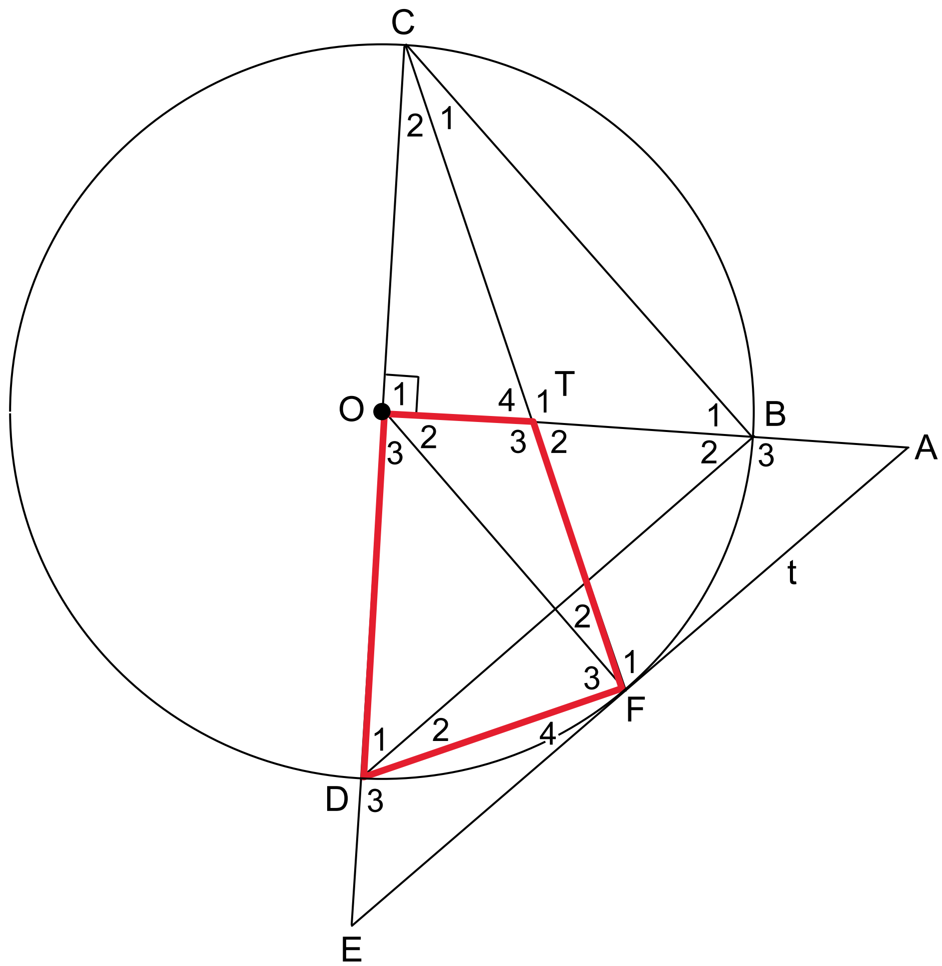
SOLUTIONS

A Warm-up Example

DBE May 2024: Q10

10.1 TODF is a cyclic quadrilateral (4)



Solutions

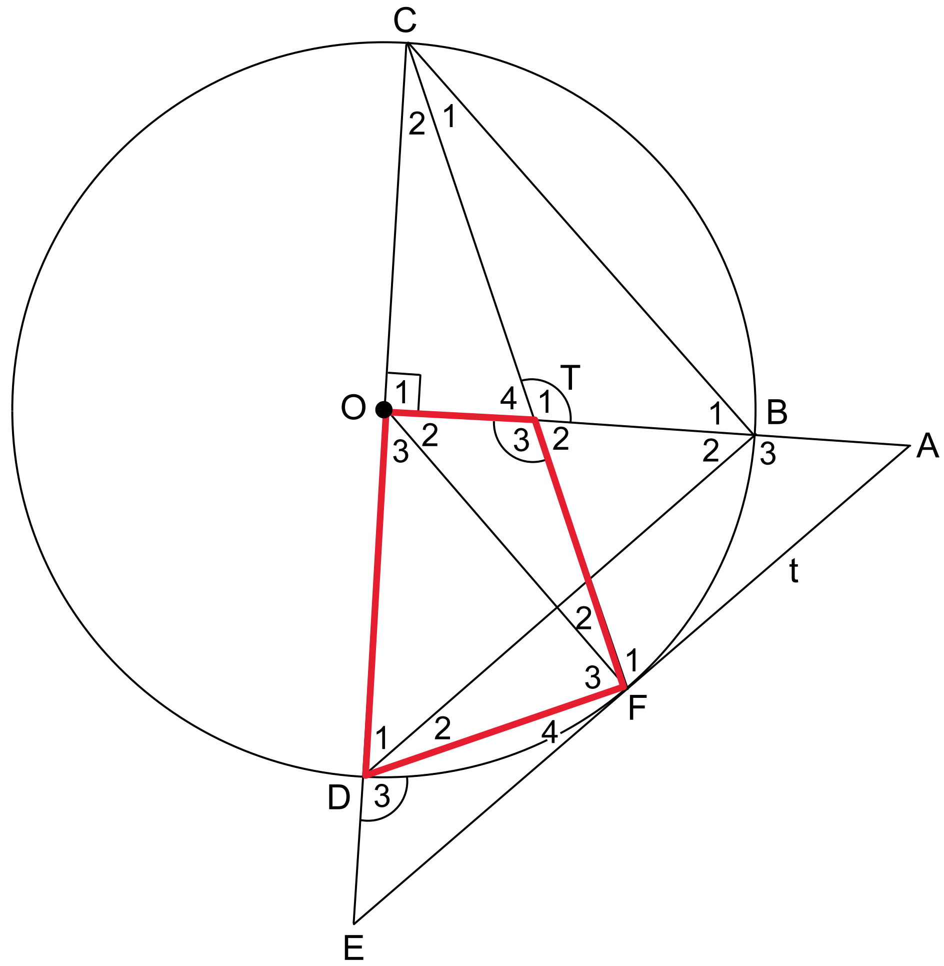
10.1 $T\hat{O}D = 90°$ ... $AO \perp CE$

& $D\hat{F}T = 90°$ ... $\angle$ *in semi-*$\odot$

$\therefore$ TODF is a cyclic quad ... *converse ext* $\angle$ *of cyclic quad*

[*or: converse opp* $\angle^s$ *of cyclic quad*]

10.2 $\hat{D}_3 = \hat{T}_1$ (3)



Solutions

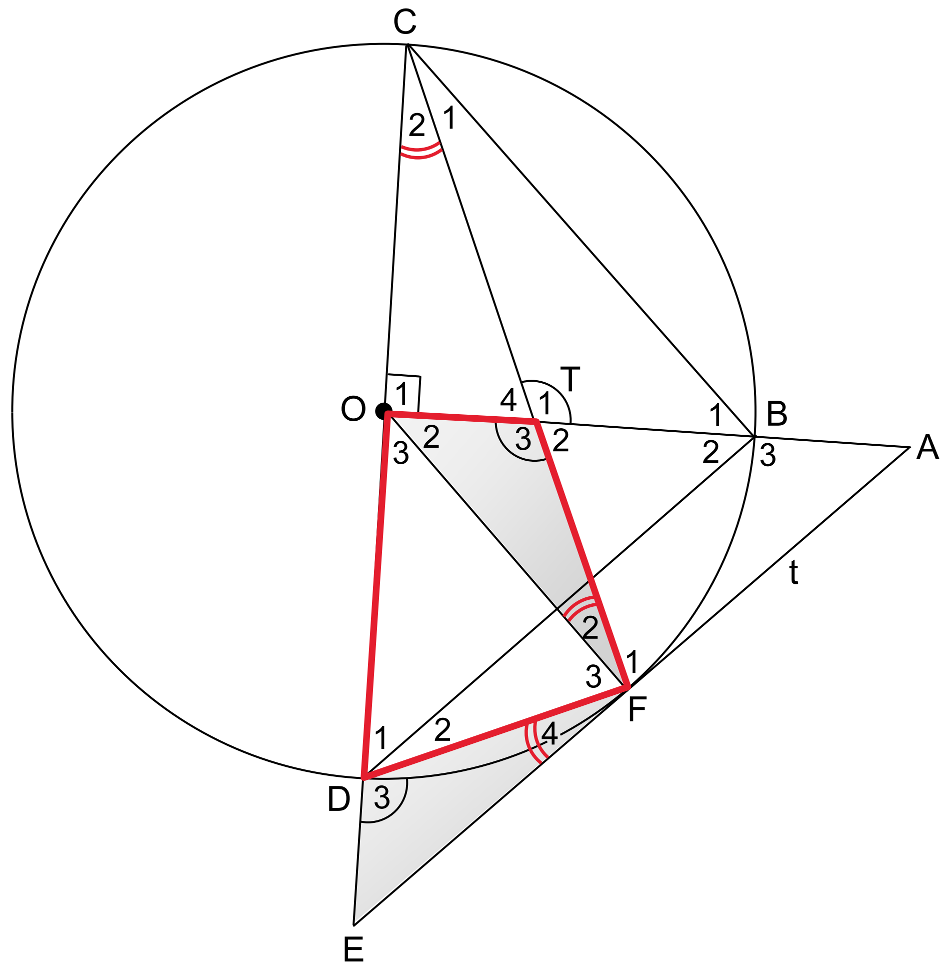
10.2 Let $\hat{D}_3 = x$

$\therefore \hat{T}_3 = x$ . . . *ext ∠ of cyclic quad*

$\therefore \hat{T}_1 = x$ . . . *vert opp* $\angle^s =$

$\therefore \hat{D}_3 = \hat{T}_1$ ➤

10.3 ΔTFO ||| ΔDFE (5)



Solutions

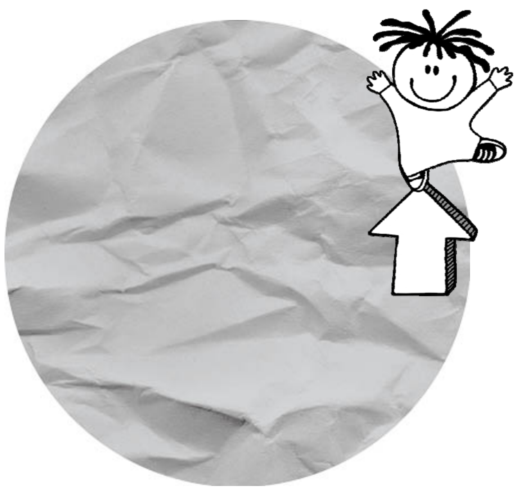
10.3 Δ^s TFO and ΔDFE

(1) $\hat{T}_3 = \hat{D}_3$ . . . *proved in 10.2*

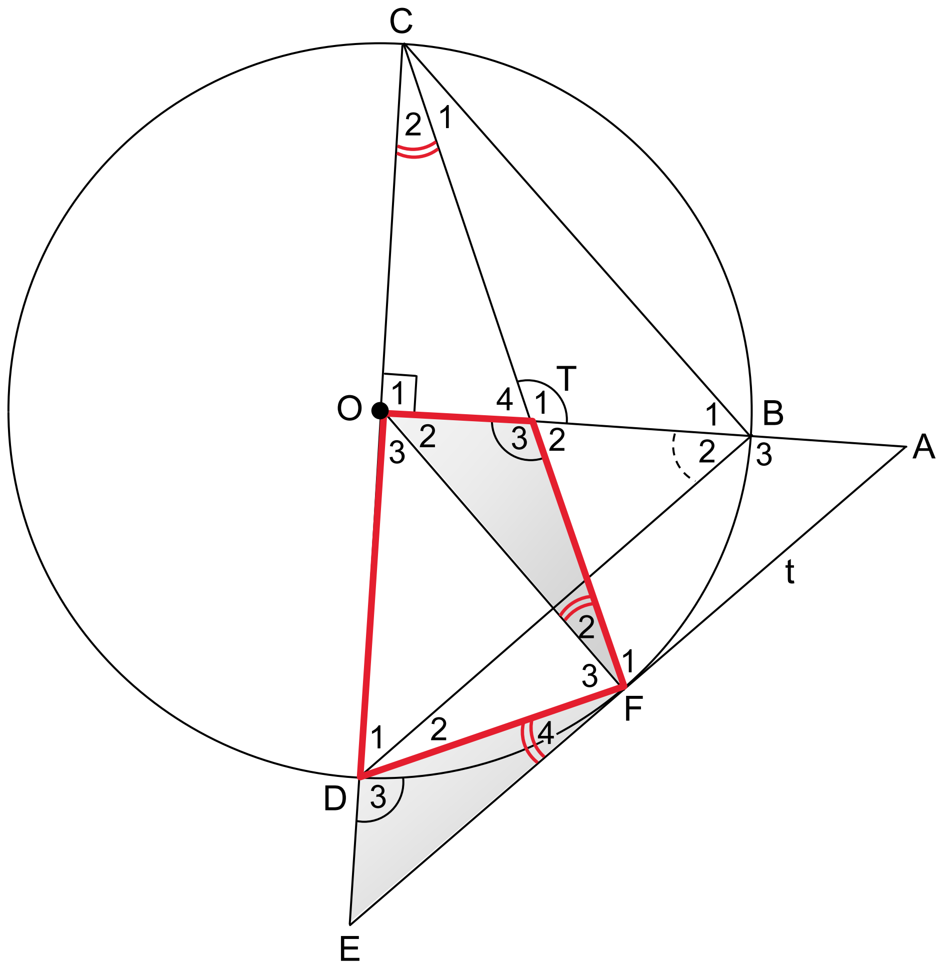
(2) $\hat{F}_4 = \hat{C}_2$ . . . *tan chord theorem*

$= \hat{F}_2$ . . . *$\angle^s$ opp equal sides (radii)*

$\therefore$ ΔTFO ||| ΔDFE ◀ . . . *equiangular Δ^s*



10.4 If $\hat{B}_2 = \hat{E}$, prove that DB || EA. (2)



Solutions

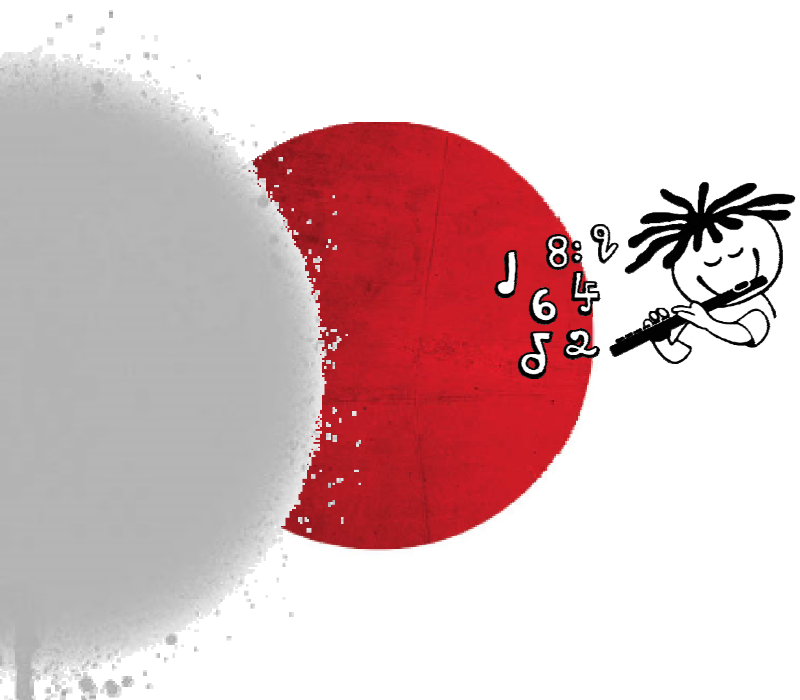
10.4 Let $\hat{E} = y$

$\therefore \hat{B}_2 = y$ . . . *given*

$\therefore \hat{D}_1 = y$ . . . $\angle^s$ *opp equal sides (radii)*

$\therefore \hat{D}_1 = \hat{E}$

$\therefore$ DB || EA ➤ . . . *corresp* $\angle^s$ =



10.5 Prove that $DO = \frac{TO.FE}{AB}$ (5)

Solutions

10.5 In ΔOEA: $\frac{DO}{ED} = \frac{BO}{AB}$ . . . *DB || EA (in 10,4); prop theorem*

$\therefore DO = \frac{BO.ED}{AB}$ . . . ❶

Now, in Δ^s TFO & DFE: $\frac{TO}{ED} = \frac{FO}{FE} \left(= \frac{TF}{DF}\right)$ . . . $|||\ \Delta^s$ *in 10.3*

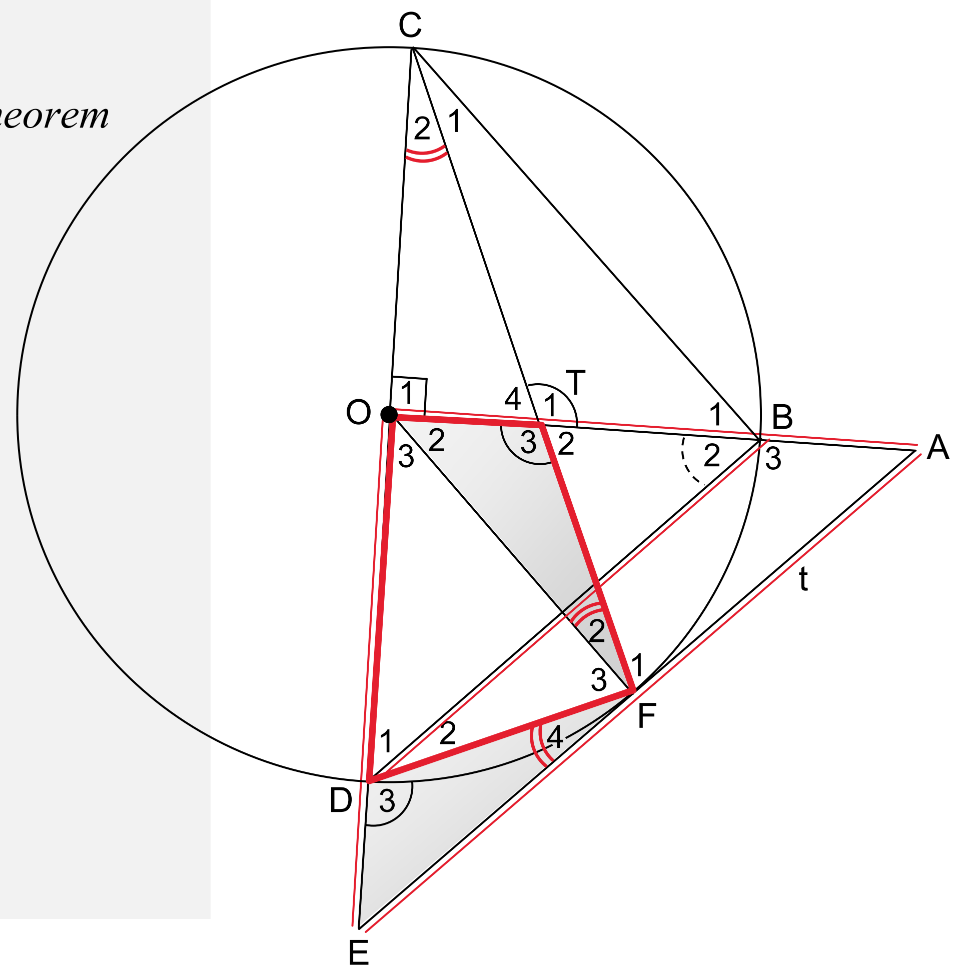
$\therefore FO.ED = TO.FE$

But FO = BO . . . *radii*

$\therefore BO.ED = TO.FE$ . . . ❷

Substitute ❷ in ❶:

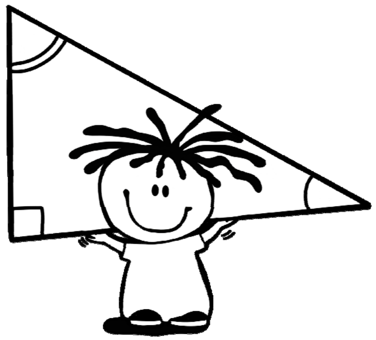
$\therefore DO = \frac{BO.ED}{AB}$ ➤



Problem Solving Solutions

1. Let $A\hat{D}E = x$

$\therefore\ A\hat{E}D = x$	$\angle$s opp = sides
$\therefore\ D\hat{A}E = 180^\circ - 2x$	$\angle$ sum of Δ
$\therefore\ E\hat{B}C = 2x$	co-int $\angle$s; AD $\parallel$ BC
$AD = BC$	opp sides of parm
$\therefore\ BE = BC$	
$B\hat{E}C + B\hat{C}E = 180^\circ - 2x$	$\angle$ sum of Δ
$\therefore\ B\hat{E}C = 90^\circ - x$	$\angle$s opp = sides
$\therefore\ D\hat{E}C = 90^\circ$	$\angle$s on a straight line
$\therefore\ EC = 12$ cm	Pythagoras



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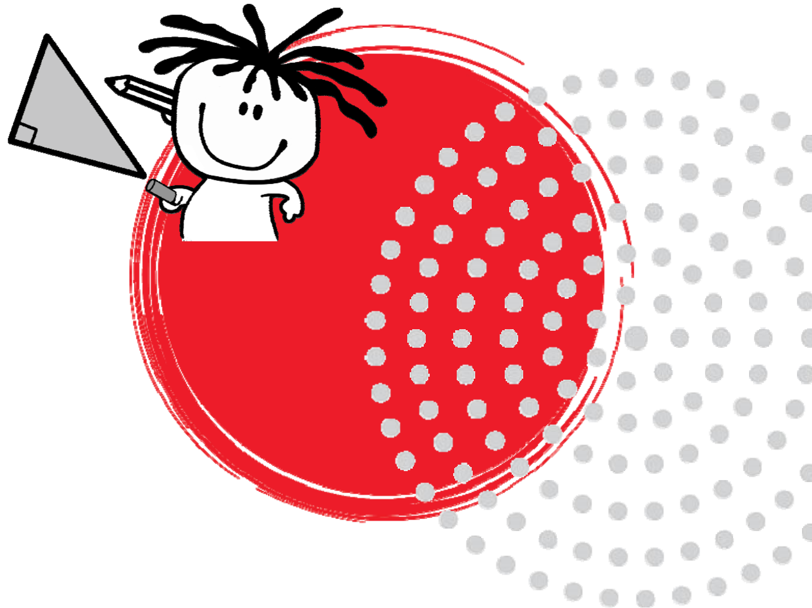
2. In ΔGHL and ΔHJM

1. $GH = HJ$	sides of square
2. $HL = JM$	sides of square and L and M are midpoints
3. $G\hat{H}L = H\hat{J}M$	angles of square
$\therefore \quad \Delta GHL \equiv \Delta HJM$	SAS



Let $H\hat{G}L = x$	
$\therefore \quad G\hat{L}H = 90^\circ - x$	$\angle$ sum of Δ
$J\hat{H}M = x$	$\Delta GHL \equiv \Delta HJM$
$\therefore \quad H\hat{N}L = 90^\circ$	$\angle$ sum of Δ
$\therefore \quad GL \perp HM$	

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3.

$PO = OS = 5$ cm	radii
$ST = TQ$	converse midpoint theorem
$QP = 6$ cm, $QS = 8$ cm and $PS = 10$ cm	
$\therefore\ P\hat{Q}S = 90^\circ$	converse Pythagoras
$OT = 3$ cm	midpoint theorem
$\therefore\ TR = 2$ cm	OR is a radius
$R\hat{T}Q = 90^\circ$	alt $\angle$s; PQ || OR

Area of PQRS $=$ Area ΔPQS + Area ΔQRS

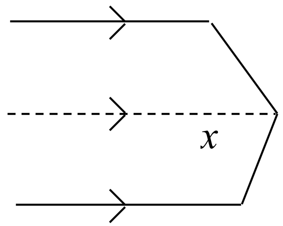
$$\therefore \text{ Area } = \frac{1}{2}\times 6\times 8+\frac{1}{2}\times 8\times 2$$

$$\therefore \text{ Area } = 32 \text{ cm}^2$$

2.2.2 $120^\circ + 110^\circ + x = 2(180^\circ)$

$\therefore\ 230^\circ + x = 360^\circ$ . . . *2 prs. co-int.* $\angle^s$

$\therefore\ x = 130^\circ$ *|| lines*



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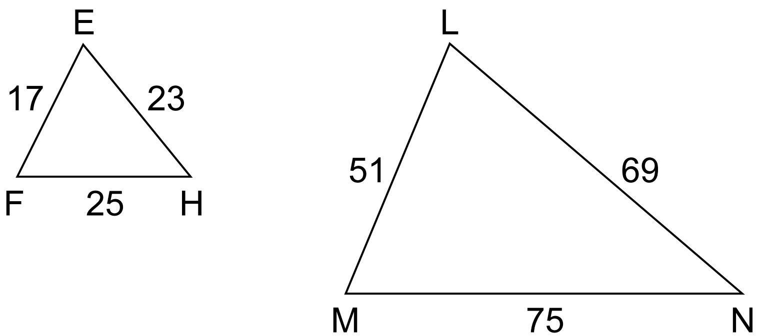
p. 7.19 Q2.2.2

5.2 Similar ; prop. sides

. . . *[17 : 23 : 25 = 51 : 69 : 75]*



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p. 7.19 Q5.2

9.

This drawing looks confusing at first. But, look at each triangle separately – the 'middle' one is just upside down! – and apply the facts to each, one at a time.

9.1 In ΔABE: P & Q are midpoints of AB & AE

$\therefore$ PQ || BE(C) . . . *midpt thm*

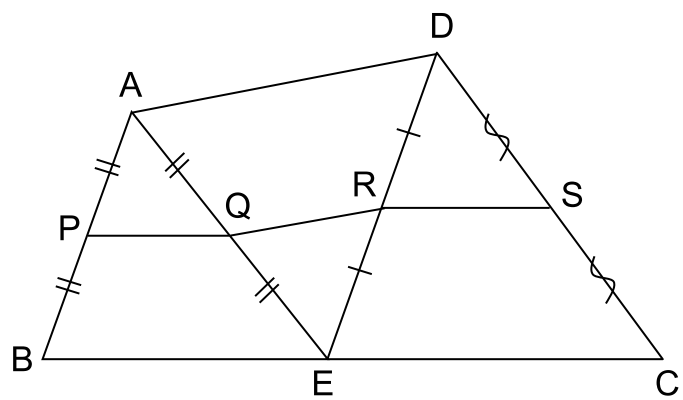
Similarly, in ΔDEC: RS || (B)EC

$\therefore$ PQ || RS . . . both are parallel to BEC

9.2 In Δ^s ABE, AED and DEC:

$$PQ + QR + RS = \frac{1}{2}BE + \frac{1}{2}AD + \frac{1}{2}EC = \frac{1}{2}(AD + BE + EC)$$

$$= \frac{1}{2}(AD + BC)$$



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7. BE ⊥ AC . . . *diagonals of a kite*

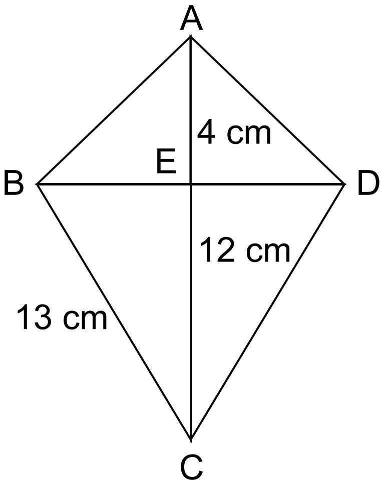
BE = 5 cm . . . *5 : 12 : 13 Δ ; Pythag.*

Area of kite $= 2.\frac{1}{2}(12+4).5$ . . . *2 × ΔACD*

$= 80 \text{ cm}^2$

OR: $\frac{1}{2}$ product of diagonals

$= \frac{1}{2}(10)(16)\text{cm}^2$ . . . *why?*



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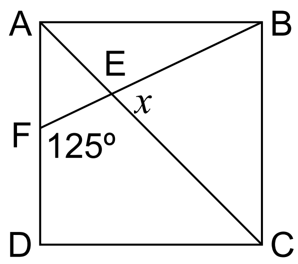
7.2 $\hat{D} = 90°$ . . . *∠ of square*

∴ $D\hat{C}A = 45°$ . . . *diag. bisects ∠*

∴ $F\hat{E}C = 100°$ . . . *sum of int. ∠s of quad.*

∴ $x = 80°$ . . . *∠s on a str. line*



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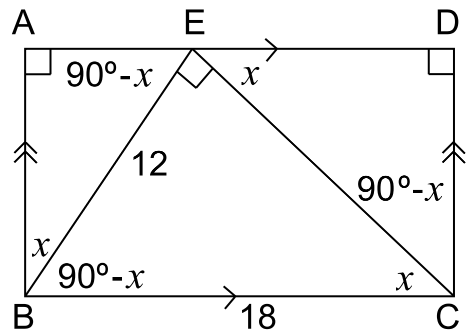
15.1



15.2 ΔABE ||| ΔECB ||| ΔDEC

NB: The order of the letters!

15.3.1 ΔABE ||| ΔECB

➡ $\frac{AE}{BE} = \frac{BE}{BC}$ . . . *sides in proportion*

×BE) ∴ $AE = \frac{BE^2}{BC} = \frac{12^2}{18} = 8 \text{ cm}$

15.3.2 $AB^2 = 12^2 - 8^2 = 80$ . . . *Theorem of Pythag*

∴ $AB = \sqrt{80}$

$\simeq 8{,}94$ cm

15.4 Area of rect. ABCD $= 8{,}94 \times 18 \simeq 161 \text{ cm}^2$

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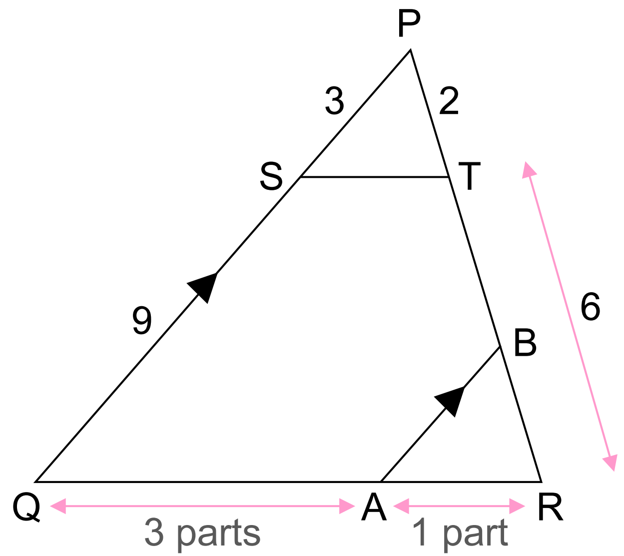
Worked Example 10



1. In ΔPQR: $\frac{\mathbf{PS}}{\mathbf{SQ}} = \frac{3}{9} = \frac{1}{3}$ & $\frac{\mathbf{PT}}{\mathbf{TR}} = \frac{2}{6} = \frac{1}{3}$

$$\therefore \frac{\mathbf{PS}}{\mathbf{SQ}} = \frac{\mathbf{PT}}{\mathbf{TR}}$$

$\therefore$ ST || QR ◀ . . . *converse of proportion thm*

2. In ΔRPQ: $\frac{\mathbf{RB}}{\mathbf{RP}} = \frac{RA}{RQ} = \frac{\mathbf{1}}{\mathbf{4}}$ . . . *proportion theorem ; AB || QP*

$RA : AQ = 1 : 3$

$\therefore$ RB $= \frac{1}{4}$RP

$=$ 2 units . . . $RP = PT + TR = 8$ *units*

$\therefore$ TB = 4 units ◀

Gr 12 Maths '3-in-1'
p. 1.47 (yet to be published)

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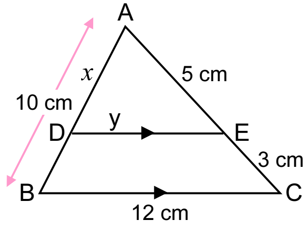
Similar Δ^s vs.
Proportion Theorem Application

1 Find the values of x and y in the figure alongside.



Answer

- In ΔABC: $\frac{x}{10} = \frac{5}{8}$ . . . *DE || BC; proportion theorem*

  $\times$ 10) $\therefore\ x = 6\frac{1}{4}$ cm ◄

- ΔADE ||| ΔABC ➡ $\frac{y}{12} = \frac{5}{8}$ . . . $\frac{DE}{BC} = \frac{AE}{AC}$; *proportional sides*

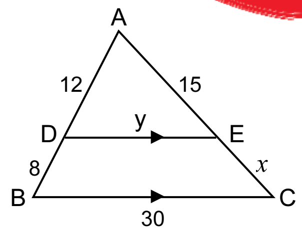
  $\times$ 12) $\therefore$ y = $7\frac{1}{2}$ cm ◄

Note:

Distinguish between the applications of the **similar** Δ^s and **proportion** theorems!

(See next column.)

2 Find x and y in the sketch alongside



▶ The Proportion theorem (finding x)

In ΔABC: DE || BC

The unknown ➡ $\frac{x}{15} = \frac{8}{12}$

$\times$ 15) $\therefore\ x$ = 10 units

The proportion theorem does NOT refer to the lengths of the parallel lines, only to AB and AC and their segments.

▶ Similar triangles theorem (finding y)

In Δ^s ADE and ABC:

(1) $\hat{A}$ is common

(2) $A\hat{D}E = \hat{B}$ ... *corresponding $\angle^s$; DE || BC*

[& $A\hat{E}D = \hat{C}$ ... *corresponding $\angle^s$; DE || BC*]

In the same figure above, ΔABC can be seen as an enlargement of ΔADE and the sides of these triangles are proportional.

$\therefore \frac{y}{30} = \frac{12}{\mathbf{12+8}}$ $\left(or\ \frac{15}{\mathbf{15+10}}\right)$

$\therefore$ ΔADE ||| ΔABC . . . *equiangular Δ^s*

$\therefore \frac{DE}{BC} = \frac{AD}{\mathbf{AB}}$ or $\frac{AE}{\mathbf{AC}}$ . . .

Note: $\frac{DE}{BC} \neq \frac{AD}{DB}$ or $\frac{AE}{EC}$ because BC is a side of ΔABC, while DB and EC are not.

$\therefore \frac{y}{30} = \frac{12}{20}$

$\times$ 30) $\therefore$ y = 18 units

It is only by using the similarity of the triangles, that we can relate the lengths of the parallel sides to the lengths of the other 2 sides of the triangles.

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