KZN 2024 Maths Subject Advisors Workshop EUCLIDEAN GEOMETRY Problem Solving



BREAK THE 70% CEILING

LEARN HOW – Remember for a moment LEARN WHY – Remember for a life time Compiled by Anne Eadie





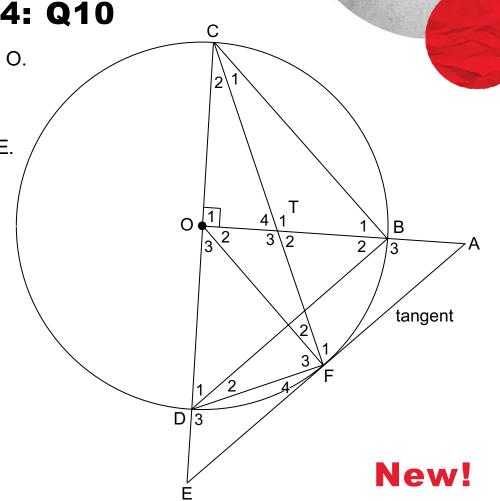
A Warm-up Example DBE May 2024: Q10



- In the diagram, COD is the diameter of the circle with centre O.
- EA is a tangent to the circle at F.
- AO \perp CE.
- Diameter COD produced intersects the tangent to the circle at E.
- OB produced intersects the tangent to the circle at A.
- CF intersects OB in T.
- CB, BD, OF and FD are drawn.

Prove, with reasons, that:

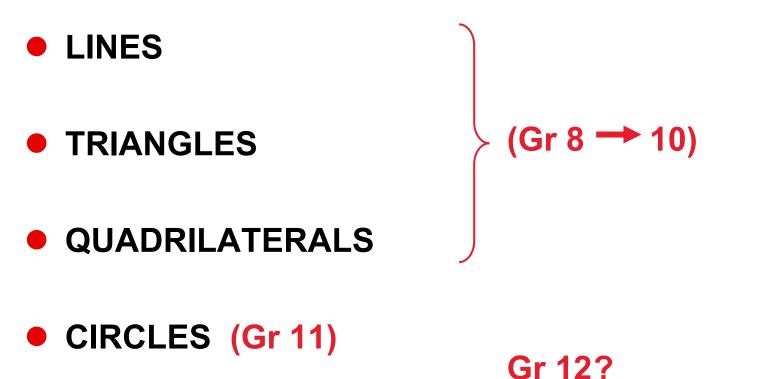
10.1	TODF is a cyclic quadrilateral	(4)
10.2	$\hat{D}_3 = \hat{T}_1$	(3)
10.3	Δ TFO Δ DFE	(5)
10.4	If $\hat{B}_2 = \hat{E}$, prove that DB EA.	(2)
10.5	Prove that DO = $\frac{\text{TO.FE}}{\text{AB}}$	(5) [19]



KZN Grade 12 2024 ATP

TERM 2					
NUMBER OF DAYS	DATE STARTED	DATE COMPLETED	ΤΟΡΙϹ	CURRICULUM STATEMENT	
03 – 10/04 (6 days)			EUCLIDEAN GEOMETRY	 Revise earlier work on the necessary and sufficient conditions for polygons to be similar. Prove (accepting results established in earlier grades) that a line drawn parallel to one side of a triangle divides the other two sides proportionally (and the Midpoint Theorem as a special case of this theorem). Solve proportionality problems and prove riders. 	
11 – 18/04 (6 days)			EUCLIDEAN GEOMETRY	 4. Prove (accepting results established in earlier grades): 4.1 that equiangular triangles are similar; 4.2 that triangles with sides in proportion are similar; and 4.3 the Pythagorean Theorem by similar triangles. 5. Solve similarity problems and prove riders. 	

THE ANSWER SERIES CONTENT FRAMEWORK: Gr 8 – 12





Gr 12

• THEOREM OF PYTHAGORAS (Gr 8)

• SIMILAR Δ^{s} (Gr 9)



• MIDPOINT THEOREM (Gr 10)



Ratio Proportion Area



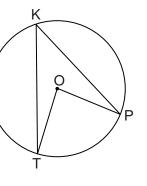
Remember these?

EUCLIDEAN GEOMETRY (41%): DBE NOV 2023 PAPER 2

QUESTION 8 60%

In the diagram, O is the centre of the circle. 8.1 55%

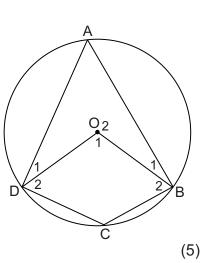
> Use the diagram alongside to prove the theorem which states that the angle subtended by a chord at the centre of the circle is equal to twice the angle subtended by the same chord at the circumference, that is, prove that $TOP = 2T\hat{K}P$.



(5)

8.2 In the diagram, O is the centre **59%** of the circle and ABCD is a cyclic quadrilateral. OB and OD are drawn.

> If $\hat{O}_1 = 4x + 100^\circ$ and $\hat{C} = x + 34^{\circ}$, calculate, giving reasons, the size of x.



8.3 In the diagram, O is the centre 65% of the larger circle. OB is a

diameter of the smaller circle. Chord AB of the larger circle intersects the smaller circle at M and B.

- 8.3.1 Write down the size of OMB. Provide a reason.
- 8.3.2 If AB = $\sqrt{300}$ units and OM = 5 units, calculate, giving reasons, the length of OB.

(4) [16]

(2)

- Gr 12 Maths Toolkit: DBE Past Papers, p. 43
- TAS Website: www.theanswer.co.za
- Diagnostic Report: Questions/Memos/Comments
- 2023 Exam Reviews



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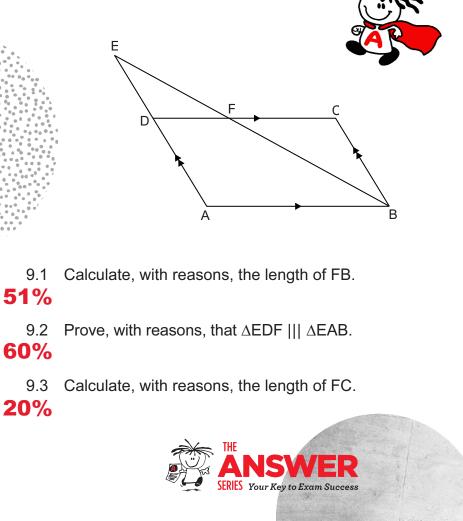
QUESTION 9 44%

In the diagram, ABCD is a parallelogram with AB = 14 units.

AD is produced to E such that AD : DE = 4 : 3.

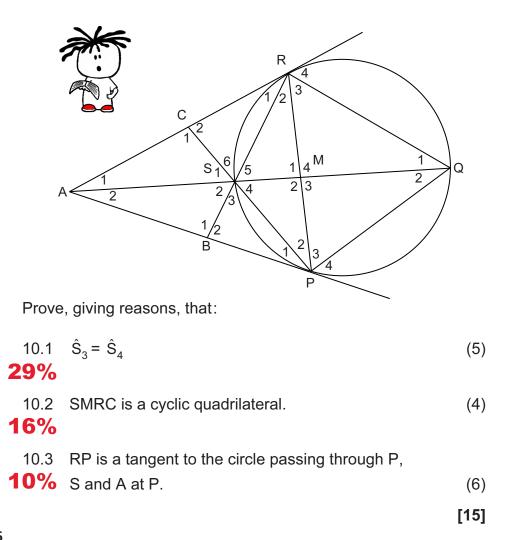
EB intersects DC in F.

EB = 21 units.



QUESTION 10 18%

In the diagram, PQRS is a cyclic quadrilateral such that PQ = PR. The tangents to the circle through P and R meet QS produced at A. RS is produced to meet tangent AP at B. PS is produced to meet tangent AR at C. PR and QS intersect at M.



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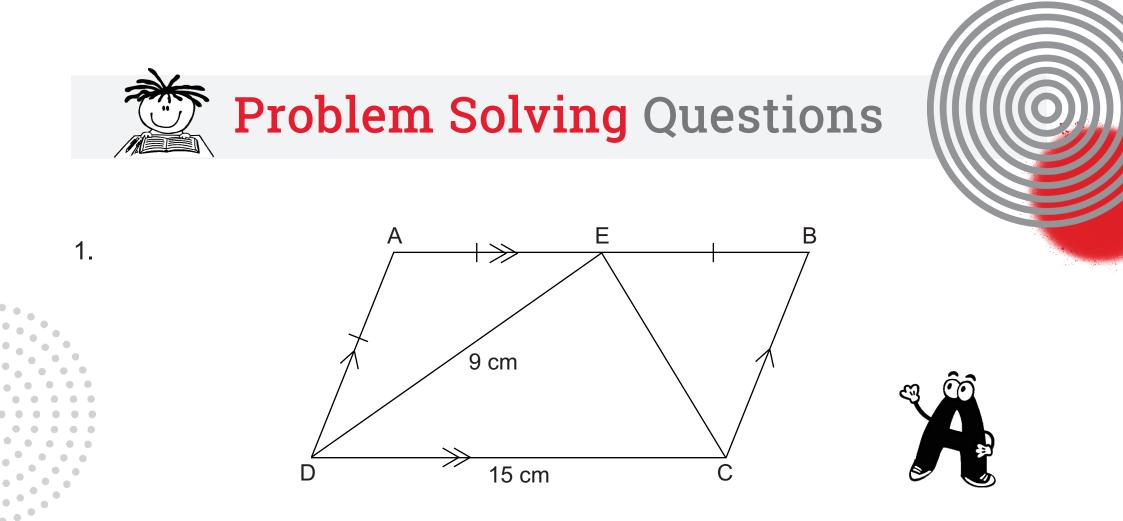
6

(3)

(3)

(3)

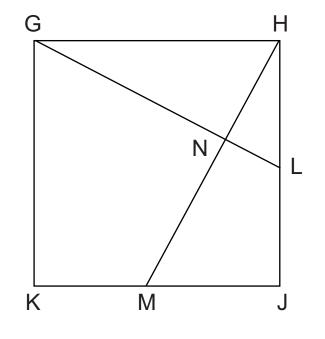
[9]



ABCD is a parallelogram with AD = AE = EB. DE = 9 cm and DC = 15 cm.

Determine the length of EC.







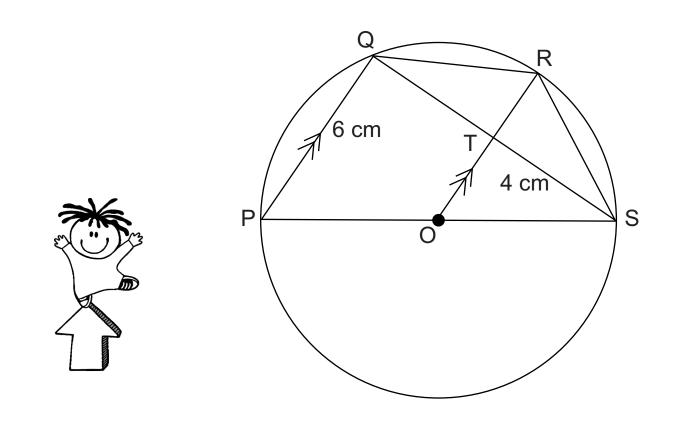
GHJK is a square with L and M the midpoints of HJ and JK respectively.

Prove that $GL \perp HM$.

2.







P, Q, R and S lie on the circumference of circle O.

PQ = 6 cm, ST = 4 cm, and the radius of the circle is 5 cm.

Determine the area of quadrilateral PQRS.





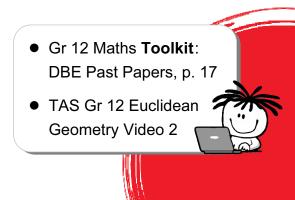
\odot Geom, no tangents

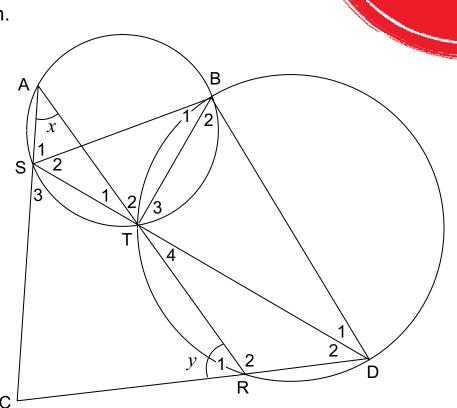
Example 1 (DBE Nov 2018 Q9.2) 44%

- In the diagram, a smaller circle ABTS and a bigger circle BDRT are given.
- BT is a common chord. Straight lines STD and ATR are drawn.
- Chords AS and DR are produced to meet in C, a point outside the two circles.
- BS and BD are drawn.

 $\hat{A} = x$ and $\hat{R}_1 = y$.

- 1.1 Name, giving a reason, another angle equal to:
 - (a) x (b) y (2)(2)
- 1.2 Prove that SCDB is a cyclic quadrilateral.
- 1.3 It is further given that $\hat{D}_2 = 30^\circ$ and $\hat{AST} = 100^\circ$. Prove that SD is not a diameter of circle BDS. (4)







(3)

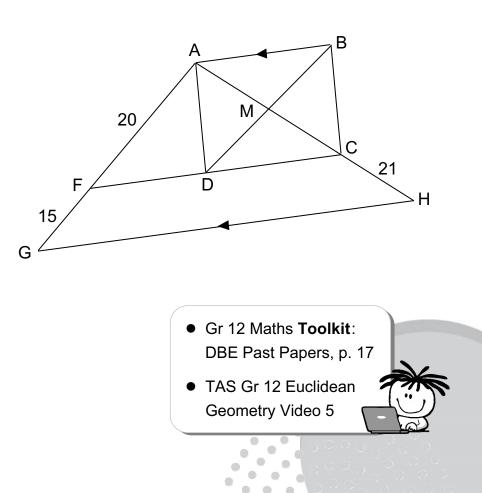


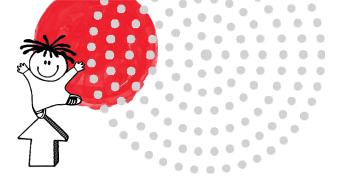


Proportionality

Example 2 (DBE Nov 2018 Q8.2) 59%

- In the diagram, $\triangle AGH$ is drawn.
- F and C are points on AG and AH respectively such that AF = 20 units, FG = 15 units and CH = 21 units.
- D is a point on FC such that ABCD is a rectangle with AB also parallel to GH.
- The diagonals of ABCD intersect at M, a point on AH.
- 2.1 Explain why FC \parallel GH. (1)
- 2.2 Calculate, with reasons, the length of DM. (5)







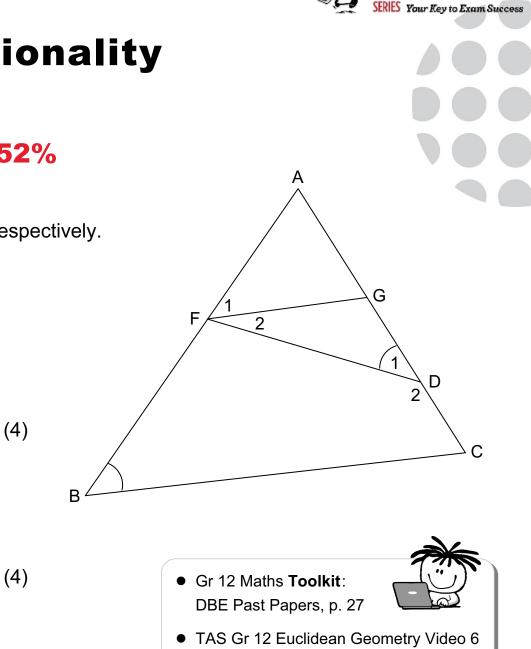
Proportionality

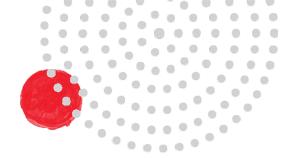
Example 3 (DBE Nov 2020 Q8.2) 52%

3. In $\triangle ABC$, F and G are points on sides AB and AC respectively.

D is a point on GC such that $\hat{D}_1 = \hat{B}$.

- If AF is a tangent to the circle passing (a) through points F, G and D, then prove, giving reasons, that FG || BC.
- (b) If it is further given that $\frac{AF}{FR} = \frac{2}{5}$, AC = 2x - 6 and GC = x + 9, then calculate the value of x.





Similarity



Example 4 (DBE Nov 2019 Q8.2) 25%

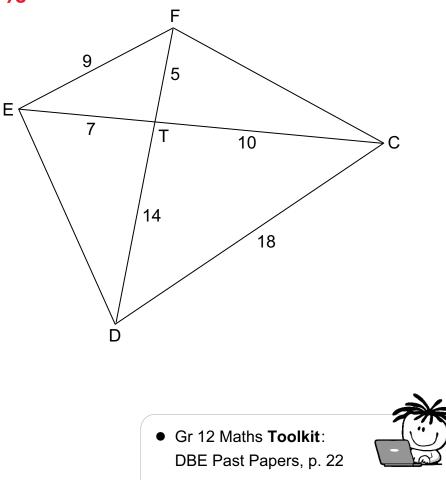
- In the diagram, the diagonals of quadrilateral CDEF intersect at T.
- EF = 9 units, DC = 18 units, ET = 7 units,

TC = 10 units, FT = 5 units and TD = 14 units.

Prove, with reasons, that:

- 4.1 $E\hat{F}D = E\hat{C}D$ (4)
- $4.2 \quad D\hat{F}C = D\hat{E}C \tag{3}$







Mixed

Example 5 (DBE Nov 2018 Q10) 31%

- In the diagram, ABCD is a cyclic quadrilateral such that $AC \perp CB$ and DC = CB.
- AD is produced to M such that AM \perp MC.

Let $\hat{B} = x$.

5.1 Prove that:

48%

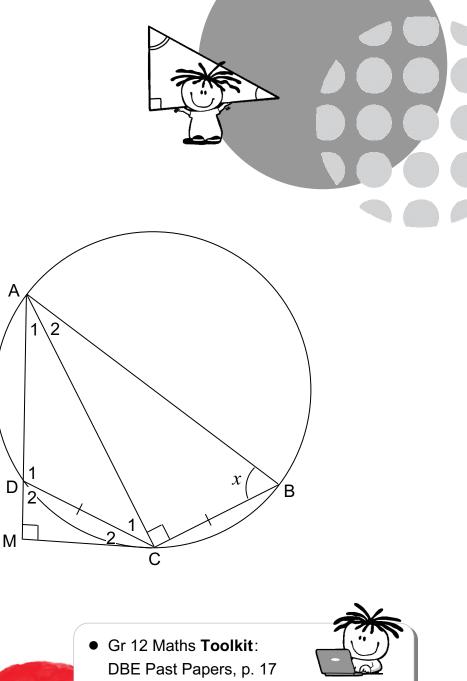
- (a) MC is a tangent to the circle at C. (5)
- (b) $\triangle ACB \parallel \mid \triangle CMD$ (3)

5.2 Hence, or otherwise, prove that:

18%

(a)
$$\frac{CM^2}{DC^2} = \frac{AM}{AB}$$
 (6)

(b)
$$\frac{AM}{AB} = \sin^2 x$$
 (2)



• TAS Gr 12 Euclidean Geometry Video 7

Example 6 (DBE Nov 2020 Q10) 43%

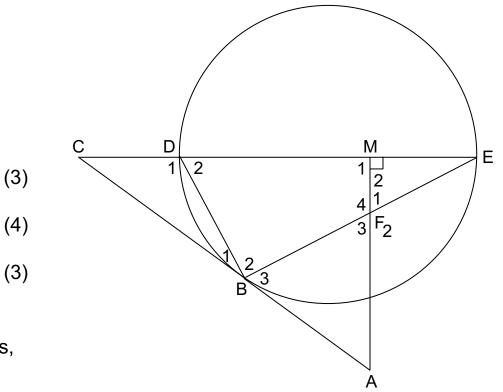
• In the diagram, a circle passes through D, B and E.

- Diameter ED of the circle is produced to C and AC is a tangent to the circle at B.
- M is a point on DE such that AM \perp DE.
- AM and chord BE intersect at F.
- 6.1 Prove, giving reasons, that:

55%

- (a) FBDM is a cyclic quadrilateral
- (b) $\hat{B}_3 = \hat{F}_1$
- (c) $\triangle CDB \parallel \Delta CBE$
- 6.2 If it is further given that CD = 2 units and DE = 6 units,26% calculate the length of:
 - (a) BC (b) DB (3)(4)

Gr 12 Maths Toolkit: DBE Past Papers, p. 27
TAS Gr 12 Euclidean Geometry Video 8

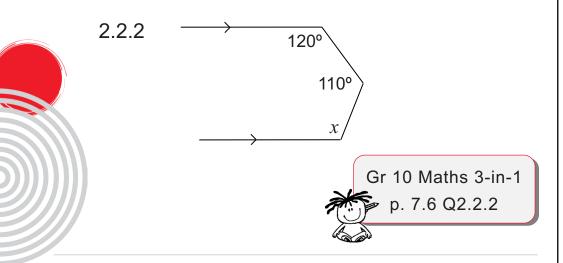




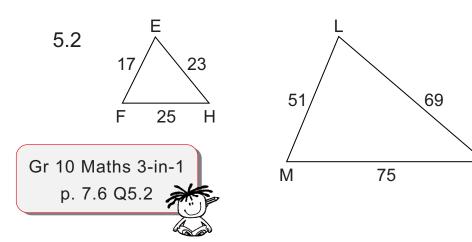
\angle^{s} , Lines, Δ^{s}



2.2 Calculate, with reasons, the values of *x*.

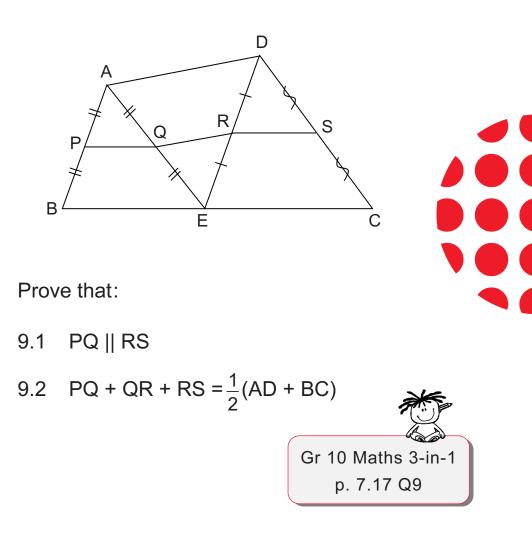


5. State whether the following pairs of triangles are congruent or similar, giving reasons for your choice.



9. ABCD is a quadrilateral.

E is a point on BC. P, Q, R and S are the midpoints of AB, AE, DE and DC respectively.



16

Ν

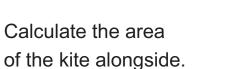


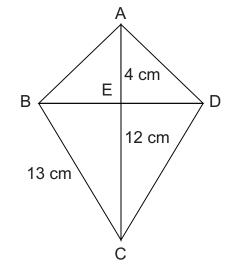
Gr 10 Maths 3-in-1

, p. 7.6 Q7

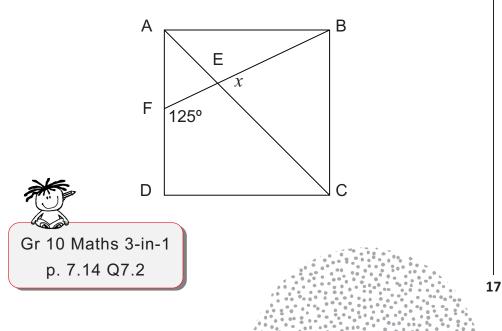
7.

Quadrilaterals

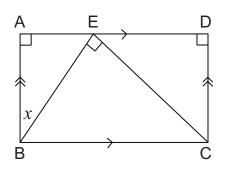




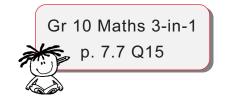
7. Calculate the value of x giving reasons, given that ABCD is a square and $B\hat{F}D = 125^{\circ}$.



- 15.1 Make a neat copy of this sketch and fill in all the other angles in terms of *x*.
 - Reasons are not required.



- 15.2 Complete the following statement: $\Delta ABE \parallel \mid \Delta \dots \mid \mid \Delta \dots$
- 15.3 If BC = 18 cm and BE = 12 cm, calculate the length of
 - 15.3.1 AE
 - 15.3.2 AB correct to two decimals.
- 15.4 Hence calculate the area of rectangle ABCD to the nearest cm^2 .

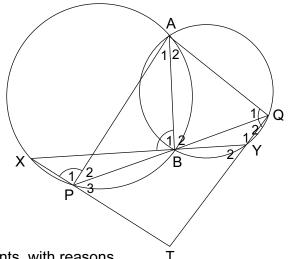




Circles

EXAMPLE 7

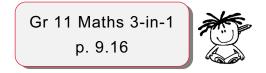
Don't be put off by this drawing! Direct your focus to one situation at a time ③



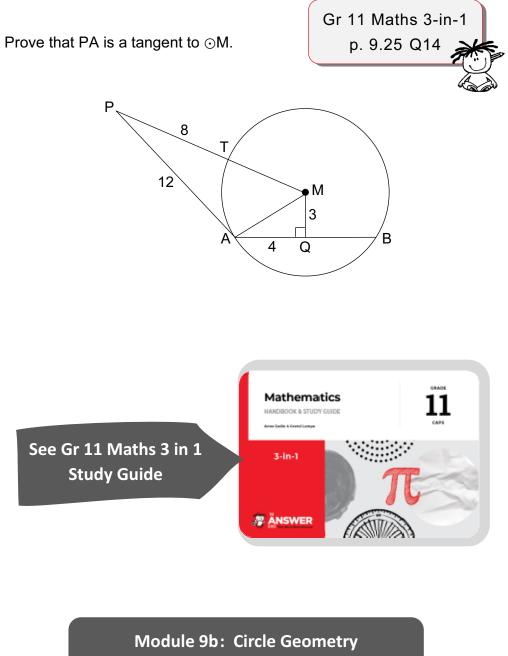
Make statements, with reasons,

- 1. In $\odot XPBA$: about \hat{P}_1 and \hat{B}_1
- 2. In $\odot ABYQ$: about \hat{B}_1 and $A\hat{Q}Y$
- 3. In quadrilateral APTQ: about \hat{P}_1 and $A\hat{Q}T$
- 4. What can you conclude about quadrilateral APTQ?

Mark the \angle^s on the drawing as you proceed.







• Exercises

• Full Solutions

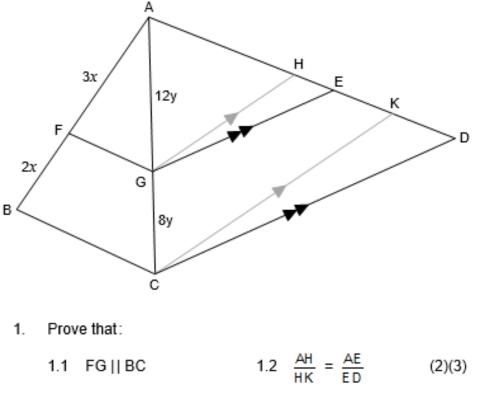
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• Notes

Proportion Theorem

Example 9

In the diagram, $\triangle ABC$ and $\triangle ACD$ are drawn. F and G are points on sides AB and AC respectively such that AF = 3x, FB = 2x, AG = 12y and GC = 8y. H, E and K are points on side AD such that GH || CK and GE || CD.

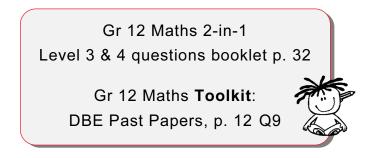


 If it is further given that AH = 15 and ED = 12, calculate the length of EK.

Study and analyse the diagram . . .



- Notice that there are 3 △^s on which to focus.
 And, in △ACD, 2 pairs of || lines. Highlight these in colour!
 (And, the first question requires proof of || lines.)
- Clearly, only 2 theorems are involved: the proportion theorem and its converse (theorem) (Study these <u>2</u> theorem statements well!)



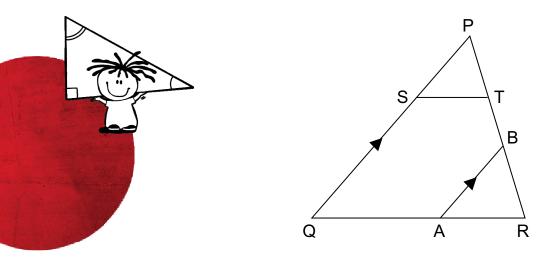


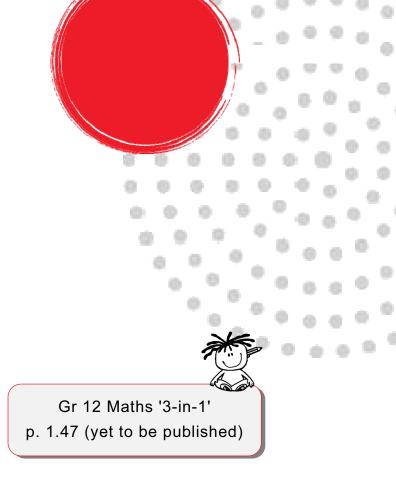


(5)

Worked Example 10

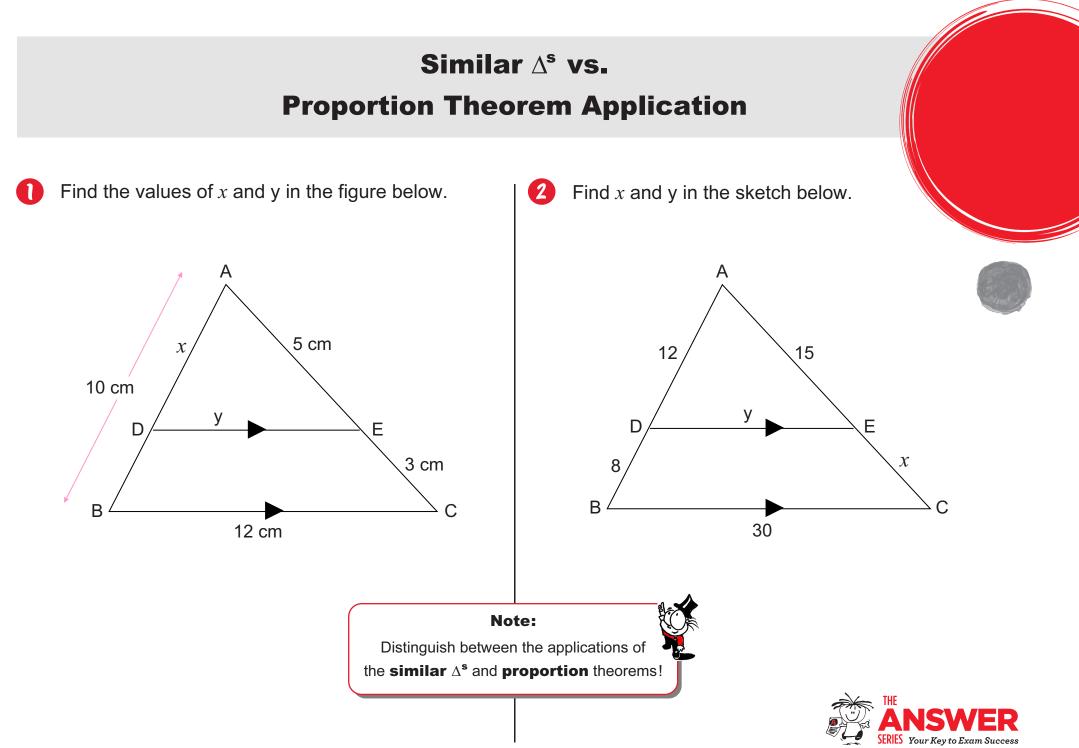
In \triangle PQR the lengths of PS, SQ, PT and TR are 3, 9, 2 and 6 units respectively.





- 1. Give a reason why ST || QR.
- 2. If AB || QP and RA: AQ = 1:3, calculate the length of TB.





Similar Δ^{s}



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EXAMPLE 11 (National November 2017 P2, Q10) **34%**

In the diagram, W is a point on the circle with centre O.

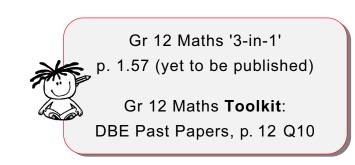
V is a point on OW.

Chord MN is drawn such that MV = VN.

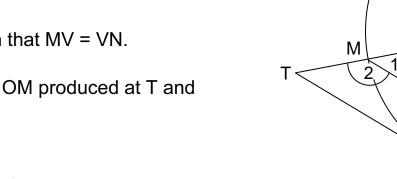
The tangent at W meets OM produced at T and ON produced at S.

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(a) Give a reason why OV \perp MN. 44%
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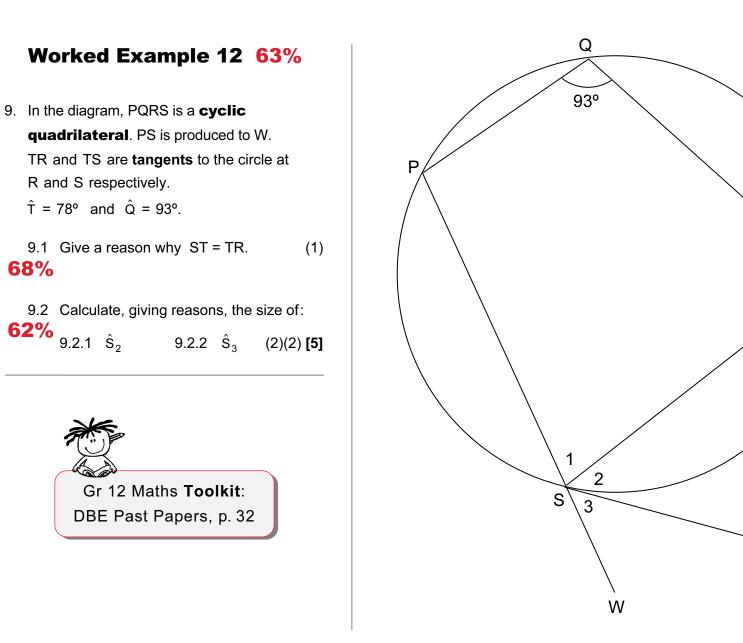
- (b) Prove that:
- 24%
 - (i) MN || TS
 - (ii) TMNS is a cyclic quadrilateral
 - (iii) OS.MN = 20N.WS

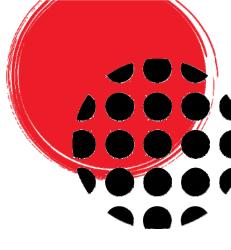






'A Mix' from DBE Nov 2021





Your Key to Exam Succes

R

2

78°

Worked Example 13 24%

10. In the diagram, BE and CD are diameters of a circle having M as centre. Chord AE is drawn to cut CD at F. **AE** \perp **CD**. Let $\hat{C} = x$.

10.1 Give a reason why AF = FE. (1) 47%

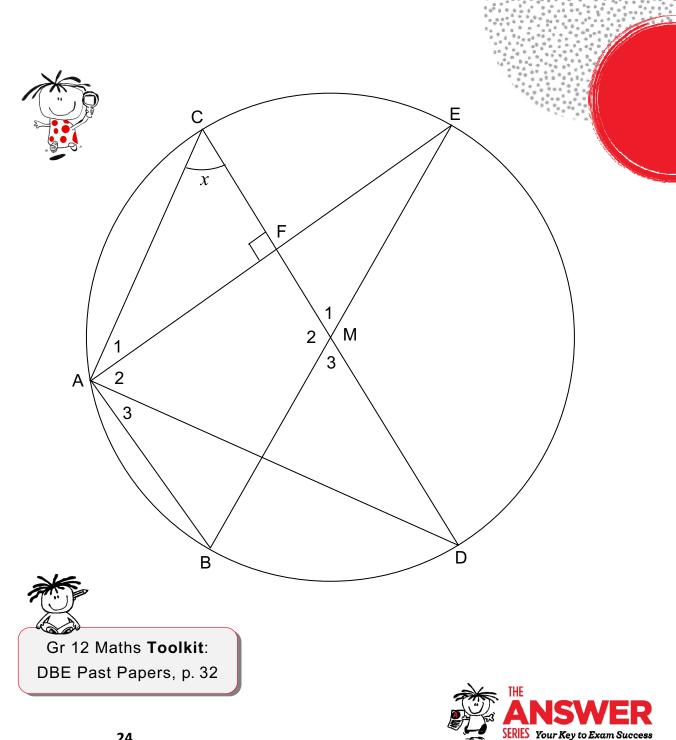
10.2 Determine, giving reasons, the size **37%** of \hat{M}_1 in terms of x. (3)

10.3 **Prove**, giving reasons, that AD is 37% a tangent to the circle passing through A, C and F. (4)

10.4 Given that CF = 6 units and **9%** AB = 24 units, calculate, giving reasons, the length of AE.

[13]

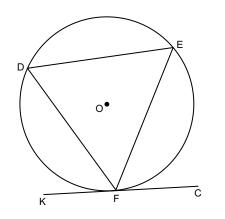
(5)



Worked Example 14 34%

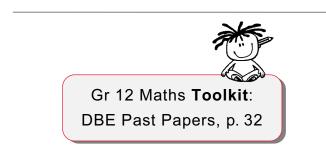
11.1 In the diagram, chords DE, EF and DF are57% drawn in the circle with centre O.

KFC is a tangent to the circle at F.



Prove the theorem which states that $D\hat{F}K = \hat{E}$.

(5)





11.2 In the diagram, PK is a tangent to the
33% circle at K. Chord LS is produced to P. N and M are points on KP and SP respectively such that MN || SK. Chord KS and LN intersect at T.

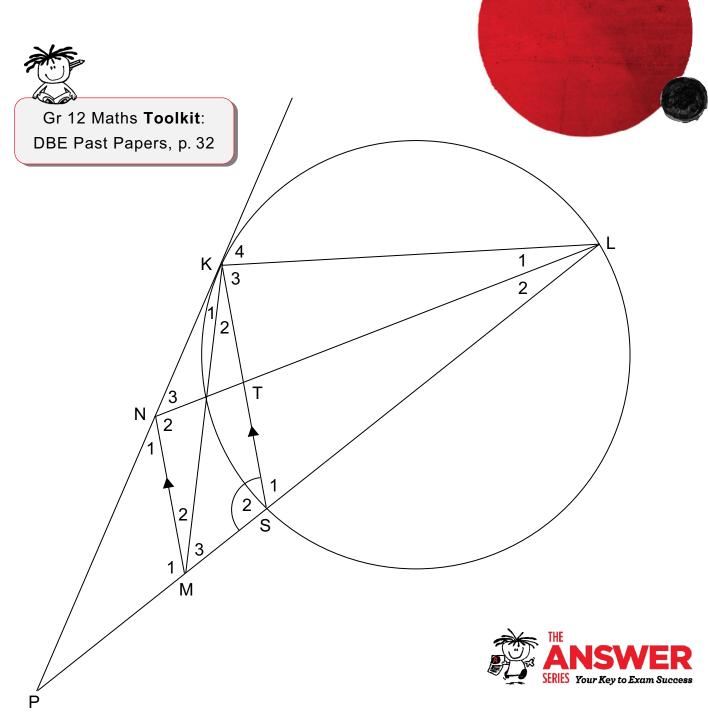
11.2.1 **Prove**, giving reasons, that:

(a) $\hat{K}_4 = N\hat{M}L$ (4) (b) KLMN is a cyclic quadrilateral. (1)

- 11.2.2 Prove, giving reasons, that Δ LKN ||| Δ KSM. (5)
- 11.2.3 If LK = 12 units and 3KN = 4SM, determine the length of KS.

(4)

11.2.4 If it is further given that
NL = 16 units, LS = 13 units
and KN = 8 units, determine,
with reasons, the length of LT. (4)





The converse theorem statements have been highlighted in yellow

LINES		If three sides of one triangle are respectively equal to	SSS	
The adjacent angles on a straight line are supplementary.	\angle^{s} on a str linep	three sides of another triangle, the triangles are congruent.		
If the adjacent angles are supplementary, the outer arms of these angles form a straight line <mark>.</mark>	adj ∠ ^s supp	If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent.	SAS OR S∠S	
The adjacent angles in a revolution add up to 360°.	\angle^{s} around a pt OR \angle^{s} in a rev			
Vertically opposite angles are equal.	vert opp \angle^{s}	If two angles and one side of one triangle are respectively equal to two angles and the corresponding	AAS OR ∠∠S	
If AB CD, then the alternate angles are equal.	alt ∠ ^s ; AB CD	side in another triangle, the triangles are congruent.		
If AB CD, then the corresponding angles are equal.	corresp ∠ ^s ; AB CD	If in two right angled triangles, the hypotenuse and one side	RHS OR 90⁰HS	
If AB CD, then the co-interior angles are supplementary.	co-int ∠ ^s ; AB CD	of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent.		
If the alternate angles between two lines are equal, then the lines are parallel.	alt ∠ ^s =	The line segment joining the midpoints of two sides of a triangle is parallel to the third side and equal to half the	Midpt Theorem	
If the corresponding angles between two lines are equal, then the lines are parallel.	corresp ∠ ^s =	length of the third side.		
If the co-interior angles between two lines are supplementary, then the lines are parallel.	co-int ∠ ^s supp	The line drawn from the midpoint of one side of a triangle, parallel to another side, bisects the third side.	line through midpt to 2 nd side	
TRIANGLES		A line drawn parallel to one side of a triangle divides the other two sides proportionally.	line one side of Δ OR prop theorem; name lines	
The interior angles of a triangle are supplementary.		If a line divides two sides of a triangle in the same proportion, then the line is parallel to the third side.	line divides two sides of Δ in	
The exterior angle of a triangle is equal to the sum of the interior opposite angles.	$ext \angle of \Delta$	If two triangles are equiangular, then the corresponding	prop	
The angles opposite the equal sides in an isosceles triangle are equal.	\angle^{s} opp equal sides	sides are in proportion (and consequently the triangles are similar).	Δ^s OR equiangular Δ^s	
The sides opposite the equal angles in an isosceles triangle are equal.	sides opp equal \angle^{s}	If the corresponding sides of two triangles are proportional, then the triangles are equiangular (and	aidea of A in pro-	
In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.	Pythagoras OR Theorem of Pythagoras	consequently the triangles are similar).	sides of Δ in prop	
If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.	<mark>Converse</mark> Pythagoras OR Converse Theorem of Pythagoras	If triangles (or parallelograms) are on the same base (or on bases of equal length) and between the same parallel lines, then the triangles (or parallelograms) have equal areas.	same base; same height OF equal bases; equal height	

QUADRILATERALS

CIRCLES

GROUP I

The interior angles of a quadrilateral add up to 360°.	sum of \angle^{s} in quad
The opposite sides of a parallelogram are parallel.	opp sides of m
If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.	opp sides of quad are OR converse opp sides of m
The opposite sides of a parallelogram are equal in length.	opp sides of m
If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.	opp sides of quad are = OR converse opp sides of a parm
The opposite angles of a parallelogram are equal.	opp ∠ ^s of m
If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.	opp ∠ ^s of quad are = OR converse opp angles of a parm
The diagonals of a parallelogram bisect each other.	diag of m
If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.	diags of quad bisect each other OR <mark>converse</mark> diags of a parm
If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram.	pair of opp sides = and
The diagonals of a parallelogram bisect its area.	diag bisect area of m
The diagonals of a rhombus bisect at right angles.	diags of rhombus
The diagonals of a rhombus bisect the interior angles.	diags of rhombus
All four sides of a rhombus are equal in length.	sides of rhombus
All four sides of a square are equal in length.	sides of square
The diagonals of a rectangle are equal in length.	diags of rect
The diagonals of a kite intersect at right-angles.	diags of kite
A diagonal of a kite bisects the other diagonal.	diag of kite
A diagonal of a kite bisects the opposite angles.	diag of kite

0	The tangent to a circle is perpendicular to the radius/diameter of the circle at the point of contact.	tan ⊥ radius tan ⊥ diameter
0	If a line is drawn perpendicular to a radius/diameter at the point where the radius/diameter meets the circle, then the line is a tangent to the circle.	line ⊥ radius OR converse tan ⊥ radius OR converse tan ⊥ diameter
	The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.	line from centre to midpt of chord
0	The line drawn from the centre of a circle perpendicular to a chord bisects the chord.	line from centre \perp to chord
	The perpendicular bisector of a chord passes through the centre of the circle.	perp bisector of chord
x 0 2x	The angle subtended by an arc at the centre of a circle is double the size of the angle subtended by the same arc at the circle (on the same side of the chord as the centre)	∠at centre = 2 × ∠ at circumference nverse
Ō	The angle subtended by the diameter at the circumference of the circle is 90°.	\angle^{s} in semi circle OR diameter subtends right angle OR \angle in $\frac{1}{2}$ \odot
0	If the angle subtended by a chord at the circumference of the circle is 90°, then the chord is a diameter.	chord subtends 90° OR <mark>converse</mark> ∠ ^s in semi circle

GROUP II		GROUP III			
Angles subtended by a chord of the circle, on the same side of the chord, are equal	\angle^{s} in the same seg	The opposite angles of a cyclic quadrilateral are supplementary (i.e. <i>x</i> and <i>y</i> are supplementary) opp \angle^{s} of cyclic quad			
If a line segment joining two points subtends equal angles at two points on the same side of the line segment, then the four points are concyclic.	line subtends equal ∠ ^s OR <mark>converse</mark> ∠ ^s in the same seg	$ \begin{array}{c} x \\ 180^{\circ}-x \end{array} $ If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic. opp \angle^{s} quad sup OR converse opp \angle^{s} of cyclic quad			
(This can be used to prove that the four points are concyclic).		The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle. $ext au$ of cyclic quad			
Equal chords subtend equal angles at the circumference of the circle.	equal chords; equal ∠ ^s	x If the exterior angle of a quadrilateral is equal to the interior opposite angle of the quadrilateral, then the ext \angle = int opp \angle x x x x x			
Equal chords subtend equal angles at the centre of the circle.	equal chords; equal \angle^{s}	quadrilateral is cyclic. converse ext \angle of cyclic quad			
		GROUP IV			
Equal chords in equal circles subtend equal angles at the circumference of the circles.	equal circles; equal chords; equal ∠ ^s	A Two tangents drawn to a circle from the same point outside the circle are small in length (AP = 40)			
$\left(\begin{array}{c} x \\ x \end{array} \right) \left(\begin{array}{c} x \\ x \end{array} \right)$ equal angles at the circumference of		A Two tangents drawn to a circle Tans from common pt from the same point outside the OR			
equal angles at the circumference of the circles. Equal chords in equal circles subtend equal angles at the centre of the circles. (A and B indicate the centres of	equal ∠ ^s equal circles; equal chords; equal ∠ ^s s are	B from the same point outside the circle are equal in length (AB = AC)Tans from common pt OR Tans from same ptThe angle between the tangent to a circle and the chord drawn from the point of contact is equal to thetan chord theorem			

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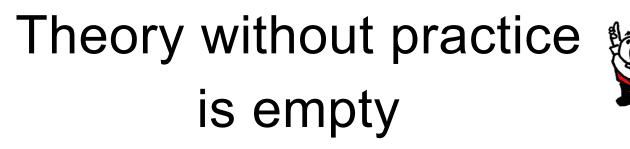
Euclidean Geometry

References to TAS Maths books



Gr 10 Maths 3-in-1 (Module 7)		SERIES Your Key to Exam	
# 1: Lines, angles & triangles: revision • vocabulary & facts	es, angles & triangles: revision • vocabulary & facts		
# 2: Quadrilaterals: revision \bullet definitions \bullet theorems \bullet areas	$7.8 \rightarrow 7.15$		
# 3: Midpoint theorem	$7.16 \rightarrow 7.17$		
# 4: Polygons: definitions & types • interior angles • exterior angles		7.18	
Note: The Gr 10 Exemplar Exams and Memos are at the end of the book			
Gr 11 Maths 3-in-1 (Module 9)			
# 1: Revision from earlier grades		$9.1 \rightarrow 9.5$	
# 2: Circle Geometry		$9.6 \rightarrow 9.26$	
Note: The Gr 11 Exemplar Exams and Memos are at the end of the book			
Gr 12 Maths 2-in-1 (Module 10)			
# 1: Circle Geometry		$36 \rightarrow 40$	
# 2: Proportion Theorem See Challenging Questions booklet:		$40 \rightarrow 42$	
# 3: Similar Triangles pages $29 \rightarrow 38$	$42 \rightarrow 43$		
#4: Mixed		43	
Back pages: Circle Geometry, Proportion and Similar Triangles Theorem PROOFS		i → iii	
Grouping of Circle Geometry Theorems	a an n 140	viii	
Converse Theorems in Circle Geometry	-	ix	
Theorem Statements & Acceptable Reasons	practice.	$x \rightarrow xii$	
Gr 12 Maths Toolkit: DBE Past Papers			
Back pages: Circle Geometry, Proportion and Similar Triangles Theorem PROOFS	Circle Geometry, Proportion and Similar Triangles Theorem PROOFS		
Grouping of Circle Geometry Theorems Theorem Statements & Accentable Beasons See the Topic Guide: DBE	En 2	xiii	
Theorem Statements & Acceptable Reasons		$xiv \rightarrow xvi$	





Practice without theory is blind

Philosopher, Immanuel Kant (18th century philosopher)

