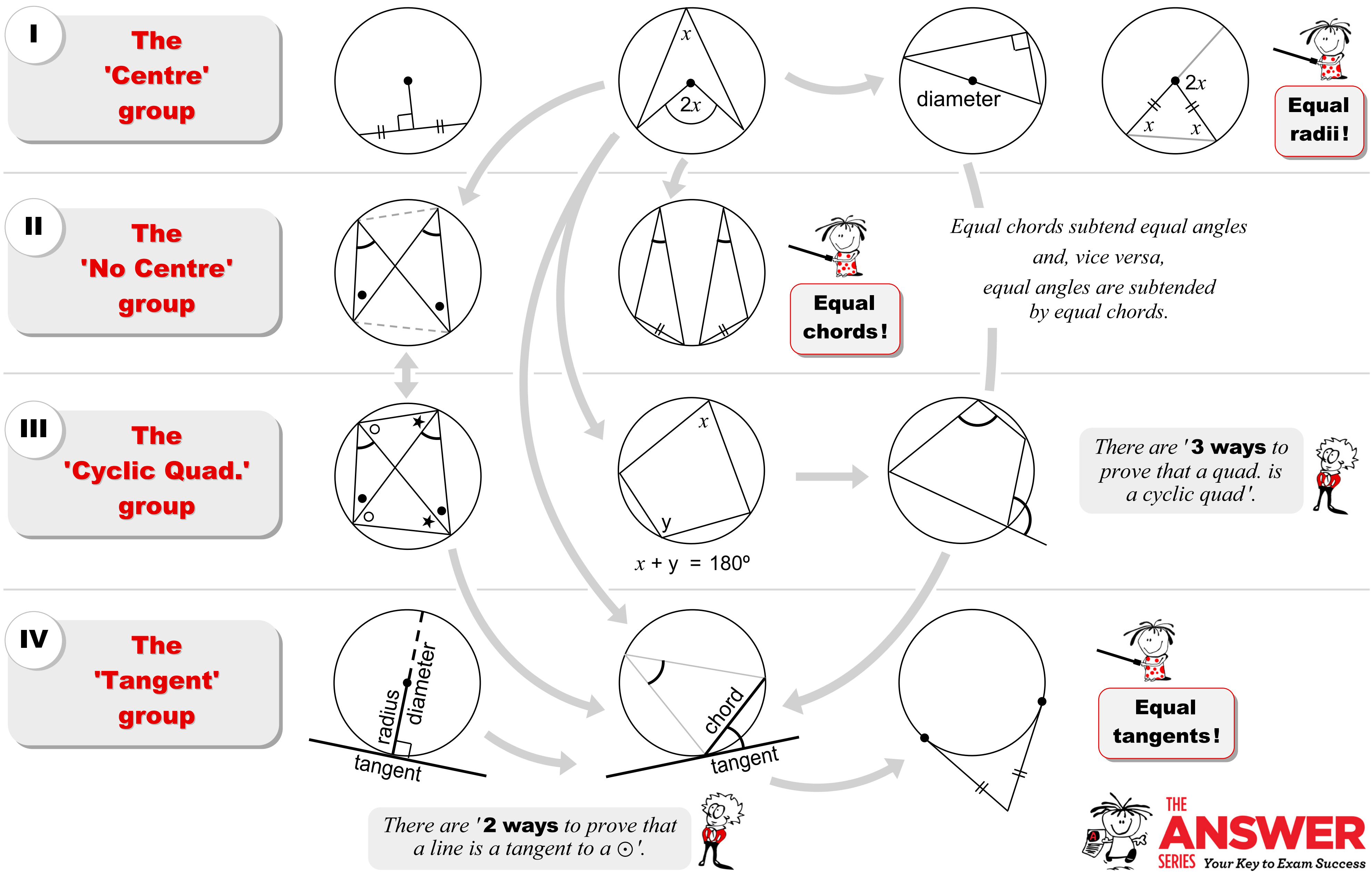


Equal tangents!

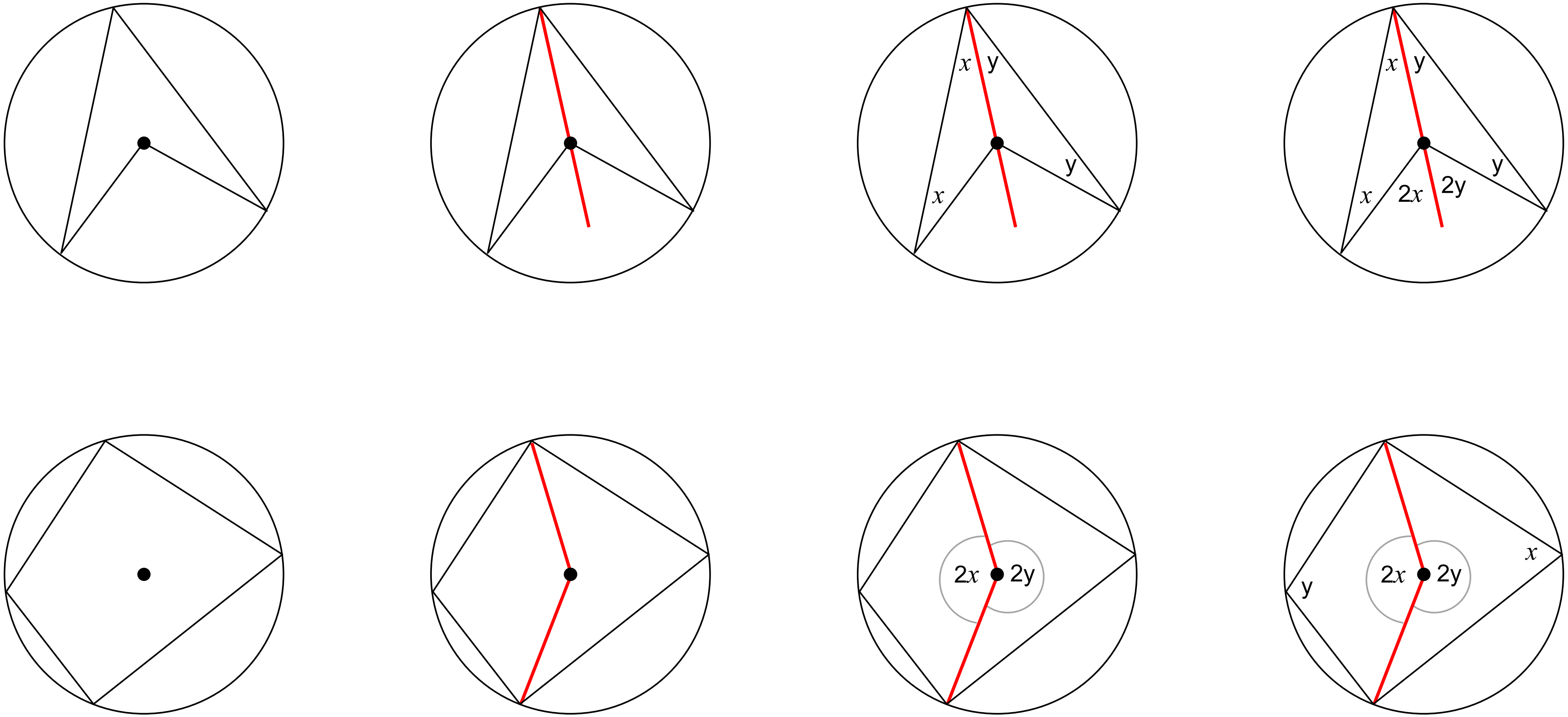
*There are '**2 ways** to prove that a line is a tangent to a ⊙'.*

GROUPING OF CIRCLE GEOMETRY THEOREMS

The grey arrows indicate how various theorems are used to prove subsequent ones



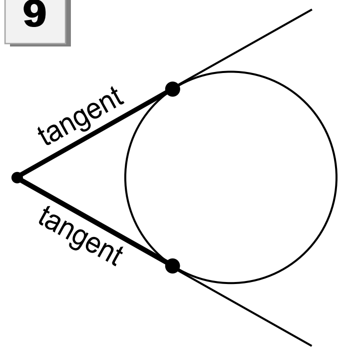
PROVING THEOREMS



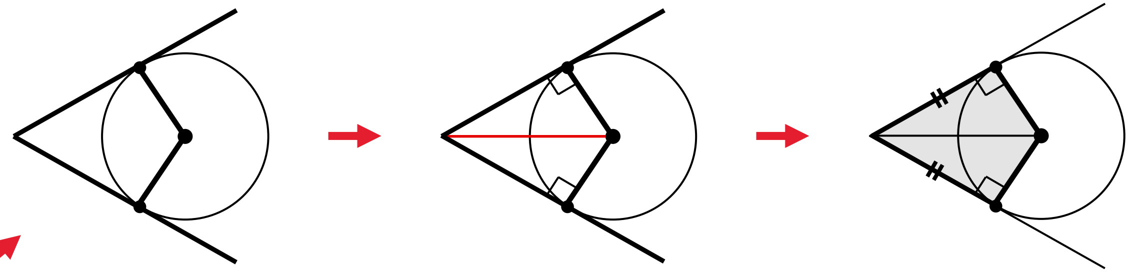
FURTHER ⊙ THEOREM PROOFS: A Visual presentation, continued . . .

The Situation | **Construction** | **The LOGIC . . .**

9



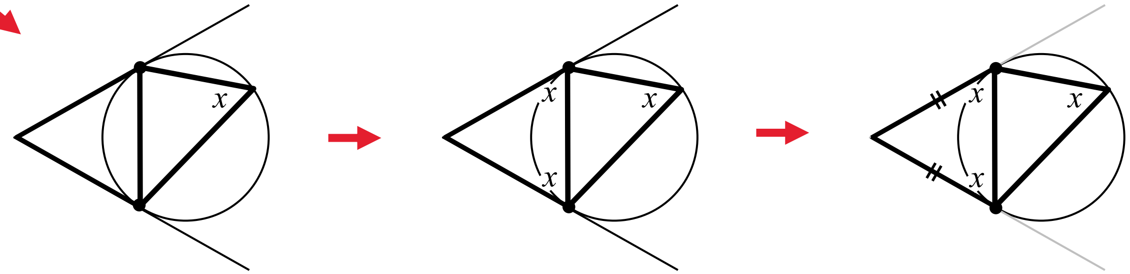
Method 1:
radii



radii ⊥ tangents

congruent Δ^s

Method 2:
chord and inscribed ∠



tan-chord theorem **4**

sides opposite equal base $\angle^s$ in Δ

Theorem Statement

Tangents to a circle **from a common point** are equal.

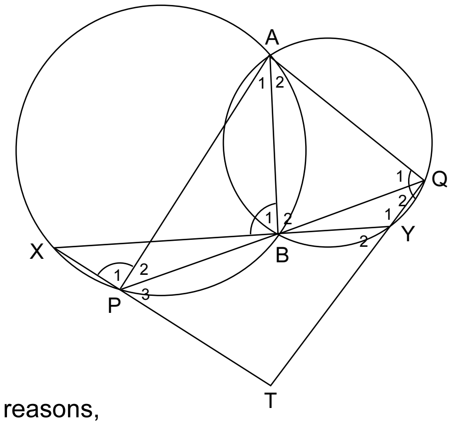
Again, the **bolded words** are the 'approved reasons' to use.

Proofs **6** to **9** are not examinable, but, the **LOGIC** is crucial when studying geometry.

Circles

EXAMPLE 7

Don't be put off by this drawing!
Direct your focus to one situation at a time ☺



Make statements, with reasons,

1. In ⊙XPBA: about $\hat{P}_1$ and $\hat{B}_1$

2. In ⊙ABYQ: about $\hat{B}_1$ and $A\hat{Q}Y$

3. In quadrilateral APTQ: about $\hat{P}_1$ and $A\hat{Q}T$

4. What can you conclude about quadrilateral APTQ?

Gr 11 Maths 3-in-1
p. 9.16

Mark the $\angle^s$ on the drawing as you proceed.

Answers

1. In c.q. XPBA: $\hat{P}_1 = \hat{B}_1$ . . . *arc XA subtends* $\angle^s$ *in same segment*

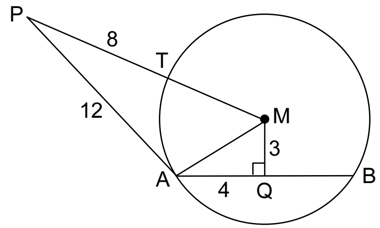
2. In c.q. ABYQ: $\hat{B}_1 = A\hat{Q}Y$ . . . *exterior* ∠ *of cyclic quad.*

3. In quad. APTQ: $\hat{P}_1 = A\hat{Q}T$ . . . *both* = $\hat{B}_1$ *above*

4. ∴ APTQ is a cyclic quad. . . . *converse of exterior* ∠ *of cyclic quad.*

EXAMPLE 8

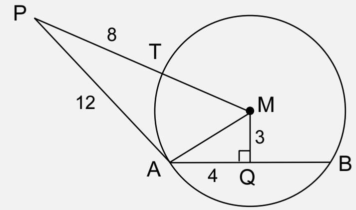
Prove that PA is a tangent to ⊙M.



Gr 11 Maths 3-in-1
p. 9.16 Q14

Answers

In right-∠^{d} ΔMAQ:



AM = 5 units . . . *3 : 4 : 5 Δ ; Pythag.*

∴ TM = 5 units . . . *TM = AM = radii*

∴ PM = 13 units

∴ ΔPAM is a 5 : 12 : 13 Δ!

i.e. $PM^2 = PA^2 + AM^2$

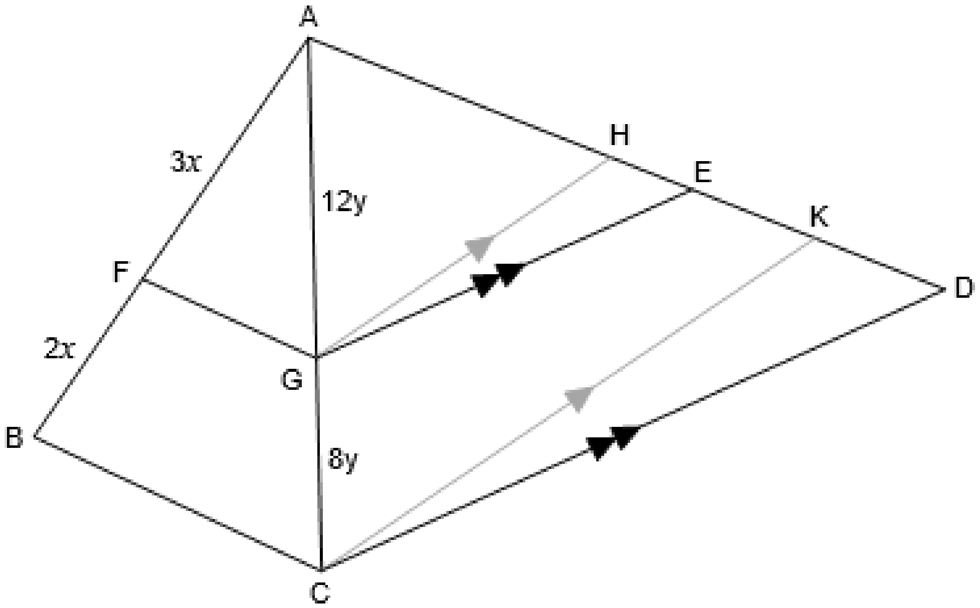
∴ $P\hat{A}M = 90°$ . . . *converse of Pythag.*

∴ **PA is a tangent to ⊙M** . . . *converse tan chord theorem*

Proportion Theorem

Example 9

In the diagram, ΔABC and ΔACD are drawn. F and G are points on sides AB and AC respectively such that $AF = 3x$, $FB = 2x$, $AG = 12y$ and $GC = 8y$. H, E and K are points on side AD such that GH || CK and GE || CD.



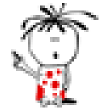
1. Prove that:

1.1 FG || BC

1.2 $\frac{AH}{HK} = \frac{AE}{ED}$ (2)(3)

Gr 12 Maths **Toolkit**:
DBE Past Papers, p. 12 Q9

Study and analyse the diagram . . .



- Notice that there are **3** Δs on which to focus.
  And, in ΔACD, **2 pairs of || lines**. Highlight these in colour!
  (And, the first question requires **proof** of || lines.)

- Clearly, only **2 theorems** are involved:
  the proportion theorem and **its converse** (theorem)
  (Study these <u>2</u> theorem statements well!)

Solution

1.1

To prove || lines, we must prove that FG divides the 2 sides of the triangle in proportion; i.e. that $\frac{AF}{FB} = \frac{AG}{GC}$.
This is the application of the **converse** proportion theorem.

In ΔABC: $\frac{AF}{FB} = \frac{3x}{2x} = \frac{3}{2}$ and $\frac{AG}{GC} = \frac{12y}{8y} = \frac{3}{2}$

$$\therefore \frac{AF}{FB} = \frac{AG}{GC}$$

$\therefore$ FG || BC ◂ . . . *line divides 2 sides of Δ in proportion –* ***converse*** *of the proportion theorem*

1.2 **In ΔACK** ➧ $\frac{AH}{HK} = \frac{AG}{GC}$ . . . *prop. thm.; GH || CK*

In ΔACD ➧ $= \frac{AE}{ED}$ ◂ . . . *prop. thm.; GE || CD*

Question

2. If it is further given that AH = 15 and ED = 12, calculate the length of EK. (5)

Solution

2. $\frac{AG}{GC} = \frac{3}{2}$ ... *from 1.1*

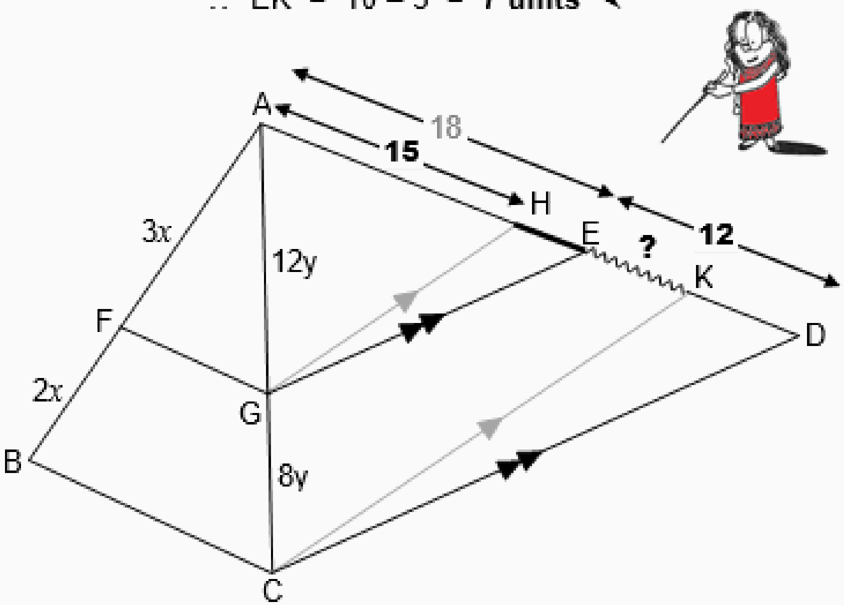
$\therefore \frac{AH}{\mathbf{HK}} = \frac{3}{2}$ and $\frac{\mathbf{AE}}{ED} = \frac{3}{2}$ ... *from 1.2*

$\therefore \frac{15}{HK} = \frac{3}{2}$ $\qquad \therefore \frac{AE}{12} = \frac{3}{2}$

$\therefore HK = 10$ $\qquad \therefore AE = 18$

$\qquad \therefore \mathbf{HE} = 3$

$\therefore EK = 10 - 3 = $ **7 units**

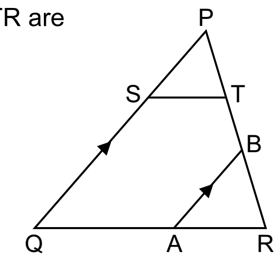


Worked Example 10

In ΔPQR the lengths of PS, SQ, PT and TR are 3, 9, 2 and 6 units respectively.

5.1 Give a reason why ST || QR.

5.2 If AB || QP and RA : AQ = 1 : 3, calculate the length of TB.

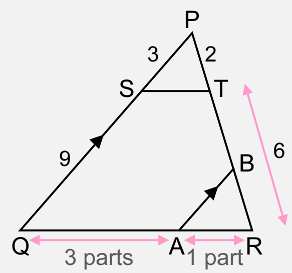


Answers

5.1 In ΔPQR: $\frac{\mathbf{PS}}{\mathbf{SQ}} = \frac{3}{9} = \frac{1}{3}$ & $\frac{\mathbf{PT}}{\mathbf{TR}} = \frac{2}{6} = \frac{1}{3}$

$\therefore \frac{\mathbf{PS}}{\mathbf{SQ}} = \frac{\mathbf{PT}}{\mathbf{TR}}$

$\therefore$ ST || QR ... *converse of proportion thm*



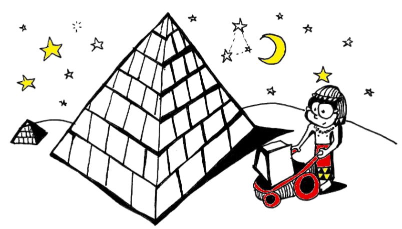
5.2 In ΔRPQ: $\frac{\mathbf{RB}}{\mathbf{RP}} = \frac{RA}{RQ} = \frac{\mathbf{1}}{\mathbf{4}}$ ... *proportion theorem ; AB || QP*

RA : AQ = 1 : 3

$\therefore RB = \frac{1}{4}RP$

$= 2$ units ... *RP = PT + TR = 8 units*

$\therefore$ TB = 4 units

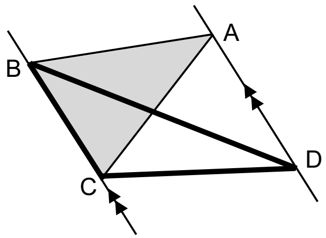


Proving the Proportion theorem

Be sure to revise the following two concepts involving areas of triangles.
These concepts are used in the proof of the proportion theorem which follows.

IMPORTANT CONCEPTS REQUIRED

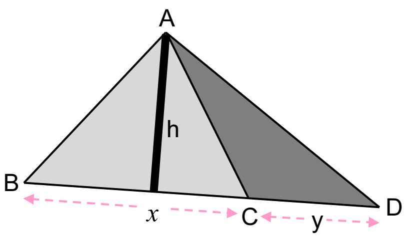
❶ Δ^s on the same base and between the same || lines have equal areas.



$\Delta ABC = \Delta DBC$ in area

These Δ^s have the same base, BC, and the same height (since they lie between the same || lines).

❷ When Δ^s have the same height, the ratio of their areas equals the ratio of their bases.

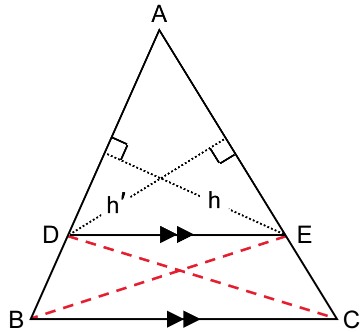


$$\frac{\text{Area of } \Delta ABC}{\text{Area of } \Delta ACD} = \frac{\frac{1}{2}x.h}{\frac{1}{2}y.h} = \frac{x}{y}$$

These Δ^s have a common vertex, A, and therefore the same height.

THE PROOF OF THE PROPORTION THEOREM

Given: ΔABC with DE || BC, D & E on AB & AC respectively.



To prove: $\frac{AD}{DB} = \frac{AE}{EC}$

Construction: Join DC & BE

Proof: $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DBE} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}DB.h} = \frac{AD}{DB}$

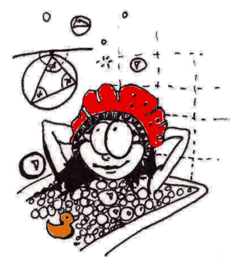
Similarly: $\frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta EDC} = \frac{AE}{EC}$ $\left(\frac{\frac{1}{2}AE.h'}{\frac{1}{2}EC.h'}\right)$

But: $\Delta DBE = \Delta EDC$ . . . *on the same base DE ; between || lines, DE & BC*

and: ΔADE is common

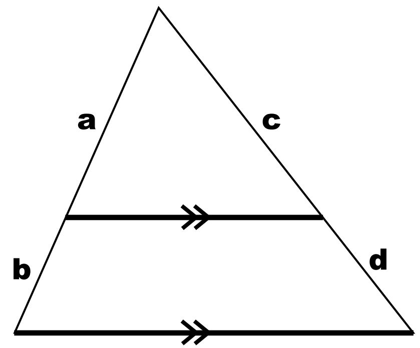
$$\therefore \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta DBE} = \frac{\text{Area of } \Delta ADE}{\text{Area of } \Delta EDC}$$

$$\therefore \mathbf{\frac{AD}{DB} = \frac{AE}{EC}}$$



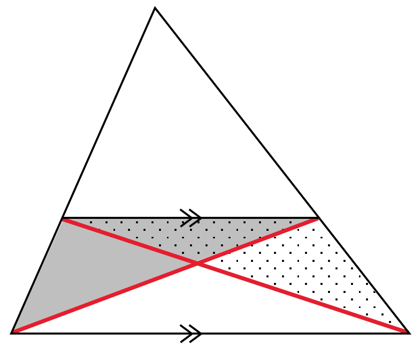
PROPORTION THEOREM PROOF: A Visual presentation

The Situation



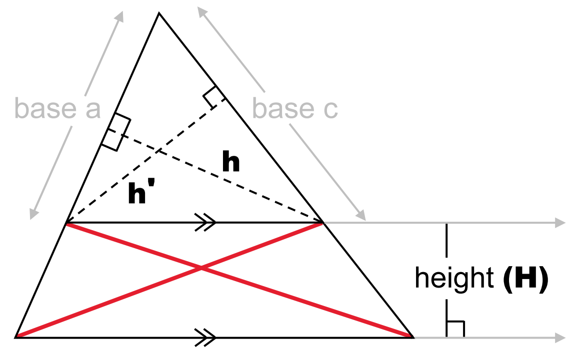
Parallel lines in a Δ

The Construction

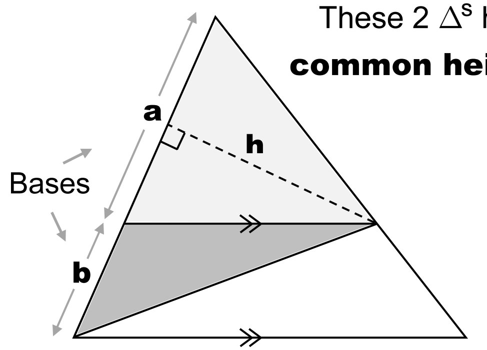


Create 2 Δ^s

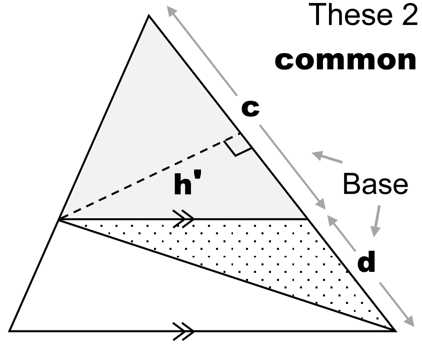
Heights of Δ^s



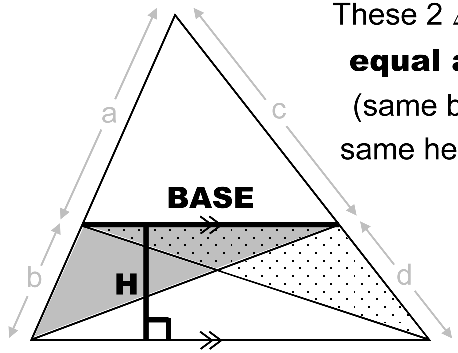
These 2 Δ^s have **common height h**



These 2 Δ^s have **common height h'**



These 2 Δ^s have **equal areas** (same base & same height **H**)



The same

$$\frac{\triangle}{\triangle} = \frac{\frac{1}{2}ah}{\frac{1}{2}bh} = \frac{a}{b}$$

$$\frac{\triangle}{\triangle} = \frac{\frac{1}{2}ch'}{\frac{1}{2}dh'} = \frac{c}{d}$$

But: $\triangle = \triangle$

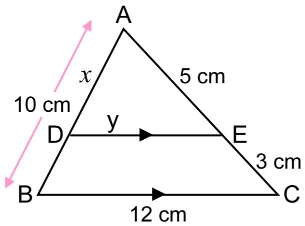
$$\therefore \frac{a}{b} = \frac{c}{d}$$

The Theorem Statement:

A line drawn parallel to one side of a triangle divides the other two sides proportionally.

Similar Δ^s vs. Proportion Theorem Application

1 Find the values of x and y in the figure alongside.



Answer

- In ΔABC: $\frac{x}{10} = \frac{5}{8}$ . . . *DE || BC; proportion theorem*

$\times$ 10) $\therefore\ x = 6\frac{1}{4}$ cm ◄

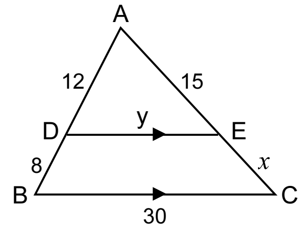
- ΔADE ||| ΔABC ➡ $\frac{y}{12} = \frac{5}{8}$ . . . $\frac{DE}{BC} = \frac{AE}{AC}$; *proportional sides*

$\times$ 12) $\therefore$ y $= 7\frac{1}{2}$ cm ◄

Note:

Distinguish between the applications of the **similar** Δ^s and **proportion** theorems! (See next column.)

2 Find x and y in the sketch alongside



▶ **The Proportion theorem** (finding x)

In ΔABC: DE || BC

The unknown ➡ $\frac{x}{15} = \frac{8}{12}$

$\times$ 15) $\therefore\ x$ = 10 units

The proportion theorem does NOT refer to the lengths of the parallel lines, only to AB and AC and their segments.

▶ **Similar triangles theorem** (finding y)

In Δ^s ADE and ABC:

(1) $\hat{A}$ is common

(2) $A\hat{D}E = \hat{B}$ … *corresponding* $\angle^s$; *DE || BC*

[& $A\hat{E}D = \hat{C}$ … *corresponding* $\angle^s$; *DE || BC*]

$\therefore$ ΔADE ||| ΔABC . . . *equiangular* Δ^s

$\therefore\ \frac{DE}{BC} = \frac{AD}{\mathbf{AB}}$ or $\frac{AE}{\mathbf{AC}}$ . . .

Note: $\frac{DE}{BC} \neq \frac{AD}{DB}$ or $\frac{AE}{EC}$ because BC is a side of ΔABC, while DB and EC are not.

$\therefore\ \frac{y}{30} = \frac{12}{20}$

$\times$ 30) $\therefore$ y = 18 units

In the same figure above, ΔABC can be seen as an enlargement of ΔADE and the sides of these triangles are proportional.

$\therefore\ \frac{y}{30} = \frac{12}{\mathbf{12+8}}$ $\left(or\ \frac{15}{\mathbf{15+10}} \right)$

It is only by using the similarity of the triangles, that we can relate the lengths of the parallel sides to the lengths of the other 2 sides of the triangles.

Similar Δ^s

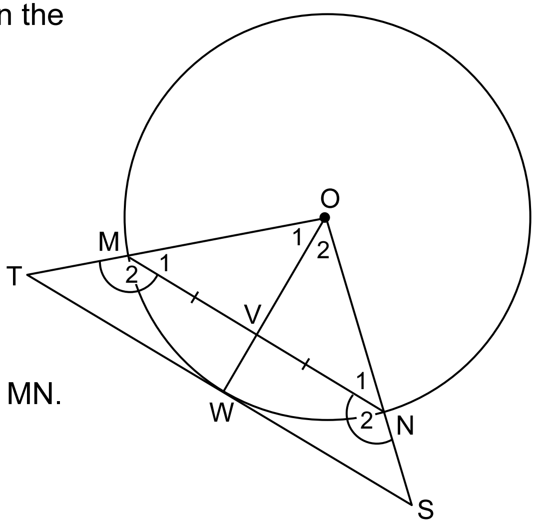
EXAMPLE 11 *(National November 2017 P2, Q10)* **34%**

In the diagram, W is a point on the circle with centre O.

V is a point on OW.

Chord MN is drawn such that MV = VN.

The tangent at W meets OM produced at T and ON produced at S.



(a) Give a reason why OV ⊥ MN.
44%

(b) Prove that:
24% (i) MN || TS

(ii) TMNS is a cyclic quadrilateral

(iii) OS.MN = 2ON.WS

Answers

(a) **Line (OV) from centre to midpoint of chord (MN)** ◂

In this case, the midpoint of the chord is given, and we can conclude that OV ⊥ MN because of that.

Note: Analyse the information and the diagram.
So far, we have used and applied a 'centre' theorem, in (a).
Another clue is the 'tangent' at W.
Think about tangent facts . . . .

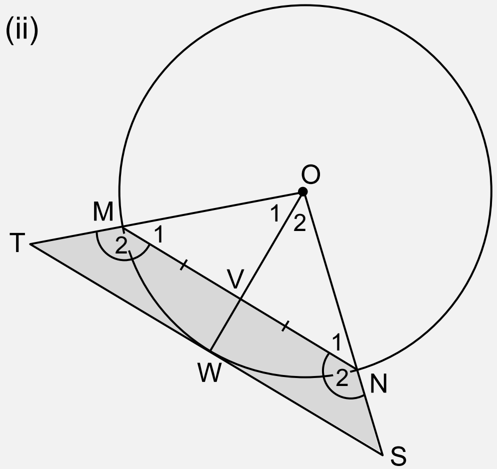
(b) (i) $O\hat{W}S = 90^o$ . . . *tangent ⊥ radius*

$\therefore$ $O\hat{V}N = O\hat{W}S$ (= 90^o)

$\therefore$ MN || TS . . . *corresp. $\angle^s$ equal*

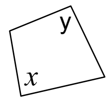
The most 'basic' way to prove lines || is: alt. or corresp. $\angle^s$ equal or co-int. $\angle^s$ suppl.

(ii)

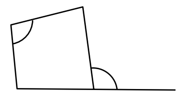


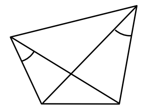
Shade the quadrilateral TMNS

There are **3 ways** to prove that a quadrilateral is a cyclic quadrilateral – choose 1:

❶


Prove that:
x + y = 180º

❷

Prove that:
Ext. ∠ = int. opp. ∠

❸

Prove that:
A side subtends equal $\angle^s$ at 2 other vertices

$\hat{M}_1 = \hat{N}_1$ ... *$\angle^s$ opposite equal radii*

$= \hat{S}$ ... *corresp. $\angle^s$; MN || TS*

We chose ❷ and proved that the exterior ∠ of **quadrilateral** TMNS = the interior opposite ∠

$\therefore$ TMNS is a cyclic quadrilateral . . . *converse ext. ∠ of cyclic quad.*

(iii)

This question looks like **ratio and proportion**.
Mark the sides on the diagram.

The sides appear to involve ΔOWS, which has VN || WS, (even though MN = 2VN)

. . . *Maybe apply the proportion theorem in this Δ?*

But, the sides in the question involve the **horizontal sides** WS and VN.

So, proportion theorem is excluded.
We will use **similar** Δ^s!

In Δ^s OVN and OWS

❶ $\hat{O}_2$ is common

❷ $O\hat{V}N = O\hat{W}S$ . . . *corresp. $\angle^s$; MN || TS*

$\therefore$ $\Delta OVN \;|||\; \Delta OWS$ . . . $\angle\angle\angle$

Let's 'arrange' the sides to suit the question.

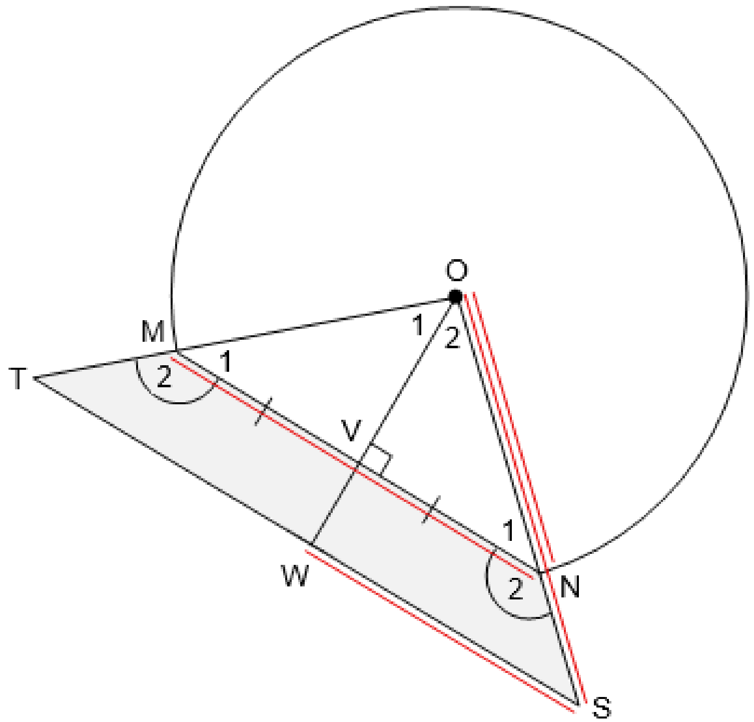
$\therefore \dfrac{OS}{ON} = \dfrac{WS}{VN} \left(= \dfrac{OW}{OV} \right)$ . . . *equiangular Δ^s*

$\therefore$ OS.VN = ON.WS

But $VN = \frac{1}{2}MN$ . . . *V midpoint MN*

$\therefore$ $OS.\frac{1}{2}MN = ON.WS$

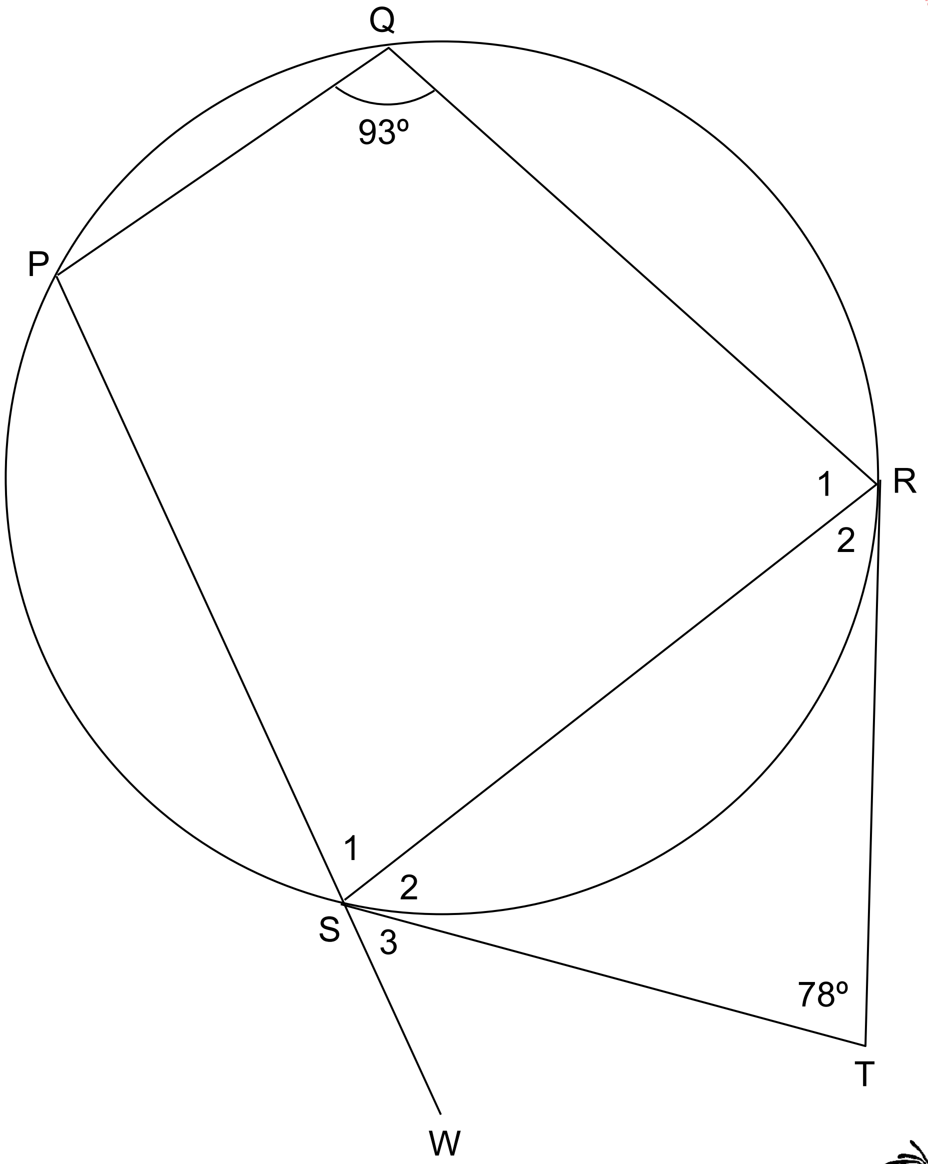
$\times 2)\therefore$ OS.MN = **2ON.WS** ◀



'A Mix' from DBE Nov 2021

Worked Example 12 63%

9. In the diagram, PQRS is a **cyclic quadrilateral**. PS is produced to W. TR and TS are **tangents** to the circle at R and S respectively. $\hat{T} = 78°$ and $\hat{Q} = 93°$.



9.1 Give a reason why ST = TR. (1)

68%

9.2 Calculate, giving reasons, the size of:

62%

9.2.1 $\hat{S}_2$ 9.2.2 $\hat{S}_3$ (2)(2) **[5]**

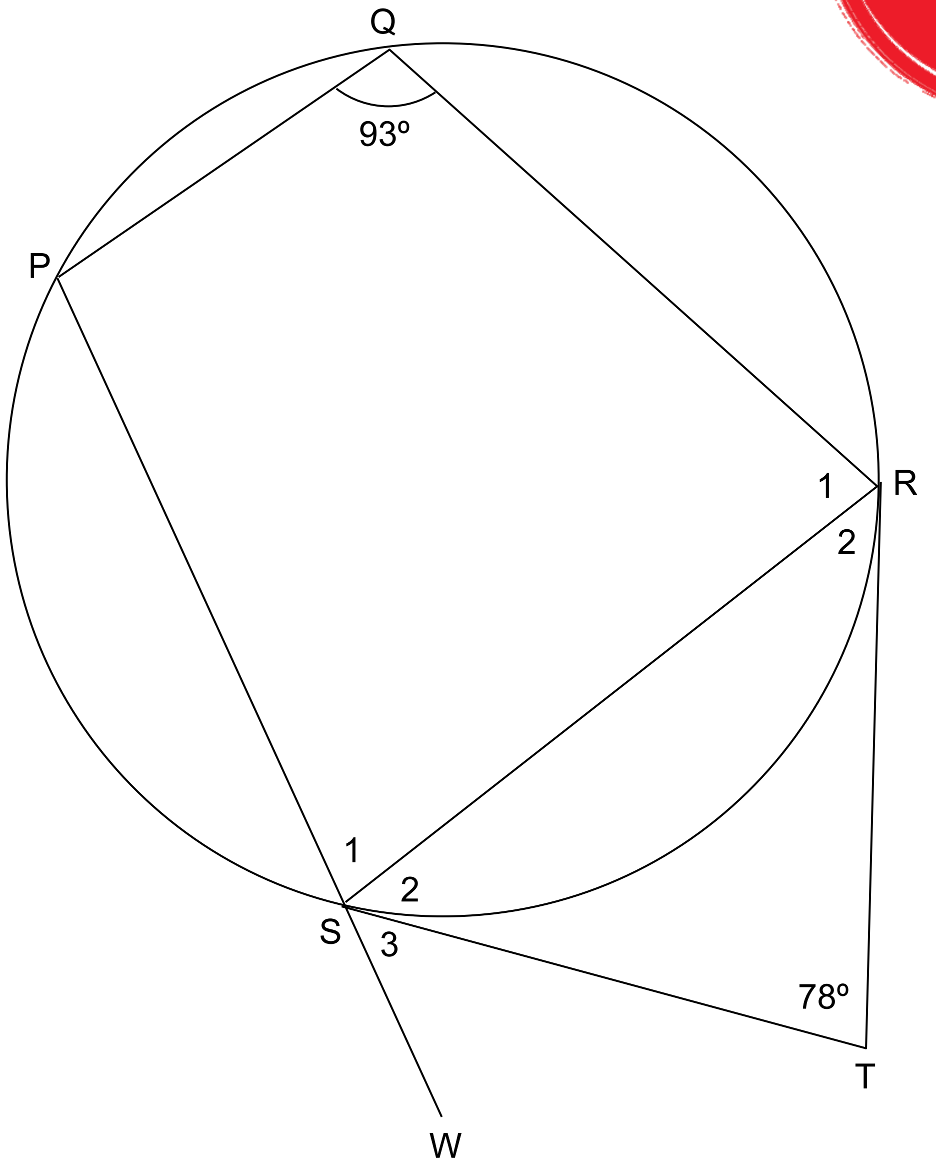
Gr 12 Maths **Toolkit**: DBE Past Papers, p. 32

9. In the diagram, PQRS is a **cyclic quadrilateral**. PS is produced to W.
TR and TS are **tangents** to the circle at R and S respectively.
$\hat{T} = 78^\circ$ and $\hat{Q} = 93^\circ$.

9.1 Give a reason why ST = TR. (1)

9.2 Calculate, giving reasons, the size of:

9.2.1 $\hat{S}_2$ 9.2.2 $\hat{S}_3$ (2)(2) **[5]**



MEMOS

9.1 Tangents from a common point.

9.2.1 $\hat{S}_2 = \hat{R}_2$ . . . *∠ˢ opposite equal sides*

$= \frac{1}{2}(180^\circ - 78^\circ)$ . . . *∠ sum of Δ*

$= \mathbf{51^\circ}$ ➤

9.2.2 $\hat{S}_3 + \hat{S}_2 = \hat{Q}$ . . . *ext. ∠ of cyc. quad.*

$\therefore\ \hat{S}_3 + 51^\circ = 93^\circ$

$\therefore\ \mathbf{\hat{S}_3 = 42^\circ}$ ➤

Worked Example 13 24%

10. In the diagram, BE and CD are **diameters** of a circle having M as **centre**. Chord AE is drawn to cut CD at F. **AE ⊥ CD**.
Let $\hat{C} = x$.

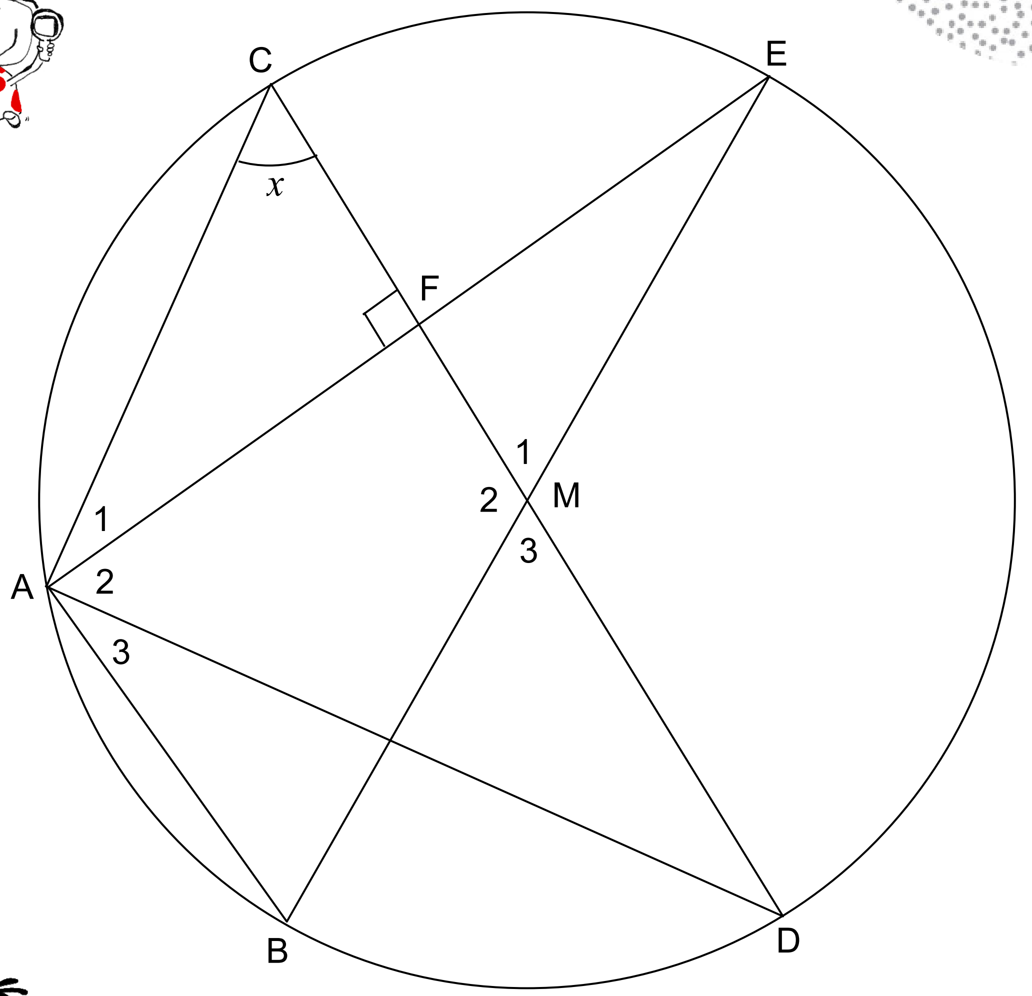
10.1 Give a reason why AF = FE. (1)
47%

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)
37%

10.3 **Prove**, giving reasons, that AD is a **tangent** to the circle passing through A, C and F. (4)
37%

10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)
9%

[13]



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10. In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. AE $\perp$ CD.
Let $\hat{C} = x$.

10.1 Give a reason why AF = FE. (1)

10.2 Determine, giving reasons, the size of $\hat{M}_1$ in terms of x. (3)

10.3 Prove, giving reasons, that AD is a tangent to the circle passing through A, C and F. (4)

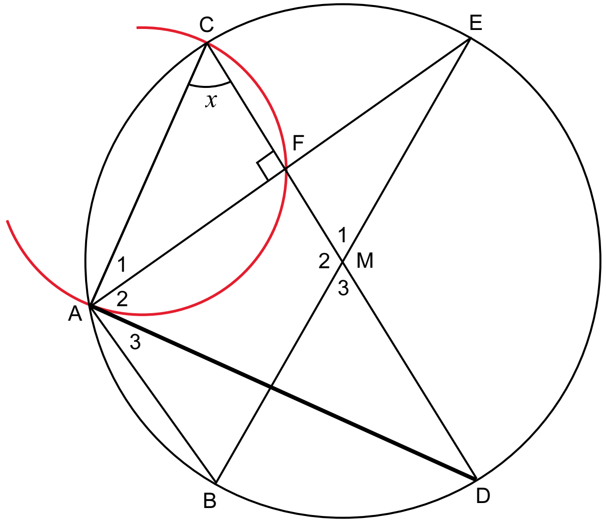
10.4 Given that CF = 6 units and AB = 24 units, calculate, giving reasons, the length of AE. (5)

[13]

MEMOS

10.1 MF $\perp$ AE, i.e. (line from centre $\perp$ to chord) ◄

10.2 $\hat{M}_1 = 2\hat{A}_1$ . . . ($\angle$ *at centre* = 2 × $\angle$ *at circ.*)
& $\hat{A}_1 = 90^\circ - x$ . . . $\angle$ *sum of* Δ
$\therefore$ $\hat{M}_1 = 2(90^\circ - x)$
$= \mathbf{180^\circ - 2x}$ ◄



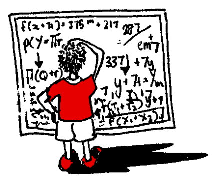
10.3 AC is a diameter of ⊙ACF . . . (*conv* $\angle$ *in semi-*⊙)
& $\hat{DAC} = 90^\circ$ . . . ($\angle$ *in semi-*⊙*; diameter CD*)
i.e. line AD $\perp$ diameter AC
$\therefore$ **AD is a tangent to ⊙ACF** ◄ . . . (*conv tan* $\perp$ *rad*)

OR: $\hat{A}_2 = x$ . . . $\angle$ *in semi-*⊙
$\therefore$ $\hat{A}_2 = \hat{C}$
$\therefore$ **AD is a tangent to ⊙ACF** ◄ . . . (*conv tan chord thm*)

10.4 In ΔEAB:
F is the midpoint of EA
& M is the midpoint of EB . . . (*line from centre of* ⊙)
$\therefore$ FM = $\frac{1}{2}$AB = 12 units . . . (*midpt theorem*)
& EB = DC = 2(6 + 12) . . . *diameter*
= 36 units
$\therefore$ In right $\angle^d$ ΔEAB:
$AE^2 = EB^2 - AB^2$ . . . (*Theorem of Pythagoras*)
$= 36^2 - 24^2$
$= 720$
$\therefore$ **AE = $\sqrt{720}$ = $12\sqrt{5}$ ≈ 26,83 units** ◄

OR: $\hat{EAB} = 90^\circ$ . . . $\angle$ *in semi-*⊙
In ΔEFM & ΔEAB
(1) $\hat{E}$ is common
(2) $\hat{EFM} = \hat{EAB}$
$\therefore$ (ΔEFM ||| ΔEAB) . . . $\angle\angle\angle$
$\therefore$ $\frac{FM}{AB} = \frac{EF}{EA} = \frac{1}{2}$
$\therefore$ FM = 12 units
$\therefore$ ME = 6 + 12 = 18 units . . . *radii equal*
$\therefore$ $EF^2 = ME^2 - FM^2$ . . . (*Theorem of Pythagoras*)
$= 18^2 - 12^2$
$= 180$
$\therefore$ EF = $6\sqrt{5}$
$\therefore$ AE = $12\sqrt{5}$ ≈ 26,83 units

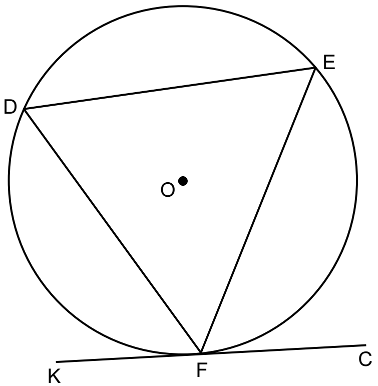
OR: In ΔEAB:
F is the midpoint of EA
& M is the midpoint of EB . . . (*line from centre of* ⊙)
$\therefore$ FM = $\frac{1}{2}$AB = 12 units . . . (*midpt theorem*)
$\therefore$ MC = 18
$\therefore$ MD = 18 . . . *radii equal*
(ΔCFA ||| ΔAFD) . . . $\angle\angle\angle$
$\therefore$ $\frac{FA}{FD} = \frac{CF}{AF}$
$\therefore$ $AF^2 = CF \cdot FD$
$= 6 \cdot 30$
$= 180$
$\therefore$ AF = $6\sqrt{5}$
$\therefore$ AE = $12\sqrt{5}$



Worked Example 14 34%

11.1 In the diagram, chords DE, EF and DF are
57% drawn in the circle with centre O.

KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$. (5)

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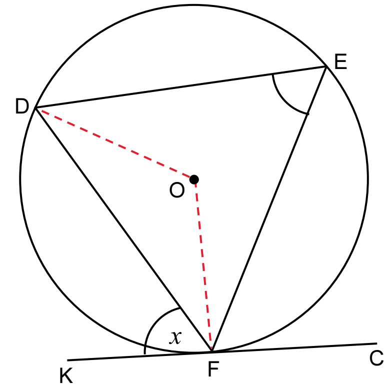
DBE Past Papers, p. 32

MEMOS

11.1 The angle between a tangent to a circle and a chord drawn from the point of contact is equal to the angle subtended by the chord in the alternate segment.

Method 1

Draw radii and use '∠ at centre' theorem.



Given: ⊙O with tangent at F and chord FE subtending $\hat{D}$ at the circumference.

RTP: $D\hat{F}K = \hat{E}$

Construction: radii OF and OD

Proof: Let $D\hat{F}K = x$

$O\hat{F}K = 90°$ . . . *radius ⊥ tangent*

$\therefore O\hat{F}D = 90° - x$

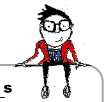
$\therefore O\hat{D}F = 90° - x$ . . . *∠ˢ opposite equal radii*

$\therefore D\hat{O}F = 2x$ . . . *sum of ∠ˢ in Δ*

$\therefore \hat{E} = x$ . . . *∠ at centre = 2 × ∠ at circumference*

$\therefore$ **$D\hat{F}K = \hat{E}$** ◄

Method 2



We use **2** 'previous' **facts involving right ∠ˢ**
❶ **tangent ⊥ diameter** . . . *so, draw a diameter!*
❷ **∠ in semi-⊙ = 90°** . . . *so, join RK!*

These proofs are logical & easy to follow.

Given: ⊙O with tangent at F and chord FE subtending $\hat{D}$ at the circumference.

RTP: $D\hat{F}K = \hat{E}$

Construction: diameter FM; join ME

Proof: $M\hat{F}K = \mathbf{90°}$ . . . *tangent ⊥ diameter*
& $M\hat{E}F = \mathbf{90°}$ . . . *∠ in semi-⊙*

Then . . .

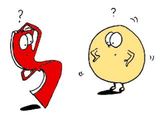
Let $D\hat{F}K = x$

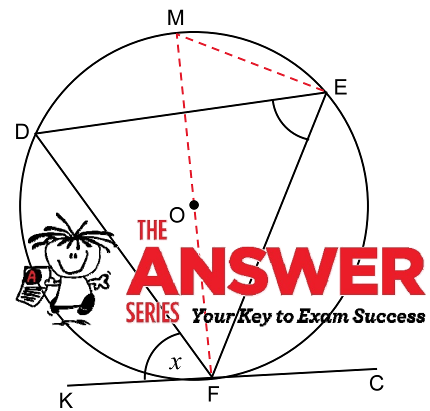
$\therefore M\hat{F}D = 90° - x$

$\therefore M\hat{E}D = 90° - x$ . . . *∠ˢ in same segment*

$\therefore D\hat{E}F = x$

$\therefore$ **$D\hat{F}K = D\hat{E}F$** ◄





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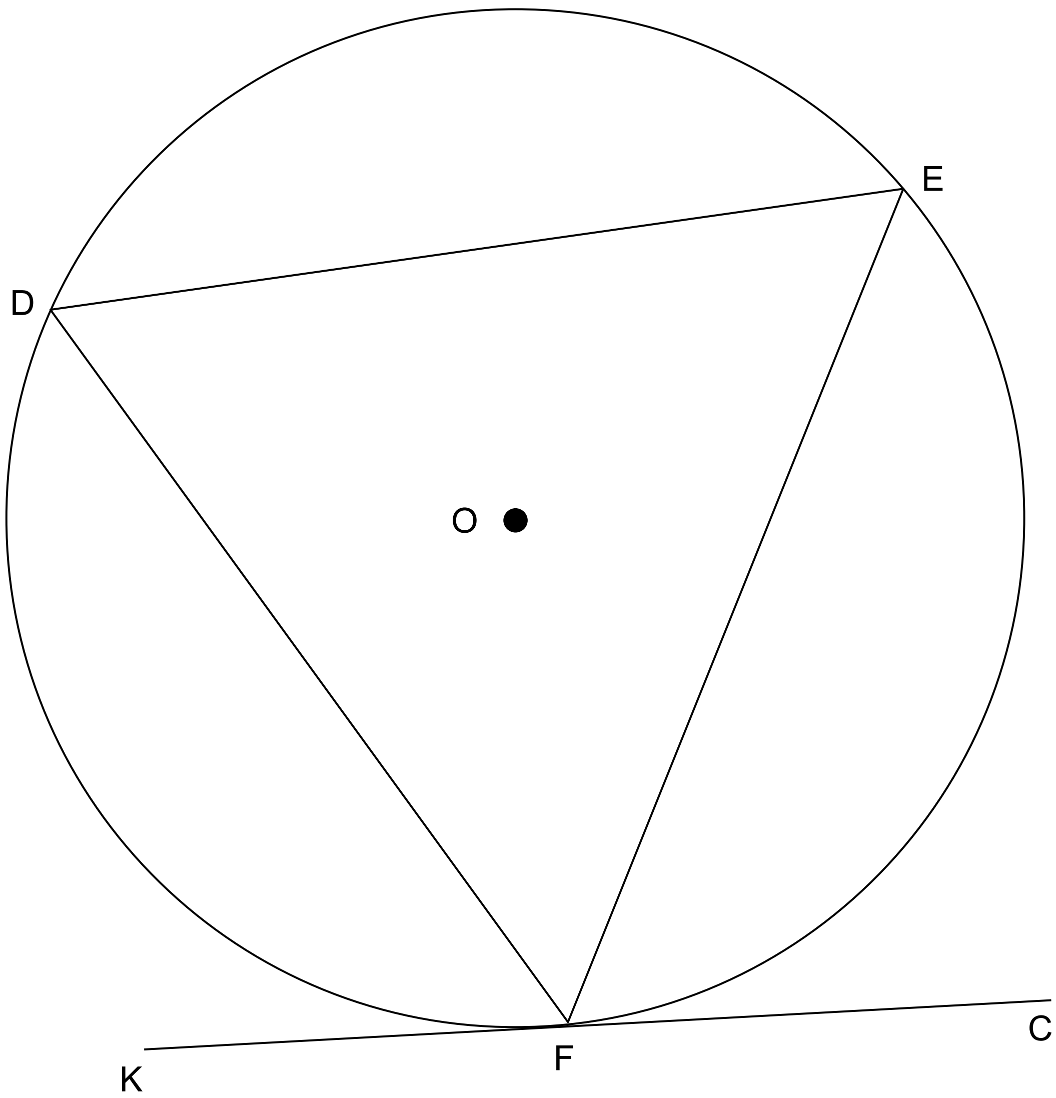
E
D
O
C
F
K

11.2 In the diagram, PK is a **tangent** to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that **MN || SK**. Chord KS and LN intersect at T.

33%

11.2.1 **Prove**, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) **KLMN is a cyclic quadrilateral.** (1)

11.2.2 Prove, giving reasons, that $\triangle LKN \parallel\!| \triangle KSM$. (5)

11.2.3 If LK = 12 units and 3KN = 4SM, determine the length of KS. (4)

11.2.4 If it is further given that NL = 16 units, LS = 13 units and KN = 8 units, determine, with reasons, the length of LT. (4)

[23]

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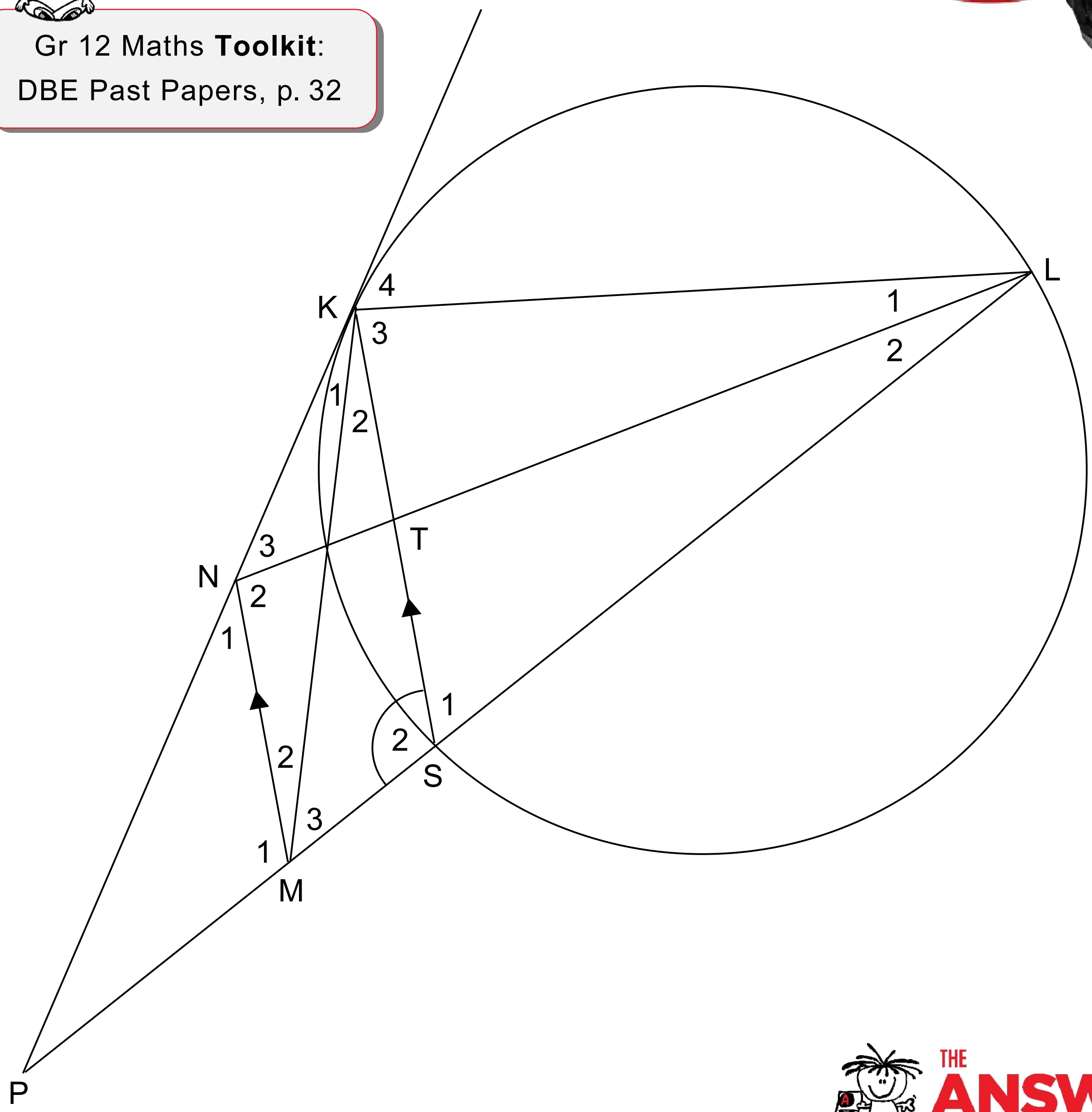


11.2 In the diagram, PK is a tangent to the circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN || SK. Chord KS and LN intersect at T.

11.2.1 Prove, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4)

(b) KLMN is a cyclic quadrilateral. (1)

11.2.2 Prove, giving reasons, that $\Delta LKN ||| \Delta KSM$. (5)

11.2.3 If LK = 12 units and 3KN = 4SM, determine the length of KS. (4)

11.2.4 If it is further given that NL = 16 units, LS = 13 units and KN = 8 units, determine, with reasons, the length of LT. (4)

[23]

MEMOS

11.2.1 (a) $\hat{K}_4 = \hat{S}_1$ ... *tan chord thm*

$= \mathbf{N\hat{M}L}$ ◄ ... *corresp ∠s; MN || SK*

(b) $\hat{K}_4 = N\hat{M}L$

∴ KLMN is a cyclic quad. ◄ ... *converse ext ∠ of c.q.*

11.2.2 $L\hat{K}N = \hat{M}_1$ ... *ext ∠ of cyclic quad. KLMN*

$= \hat{S}_2$ ... *corresp ∠s; MN || SK*

∴ In Δ^s LKN & KSM

(1) $L\hat{K}N = \hat{S}_2$

(2) $\hat{N}_3 = \hat{M}_3$ ... *∠s in same segment*

∴ **ΔLKN ||| ΔKSM** ◄ ... ∠∠∠

11.2.3 ∴ $\frac{KS}{LK} = \frac{SM}{KN}$ ... $|||\Delta^s$

3KN = 4SM

∴ $\frac{3}{4} = \frac{SM}{KN}$

∴ $\frac{KS}{12} = \frac{3}{4}$

∴ $KS = \frac{36}{4}$

= 9 units ◄

11.2.4 4SM = 3KN = 3 × 8 = 24

∴ SM = 6 units

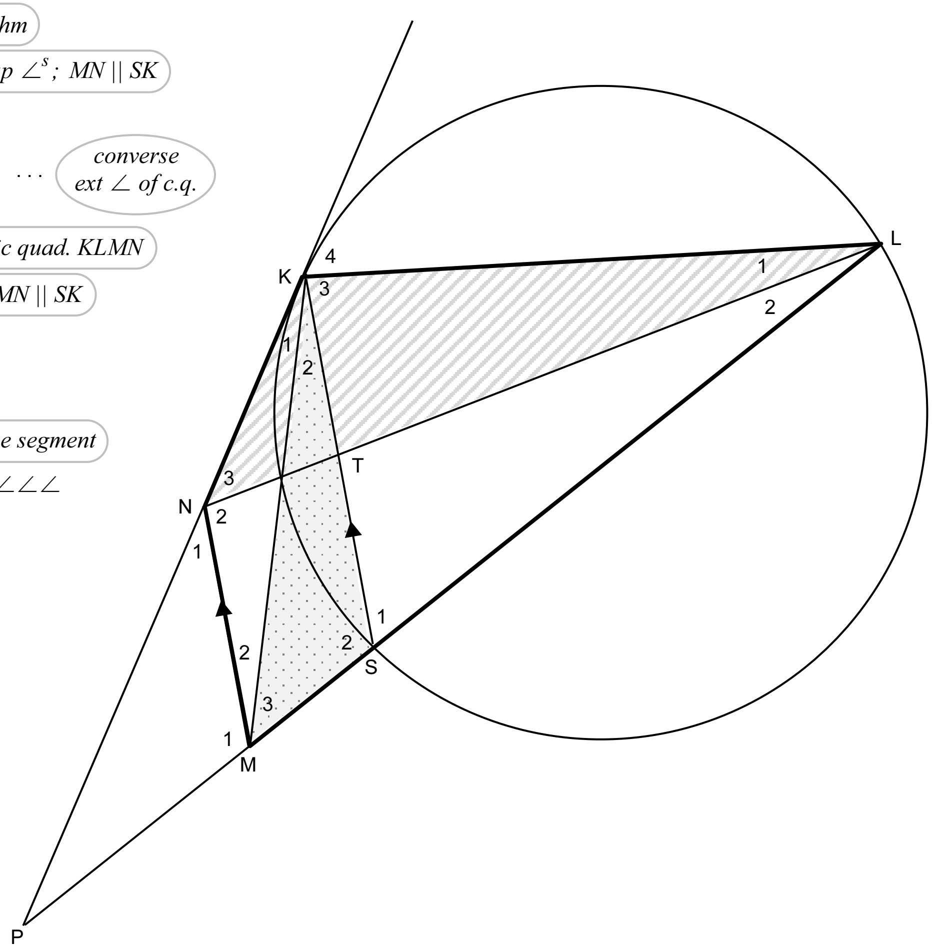
In ΔLNM: $\frac{LT}{LN} = \frac{LS}{LM}$ ... *prop thm; MN || ST*

∴ $\frac{LT}{16} = \frac{13}{13+6}$

∴ $LT = \frac{13}{19} \times 16$

$= \frac{208}{19}$

≈ **10,95 units** ◄



Euclidean Geometry

References to TAS Maths books

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Gr 10 Maths 3-in-1 (Module 7)

# 1:	Lines, angles & triangles: revision • vocabulary & facts	7.1 ⟶ 7.7
# 2:	Quadrilaterals: revision • definitions • theorems • areas	7.8 ⟶ 7.15
# 3:	Midpoint theorem	7.16 ⟶ 7.17
# 4:	Polygons: definitions & types • interior angles • exterior angles	7.18

Note: The Gr 10 Exemplar Exams and Memos are at the end of the book

Gr 11 Maths 3-in-1 (Module 9)

# 1:	Revision from earlier grades	9.1 ⟶ 9.5
# 2:	Circle Geometry	9.6 ⟶ 9.26

Note: The Gr 11 Exemplar Exams and Memos are at the end of the book

Gr 12 Maths 2-in-1 (Module 10)

# 1:	Circle Geometry	36 ⟶ 40
# 2:	Proportion Theorem	40 ⟶ 42
# 3:	Similar Triangles	42 ⟶ 43
# 4:	Mixed	43

See Challenging Questions booklet: pages 29 ⟶ 38

Back pages:	Circle Geometry, Proportion and Similar Triangles Theorem PROOFS	i ⟶ iii
	Grouping of Circle Geometry Theorems	viii
	Converse Theorems in Circle Geometry	ix
	Theorem Statements & Acceptable Reasons	x ⟶ xii

See the Topic Guide on p. 148 for further exam practice.

Gr 12 Maths Toolkit: DBE Past Papers

Back pages:	Circle Geometry, Proportion and Similar Triangles Theorem PROOFS	i ⟶ iii
	Grouping of Circle Geometry Theorems	xiii
	Theorem Statements & Acceptable Reasons	xiv ⟶ xvi

See the Topic Guide: DBE: p. 2

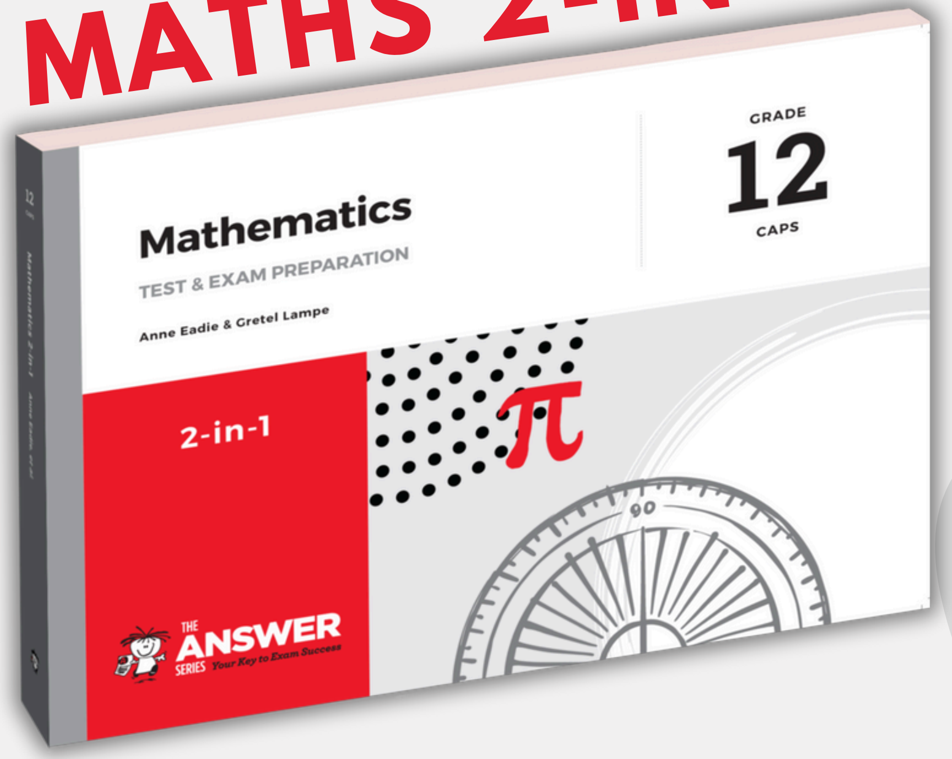
ABOUT

THE ANSWER SERIES

GRADE 12 MATHS

2-IN-1

GRADE 12
MATHS 2-IN-1

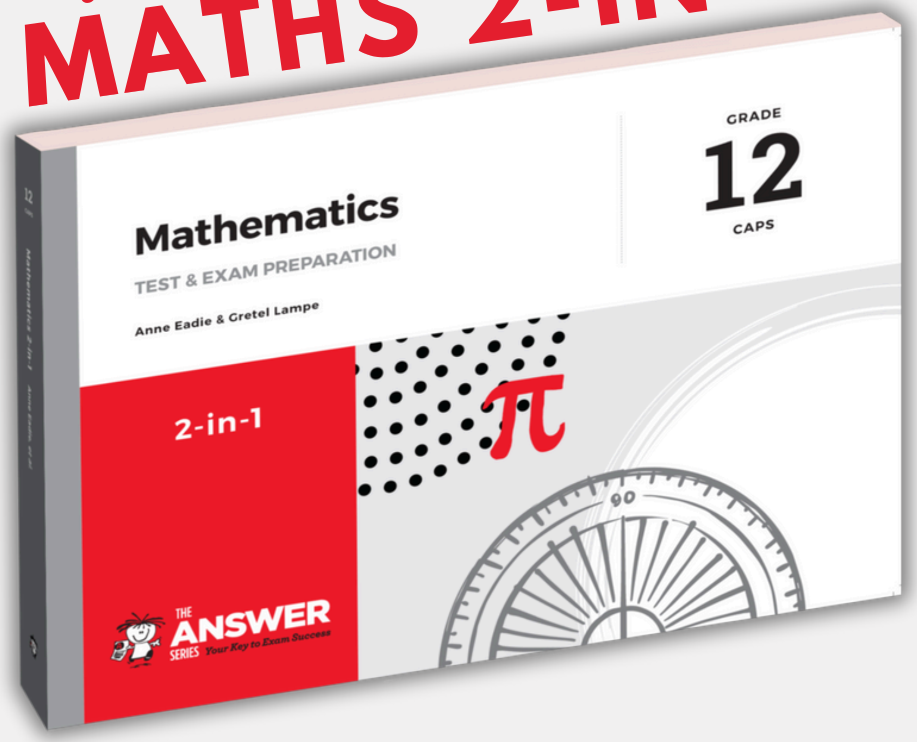


OFFERS

a **unique question and answer method** of mastering maths

a **way of thinking**

GRADE 12 MATHS 2-IN-1



DEVELOPS

- conceptual understanding
- procedural fluency and adaptability
- reasoning techniques
- a variety of strategies for problem-solving

The questions and detailed solutions have been structured into ...

SECTION 1

Separate topics

+

SECTION 2

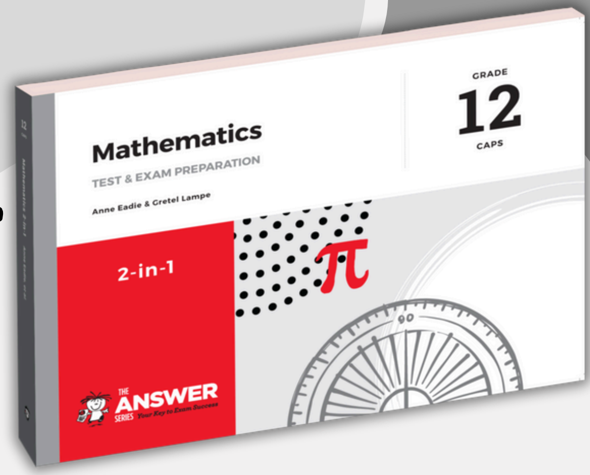
Exam papers

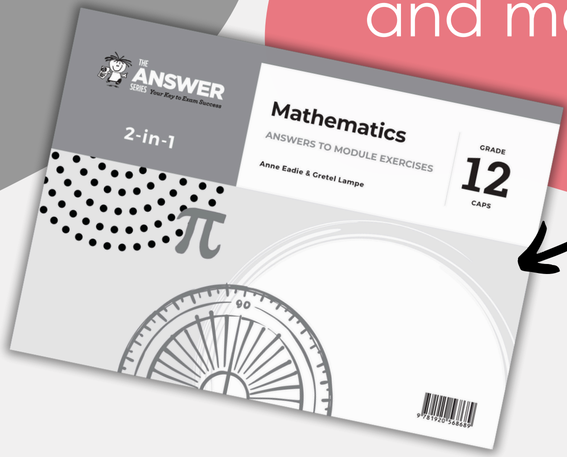
+

NEW

EXTENDED SECTION

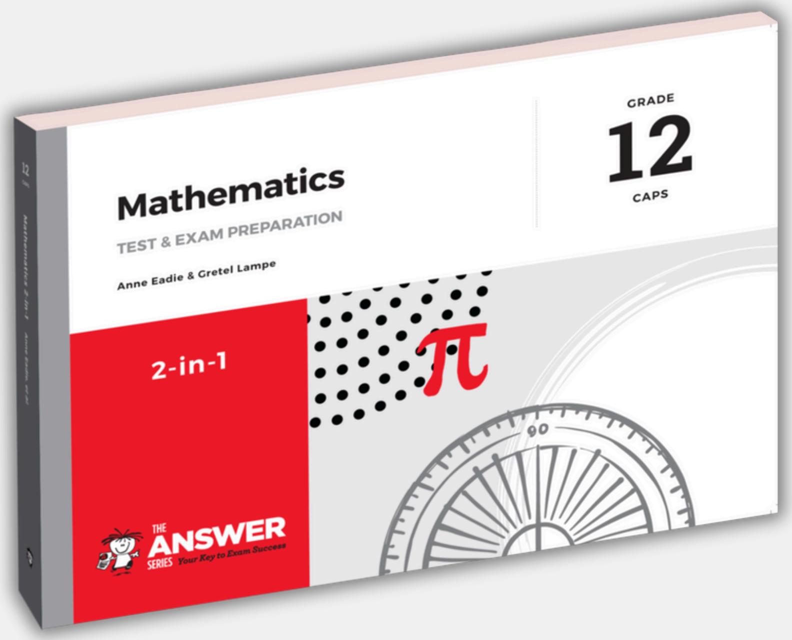
Challenging questions and memos





SECTION 1

SEPARATE TOPICS



It is important that **learners** focus on and **master one topic at a time** ***before*** **attempting past papers** which could be bewildering and demoralising.

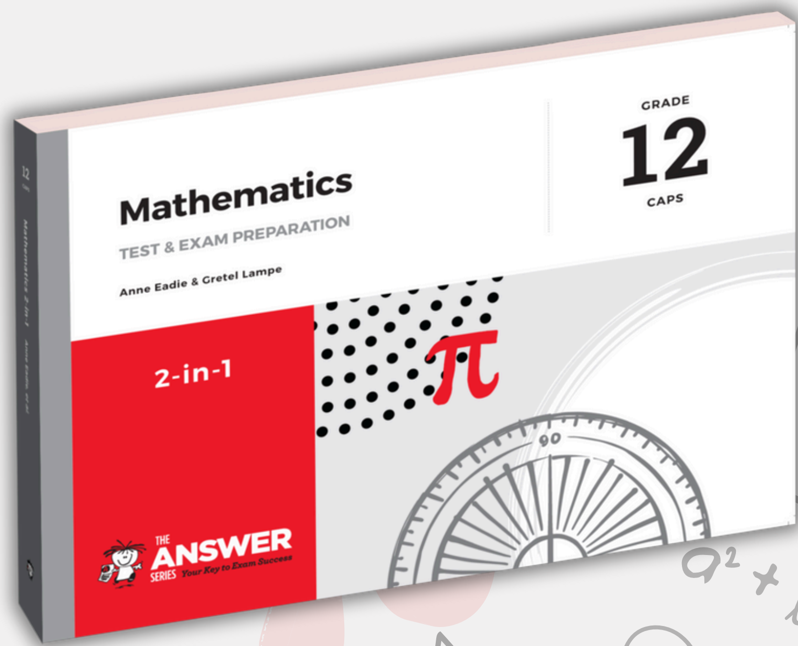
In this way they can **develop confidence** and a **deep understanding**.

SECTION 1

SEPARATE TOPICS

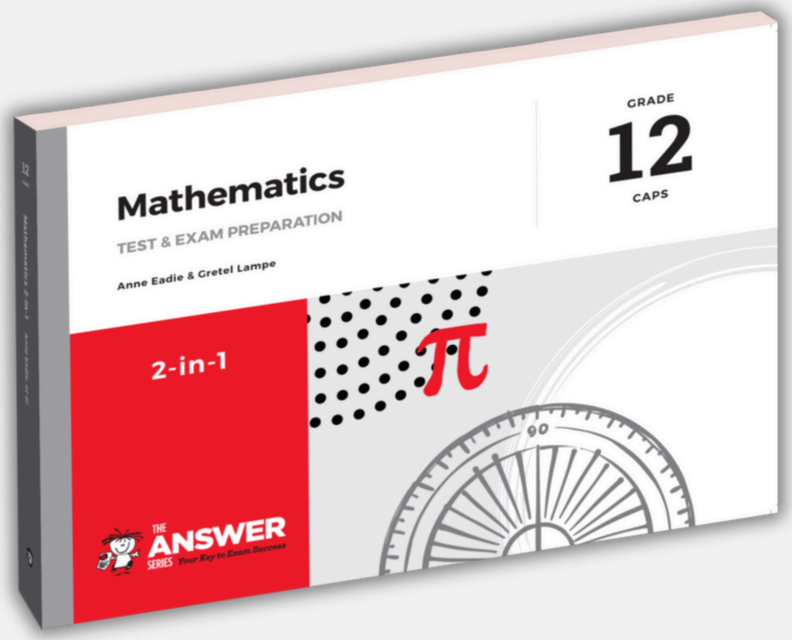
The questions in this section are designed to ...

- transition from basic concepts through to the more challenging concepts
- include critical prior learning (Gr 10 & 11) when this foundation is required for mastering the entire FET curriculum
- engage learners eagerly as they participate and thrive on their maths journey
- accommodate all cognitive levels



SECTION 2

EXAM PAPERS



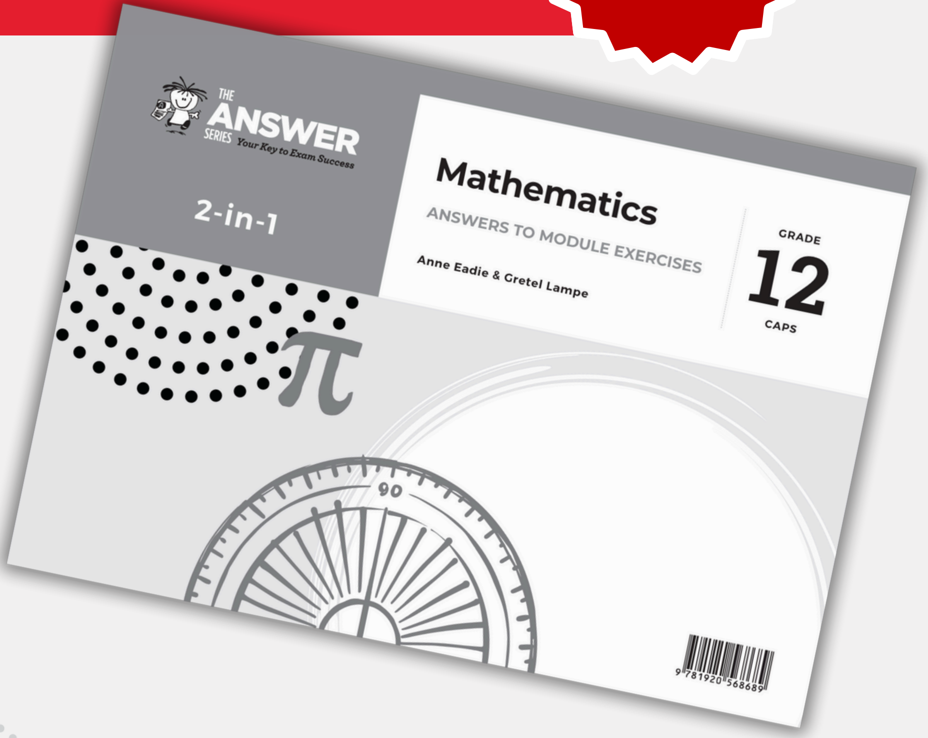
When learners have worked through the topics and grown fluent, they can then move on to the **exam papers** to experience working **a variety of questions** in one session, and to **perfect their skills**.

The **TOPIC GUIDES** will enable learners to continue mastering **one topic at a time**, even when working through the exam papers.

EXTENDED

SECTION

NEW



CHALLENGING QUESTIONS & MEMOS

These questions are **Cognitive Level 3 & 4 questions**, diagnosed as such following poor performance of learners in recent examinations.

Theory without practice is empty

Practice without theory is blind

Philosopher, Immanuel Kant (18th century philosopher)