

# Mathematics

TEST & EXAM PREPARATION

Anne Eadie & Gretel Lampe

GRADE

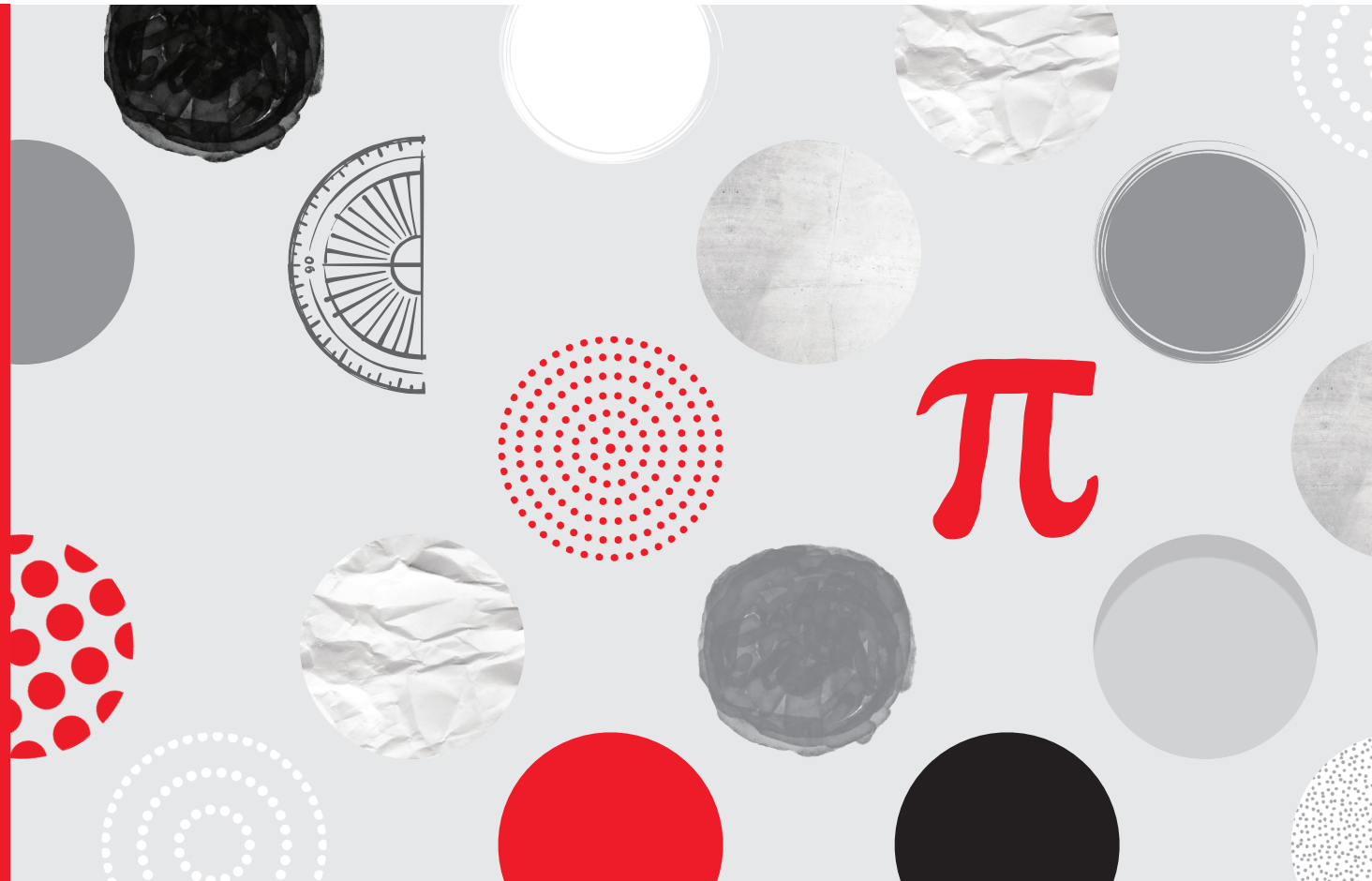
9

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# Mathematics

Anne Eadie & Gretel Lampe

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MATHS COMPANION**

Workbook 1: Terms 1 & 2

Workbook 2: Terms 3 & 4

& Answer book



## THIS STUDY GUIDE INCLUDES

1 Questions in Topics

2 Examination Papers

*Detailed solutions are provided for both sections*

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# 4 EXPONENTS

## Exponential Expressions



### Exercise 4.1

Answers on p. A13



Apply the given law provided in each of the 6 cases below to simplify the following expressions.

**Law 1:**  $a^m \times a^n = a^{m+n}$

1.1  $x^5 \times 2x^3$       1.2  $3x^2 \times 2x^3$       (1)(2)

1.3  $2^{-4} \times 2^5 \times 2^0$       (1)

**Law 2:**  $a^m \div a^n = a^{m-n}$

1.4  $2^5 \div 2^2$       1.5  $\frac{p^7}{p^3}$       1.6  $\frac{a^2}{a^{-1}}$       (1)(2)(2)

**Law 3:**  $(a^m)^n = a^{mn}$

1.7  $(2^3)^2$       1.8  $(s^3)^5$       1.9  $(3^x)^2$       (1)(1)(1)

**Law 4:**  $(ab)^m = a^m b^m$

1.10  $(2 \times 5)^2$       1.11  $(3x^3)^3$       (2)(3)

1.12  $(-4a^5b^3)^2$       (3)

**Law 5:**  $a^0 = 1 ; a \neq 0$

1.13  $5^0$       1.14  $7a^0$       (1)(1)

1.15  $3p^0 + (3p)^0$       (2)

**Law 6:**  $a^{-n} = \frac{1}{a^n} ; a \neq 0$

1.16  $2^{-3}$       1.17  $x^{-4}$       (1)(1)

1.18  $a^3 \cdot b^{-4}$       (1)

### Note

$(-3)^2 = 9$  but  $-3^2 = -9$

&

$(-3)^0 = 1$  but  $-3^0 = -1$



Now combine the laws...

Simplify the following as far as possible.

Where applicable, leave the answer with positive exponents.



### Questions including Laws 1 & 2

2.1  $3a \cdot 2ab \cdot 3abc$       2.2  $2x^3 \times -3x^5$       (2)(2)

2.3  $2a^4 \times 4a^2$       2.4  $a^{-4} \times a^7$       (2)(1)

2.5  $x^3(x^2)^3x^0x^{-2}$       2.6  $6a^5 \times a \div 3a^2$       (2)(3)

2.7  $5x^3y^5 \times 3x^{-2}y^2$       2.8  $8y^3 \times \frac{1}{4}y^2$       (2)(2)

2.9  $\frac{a^3 \cdot b^2 \cdot c}{a \cdot b^2 \cdot c^3}$       2.10  $\frac{5^2 \cdot 5^3 \cdot x^2 \cdot y^4}{5^6 \cdot x^3 \cdot y^2}$       (2)(3)

2.11  $\frac{3ab^4c^0}{18a^2bc}$       2.12  $\frac{4x^3y^7}{8x^2y^9}$       (2)(2)

2.13  $\frac{8b^{-3}c^5}{12b^2c^{-6}}$       2.14  $\frac{121x^7y^{13}}{11x^{-4}y^2}$       (3)(3)

2.15  $\frac{55x^3y^{12}}{11x^{-2}y^{17}}$       2.16  $\frac{15xy^{-2}}{3x^{-3}y^4}$       (3)(4)

2.17  $\frac{18a^{-2}b^3c}{12a^3b^5c^{-4}}$       2.18  $\frac{-39x^{17}y^{10}}{-3x^4y^{-3}}$       (3)(3)

### Questions extending to Laws 3 & 4



3.1  $(-3x^2)^2$       3.2  $(-3x^4)(2x^3)$       (2)(2)

3.3  $(2^3x^2y)^3$       3.4  $(3p^5q^3)^2$       (3)(3)

3.5  $4(-a)^2 - (-2a)^2$       3.6  $x^{-7}(x^{13} + x^{11})$       (3)(3)

3.7  $\frac{ab^2 \times a^3b}{(a^2b^2)^2}$       3.8  $\frac{ab(-2a^2b^3)^3}{-56b^3}$       (3)(4)

3.9  $\frac{(3m^3n)(-2mn^4)}{(-2mn)^4}$       3.10  $\frac{(x^2y) \times (x^3y^3)}{(x^2y)^3}$       (4)(3)

### Questions extending to Laws 5 & 6

4.1  $\frac{x^{-3}}{y^{-4}}$       4.2  $\frac{m^5 \cdot n^{-2} \cdot p^0}{m \cdot n^3}$       (1)(3)

4.3  $\frac{x \cdot y^{-2}}{x^0 \cdot y}$       4.4  $\frac{x^0}{0,25}$       (3)(2)

4.5  $\frac{(5m)^0}{5m^0}$       4.6  $(a^{-2})(-3a^0)$       (2)(3)

4.7  $(-2a^4b^3)^2(-3a^2b^0)$       (3)

4.8  $\frac{(2^2q^{-3})^0 \times (p^{-3}q)^{-2}}{(-2pq^{-1})^2}$       (4)

4.9  $\frac{(4x^3)^2(-3x)^0}{-6x^2}$       (4)



Mixed Questions

**A Summary of the Laws of Exponents using algebra (letters)**

$$a^m \times a^n = a^{m+n}$$

$$a^m \div a^n = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$(a \times b)^n = a^n \times b^n$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$$



Evaluate the following without the use of a calculator:

5.1  $3^{-2} + 3^0 + 3^2$  (2) 

5.2  $-5^2 \div \sqrt[3]{-1}$  (3)

5.3  $\sqrt{3^2 + 3^1 + 3^0 + 3^{-1} + 3^{-2}}$  (4) Remember to use the exponent laws

5.4  $\sqrt[3]{27} - \sqrt{49} + 4$  (3) 

5.5  $\sqrt{9+16}$  (1)

5.6  $\sqrt{\frac{18}{2}} - (-2)^3$  (4)

5.7  $\sqrt[3]{\frac{27}{8}} + \sqrt{2\frac{1}{4}}$  (4)

Simplify, leaving the answer with positive exponents where applicable:

5.8  $\sqrt{36x^{36} + 64x^{36}}$  5.9  $\sqrt{9x^{10} + 16x^{10}}$  (3)(3)

5.10  $\sqrt{144a^6 b^{10}} \cdot (-2ab^2)$  (3)

5.11  $\sqrt{9(a+2b)^2 x^4}$  5.12  $\sqrt{169x^{14} y^{22}}$  (3)(3)

5.13  $\sqrt{\frac{4a^3}{a^9}}$

5.14  $\left(\frac{5x^{-1}}{2}\right)^{-1} \cdot \sqrt{25x^4 y^{-2}}$

5.15  $\sqrt{\frac{(ab^2)^3}{a(-4c)^2}}$

5.16  $\frac{(3x^5 y^{-3})^2}{-6y^5} \times \frac{2x^{-4}}{x^0}$

5.17  $\frac{2xy^2}{3m} \div \frac{4y^2}{9xm}$

5.18  $\frac{3x}{y^2 z^3} \div \frac{9x^2}{2yz^4}$

5.19  $\frac{(2ac^2)^4}{4a^4} \times \frac{(a^2 b^3)^2}{(b^2 c)^3}$

5.20  $\frac{(2a^2 b^3)^4 \times (8ab^{-2})^2}{4ab^5 \times 12a^9 b^7}$

5.21  $\frac{(5x^{-3} y)^2}{4xy \times (x^2 y)^2} \div \left(\frac{5xy}{2}\right)^2$

5.22  $\frac{xy^3}{x^2} \div \frac{x^3 y}{x^4} \times \frac{x^0}{x^2 y}$

5.23  $\frac{x^2 y^3}{x^4} \times \frac{x^3 y^4}{y^7} \div \frac{x^2}{x^3}$

5.24  $\frac{x^4 y}{y^0} \div \frac{xy^3}{x^2} \div \frac{x^2 y^3}{x^{-3} y^4}$

5.25  $\frac{\sqrt{49a^6 b^{-2}} \cdot (3a^2 b^{-1})^{-2}}{2a^0 b^{-3}}$

6.1 Determine which of the following has the largest value (you may use a calculator):

(2)  $\left(\frac{1}{7}\right)^{-7}$   $7^{\frac{10}{7}}$   $\sqrt[7]{777777}$   $777,777^{\frac{1}{7}}$   $0,7^{-0,7}$



(2) 6.2 Now, without the use of a calculator, determine which is larger:  $2^{-5}$  or  $5^{-2}$ ? (3)

(3) 7. Round off each of the following to three dec. places.

7.1  $\pi \times \sqrt[5]{237}$  7.2  $\left(\frac{1}{7}\right)^{-7} \times 0,135$  (1)(1)

(4) 8. Express the number 32 as a power with a base of 4. [Hint: Remember:  $2^2 = 4$ ] (2)

(6) 9. For each of the following questions, four options have been given for the answer. Only one of the options provided is correct. In each case, write down the correct letter. (10)

9.1  $\frac{x^2}{y^2} = \dots$

A:  $\frac{x}{y}$

B:  $\frac{y}{x}$

C:  $x - y$

D: cannot be simplified



9.2  $\sqrt{36x^{16}} = \dots$

A:  $6x^8$

B:  $6x^4$

C:  $18x^8$

D:  $18x^4$



9.3  $\sqrt{\frac{64a^{16}}{b^{36}}} = \dots$

A:  $\frac{8a^8}{b^{18}}$

B:  $\frac{8a^4}{b^6}$

C:  $\frac{32a^8}{b^{18}}$

D:  $\frac{64^2 a^{32}}{b^{72}}$

9.4 If  $p = -\frac{1}{2}$  then  $-p^2 = \dots$

A:  $\frac{1}{4}$

B:  $-\frac{1}{4}$

C:  $-1$

D:  $1$


9.5  $a^{-1} + b^{-1} = \dots$

A:  $\frac{1}{a+b}$




B:  $\frac{1}{ab}$

C:  $-a - b$

D:  $\frac{1}{a} + \frac{1}{b}$

10. The following statements are both false. In each case correct the right hand side. 

10.1  $x^2 + x^2 = 2x^4$     10.2  $\frac{1}{6x^{-2}} = 6x^2$     (1)(1)

11. State whether the following are True or False.   

Give the correct solution where false.

| EXPRESSION  | TRUE OR FALSE | CORRECT SOLUTION, IF FALSE |
|---|---------------|----------------------------|
| 11.1 $2a^{-3} = \frac{1}{2a^3}$                         |               |                            |
| 11.2 $(-1)^5 = -1$                                      |               |                            |
| 11.3 $(2ab^3)^3 = 6a^3b^9$                              |               |                            |
| 11.4 $a^0 = 0$  |               |                            |
| 11.5 $2^{-1} = -2$                                      |               |                            |
| 11.6 $a^{-3}a^2 = \frac{1}{a}$                          |               |                            |
| 11.7 $\left(\frac{3}{4}\right)^{-1} = \frac{4}{3}$      |               |                            |
| 11.8 $\frac{a^3b^5}{ab^7} = \left(\frac{a}{b}\right)^2$ |               |                            |
| 11.9 $2^3 \times 2^4 = 4^7$                             |               |                            |
| 11.10 $5^2 \div 5^5 = \frac{1}{5^3}$                    |               |                            |

(10)

### Substitution



#### Exercise 4.2

Answers on p. A18

- Calculate the value of  $a \times b^c$ , where  $a = 3$ ,  $b = 2$  and  $c = -1$ . (3)
- Calculate the value of  $(ab)^c$ , where  $a = 1$ ,  $b = 5$  and  $c = -2$ . (3)
- Calculate the value of  $a^b + b^c$ , where  $a = 2$ ,  $b = -1$  and  $c = 3$ . (3)

### Exponential Equations



#### Exercise 4.3

Answers on p. A18

Solve for  $x$  in each of the following equations:

- |                     |                                |
|---------------------|--------------------------------|
| 1.1 $2^x = 2^3$     | 1.2 $5^{x-1} = 5^2$ (1)(1)     |
| 1.3 $3^{2x} = 3^6$  | 1.4 $3^x = 9$ (1)(1)           |
| 1.5 $8^x = 64$      | 1.6 $2^x = \frac{1}{4}$ (1)(1) |
| 1.7 $7^x = 1$       | 1.8 $5^{2x} = 5$ (1)(1)        |
| 1.9 $11^x = 121$    | 1.10 $7^x = 49$ (1)(1)         |
| 1.11 $3^{x-2} = 81$ | 1.12 $10^x = 0,1$ (2)(2)       |

Remember to use the exponent laws where necessary.



- |                                       |                                     |
|---------------------------------------|-------------------------------------|
| 1.13 $\left(\frac{1}{3}\right)^x = 3$ | 1.14 $9^x = 27$ (2)(3)              |
| 1.15 $2^x = 0,125$                    | 1.16 $5^x = 0,04$ (3)(3)            |
| 1.17 $3^{2x+1} = 3^{x+3}$             | 1.18 $2^x \cdot 2^3 = 32$ (2)(3)    |
| 1.19 $(4x)^2 = 64$                    | 1.20 $\frac{5^x}{5^2} = 125$ (3)(3) |
| 1.21 $8^{x+1} = \frac{1}{8}$          | 1.22 $7^{x-2} = 1$ (3)(2)           |
| 1.23 $9^{x-2} = 81$                   | 1.24 $x^3 = -8$ (3)(2)              |
| 1.25 $2x^3 = 54$                      | 1.26 $x = \sqrt[3]{27}$ (3)(3)      |
| 1.27 $x^{-1} = \frac{1}{2}$           | 1.28 $x^{-2} = \frac{4}{9}$ (2)(3)  |

- Determine the value of  $x$  if  $(p^x)^3 = p^2 \cdot p^4$  (2)
- Determine the product of  $x$  and  $y$  if  $2^x + 3^y = 41$ , where  $x$  and  $y$  are natural numbers. (3)
- Determine which sign ( $<$ ;  $>$ ;  $=$ ) should be placed in each empty box if  $x = 3$  and  $y = -2$ .

- |                                      |     |
|--------------------------------------|-----|
| 4.1 $y^2 \square x^2$                | (2) |
| 4.2 $(3^x)^3 \square (y^2)^2$        | (2) |
| 4.3 $(x \cdot y)^4 \square (4)^{xy}$ | (2) |



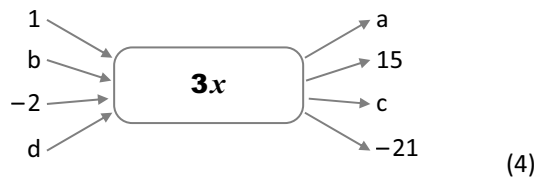


# 6 FUNCTIONS & RELATIONSHIPS (Part 1)

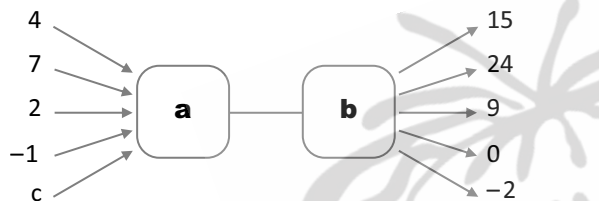
## Exercise 6.1

Answers on p. A23

1. Write down the values of a, b, c and d.



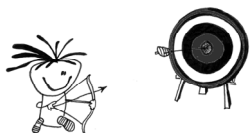
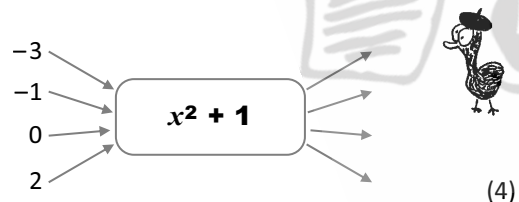
- 2.



- 2.1 Fill in the operations at a and b. (2)

- 2.2 Write down the value of c. (2)

3. Determine the output values for the following flow diagram:



4. Study the table below and answer the questions that follow:

|                   |   |   |    |   |   |   |
|-------------------|---|---|----|---|---|---|
| <b>Input (x)</b>  | 1 | 2 | 3  | 4 | 5 | 6 |
| <b>Output (y)</b> | 4 | 7 | 10 |   |   |   |

- 4.1 Complete the table. (2)

- 4.2 Draw an input-output flow diagram including a formula to describe the relationship between these input and output values, i.e. to illustrate the rule. (2)

- 4.3 Is this a linear function? Give a reason for your answer. (2)

5. Given the formula,  $y = 3x - 4$ , copy and complete the following table:

|          |    |    |   |   |   |
|----------|----|----|---|---|---|
| <b>x</b> | -2 | -1 | 0 | 1 | 2 |
| <b>y</b> |    |    |   |   |   |



6. Study the table below and answer the questions that follow:

|                   |   |   |    |   |   |   |
|-------------------|---|---|----|---|---|---|
| <b>Input (x)</b>  | 1 | 2 | 3  | 4 | 5 | 6 |
| <b>Output (y)</b> | 5 | 2 | -1 | p | q | r |

- 6.1 Write down the values of p, q and r. (3)

- 6.2 Write down a formula to describe the relationship between the input and output values. (2)

- 6.3 Draw an input-output flow diagram to illustrate the rule. (2)

- 6.4 Is this a linear function? Give a reason for your answer. (2)

7. Study the following table:

|          |    |    |   |   |   |
|----------|----|----|---|---|---|
| <b>x</b> | -2 | -1 | 0 | 1 | 3 |
| <b>y</b> | -3 | -1 | 1 | 3 | 7 |



- 7.1 Do the points form a linear or non-linear function? Give a reason. (2)

- 7.2 Write down a formula to determine the relationship between x and y. (2)

8. Use the equation  $y = -2x + 3$  to complete the row of y-values in the following table. (3)

|          |    |    |   |   |   |   |
|----------|----|----|---|---|---|---|
| <b>x</b> | -4 | -1 | 0 | 2 | 5 | 8 |
| <b>y</b> |    |    |   |   |   |   |



9. Which of the following equations describes the relationship between x and y in the table below? (1)

$$y = x - 1 ; y = x^2 - 1 ; y = 2x^2 - 2$$

|          |   |   |   |
|----------|---|---|---|
| <b>x</b> | 1 | 2 | 3 |
| <b>y</b> | 0 | 3 | 8 |



10. Water is pumped from a dam into a reservoir. The following table of values represents the volume (V) of water in the reservoir at any given time (t).

|                                 |    |    |    |    |    |    |
|---------------------------------|----|----|----|----|----|----|
| <b>Time (t) in minutes</b>      | 10 | 20 | 30 | 40 | 50 | 60 |
| <b>Volume (V) in kilolitres</b> | 7  | 12 | 17 | 22 | 27 | 32 |

- 10.1 What is the increase in volume every 10 minutes? (1)

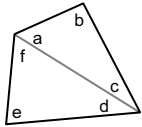
- 10.2 Hence determine the rate of increase in kilolitres per minute. (3)

- 10.3 Write down a formula that could be used to determine the volume of water in the reservoir at any given time. (2)

# QUADRILATERALS - pathways of definitions, areas and properties - A Summary

## All you need to know!

### 'Any' Quadrilateral



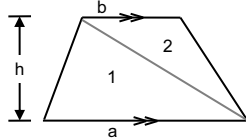
Sum of the  $\angle^s$  of any quadrilateral =  $360^\circ$

$$\begin{aligned} \text{Sum of the interior angles} &= (a + b + c) + (d + e + f) \\ &= 2 \times 180^\circ \dots (2 \Delta^s) \\ &= 360^\circ \end{aligned}$$

The arrows indicate various 'pathways' from 'any' quadrilateral to the square (the 'ultimate quadrilateral'). These pathways, which combine logic and fact, are essential to use when proving specific types of quadrilaterals.

See how the properties accumulate as we move from left to right, i.e. the first quad has no special properties and each successive quadrilateral has all preceding properties.

### A Trapezium

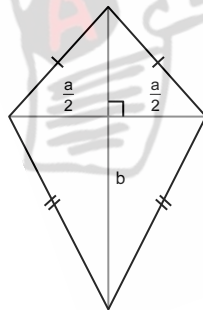


**DEFINITION:**  
Quadrilateral with 1 pair of opposite sides  $\parallel$

$$\begin{aligned} \text{Area} &= \Delta 1 + \Delta 2 \\ &= \frac{1}{2} ah + \frac{1}{2} bh \\ &= \frac{1}{2} (a + b) \cdot h \end{aligned}$$

'Half the sum of the  $\parallel$  sides  $\times$  the distance between them.'

### A Kite



**DEFINITION:**  
Quadrilateral with 2 pairs of adjacent sides equal

Given diagonals a and b

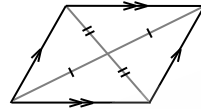
$$\text{Area} = 2\Delta^s = 2 \left( \frac{1}{2} b \cdot \frac{a}{2} \right) = \frac{ab}{2}$$

'Half the product of the diagonals'

#### THE DIAGONALS

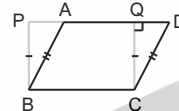
- cut perpendicularly
- ONE DIAGONAL bisects the other diagonal, the opposite angles and the area of the kite

### A Parallelogram



**DEFINITION:**  
Quadrilateral with 2 pairs opposite sides  $\parallel$

Area = base  $\times$  height

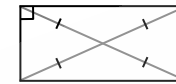


$\parallel^m$  ABCD = ABCQ +  $\Delta$ QCD  
rect. PBCQ = ABCQ +  $\Delta$ PBA  
where  $\Delta$ QCD  $\cong$   $\Delta$ PBA  $\dots$  RHS/ $90^\circ$ HS  
 $\therefore \parallel^m$  ABCD = rect. PBCQ (in area)  
= BC  $\times$  QC

#### Properties:

- 2 pairs opposite sides equal
- 2 pairs opposite angles equal
- & DIAGONALS BISECT ONE ANOTHER

### A Rectangle

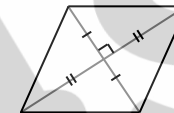


**DEFINITION:**  
A  $\parallel^m$  with one right  $\angle$

$$\text{Area} = \ell \times b$$

DIAGONALS are EQUAL

### A Rhombus



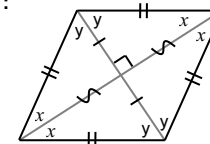
**DEFINITION:**  
A  $\parallel^m$  with one pair of adjacent sides equal

Area =  $\frac{1}{2}$  product of diagonals (as for a kite)  
or  
= base  $\times$  height (as for a parallelogram)

#### THE DIAGONALS

- bisect one another PERPENDICULARLY
- bisect the angles of the rhombus
- bisect the area of the rhombus

Note:



$$\begin{aligned} 2x + 2y &= 180^\circ \dots \angle^s \text{ of } \Delta \text{ or} \\ \rightarrow x + y &= 90^\circ \text{ co-int. } \angle^s ; \parallel \text{ lines} \end{aligned}$$

### The Square



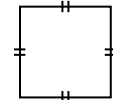
the 'ultimate' quadrilateral!

$$\text{Area} = s^2$$

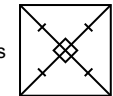
#### Properties:

It's all been said 'before'!  
Since a square is a rectangle, a rhombus, a parallelogram, a kite, ... ALL the properties of these quadrilaterals apply.

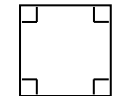
sides



diagonals



angles



Quadrilaterals play a prominent role in both Euclidean and Analytical Geometry right through to Grade 12!



# 11 GEOMETRY OF STRAIGHT LINES

## Classifying 2D Shapes



### STRAIGHT LINE GEOMETRY

#### ► Important VOCABULARY

An acute angle is one that lies between  $0^\circ$  &  $90^\circ$ .

An obtuse angle is one that lies between  $90^\circ$  &  $180^\circ$ .

A reflex angle is one that lies between  $180^\circ$  &  $360^\circ$ .

A right angle =  $90^\circ$

A straight angle =  $180^\circ$

A revolution =  $360^\circ$

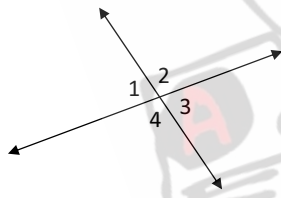


When the sum of 2 angles =  $90^\circ$ , we say the angles are complementary.

When the sum of 2 angles =  $180^\circ$ , we say the angles are supplementary.

When 2 lines intersect, 4 angles are formed:

$\hat{1}$ ,  $\hat{2}$ ,  $\hat{3}$ ,  $\hat{4}$



#### Adjacent angles

have a common vertex and a common arm, e.g.  $\hat{1}$  and  $\hat{2}$ ,  $\hat{2}$  and  $\hat{3}$ ,  $\hat{3}$  and  $\hat{4}$  or  $\hat{1}$  and  $\hat{4}$ .

#### Vertically opposite angles

lie opposite each other, e.g.  $\hat{1}$  and  $\hat{3}$  or  $\hat{2}$  and  $\hat{4}$ .

#### ► The FACTS

When 2 lines intersect:

- adjacent angles are **supplementary**
- vertically opposite angles are **equal**.



#### Angle notation

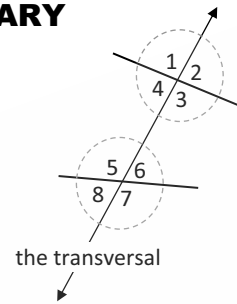
We usually use letters, not numbers for angles. On this page the numbering is intended to show the connections between angles.



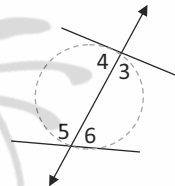
#### ► Important VOCABULARY

When 2 lines are cut by another line (a transversal), two families of angles are formed:

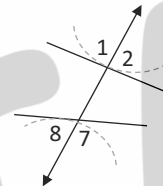
$\hat{1}$ ,  $\hat{2}$ ,  $\hat{3}$ ,  $\hat{4}$  and  $\hat{5}$ ,  $\hat{6}$ ,  $\hat{7}$ ,  $\hat{8}$



These are **interior** angles.

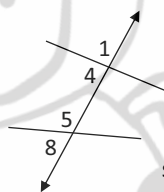


These are **exterior** angles.



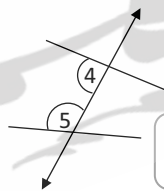
Both these groups are '**co-**' angles

i.e. they are on the same side of the transversal



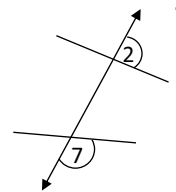
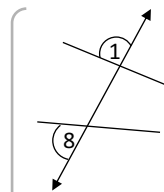
These are pairs of **co-interior** angles.

*They are NOT necessarily supplementary.*

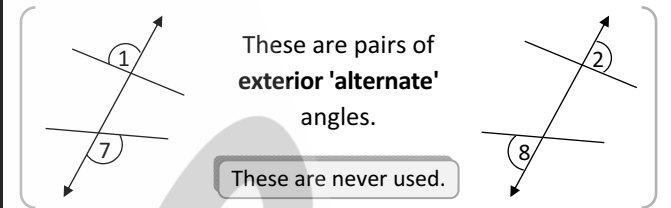
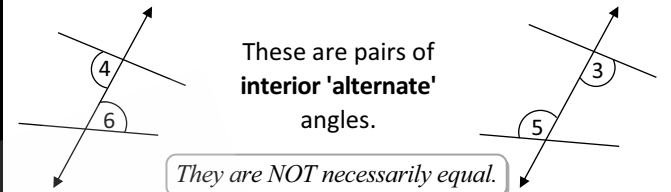


These are pairs of **co-exterior** angles.

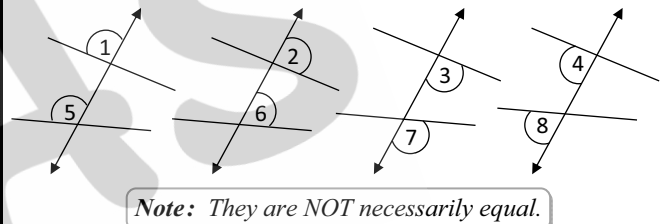
*These are never used.*



#### Angles that 'alternate' are on opposite sides of the transversal.

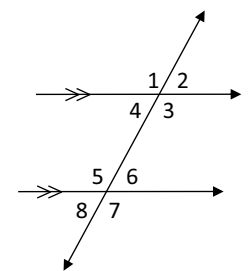


These pairs of angles **correspond**.



#### ► The FACTS

When 2 **PARALLEL** lines are cut by a transversal, then the **corresponding angles** are **equal**, the (interior) **alternate angles** are **equal**, and the **co-interior** angles are **supplementary**.



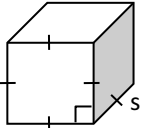
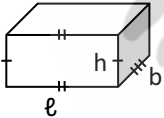
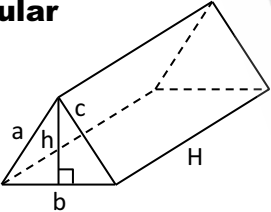
& CONVERSELY:

If the **corresponding angles** are **equal**, or if the (interior) **alternate angles** are **equal**, or if the **co-interior** angles are **supplementary**, then the lines are **parallel**.



Recognise these angles in unfamiliar situations.

# 18 VOLUME & TOTAL SURFACE AREA OF 3D OBJECTS: FORMULAE

| 3D Objects  | TSA = sum of areas of all surfaces   | Volume = Area of base × Height  |
|---|--|---|
|   | The total exterior area of all the exposed surfaces of a 3D shape.   | The 3D space that a 3D object occupies.   |
| <b>Cube</b><br>s = side<br>  | Total Surface Area<br>$= (\text{side} \times \text{side}) \times 6$<br>$= 6(\text{side})^2$<br>$\therefore \text{TSA} = 6s^2$  | Volume<br>$= \text{side} \times \text{side} \times \text{side}$<br>$= (\text{side})^3$<br>$\therefore V = s^3$  |
| <b>Rectangular Prism</b><br>ℓ = length<br>b = breadth<br>h = height<br>  | Total Surface Area<br>$= 2(\text{length} \times \text{breadth}) + 2(\text{length} \times \text{height}) + 2(\text{breadth} \times \text{height})$<br>$\therefore \text{TSA} = 2\ell b + 2\ell h + 2bh$   | Volume<br>$= \text{length} \times \text{breadth} \times \text{height}$<br>$\therefore V = \ell \times b \times h$   |
| <b>Triangular Prism</b><br><br>a = side <sub>1</sub><br>b = side <sub>2</sub> (base of the triangle face)<br>c = side <sub>3</sub><br>h = ⊥ height of Δ<br>H = height of prism (distance between the 2 bases) | Total Surface Area<br>$= 2\left(\frac{b \times h}{2}\right) + (\text{side}_1 \times \text{prism height}) + (\text{side}_2 \times \text{prism height}) + (\text{side}_3 \times \text{prism height})$<br>$\therefore \text{TSA} = 2\left(\frac{b \times h}{2}\right) + (a \times H) + (b \times H) + (c \times H)$ | Volume<br>$= \text{area of base} \times \text{height of prism}$<br>$\therefore V = \left(\frac{b \times h}{2}\right) \times H$<br>Remember:<br>Area of Δ<br>$= \frac{1}{2}b \times h$ OR $\frac{b \times h}{2}$ |

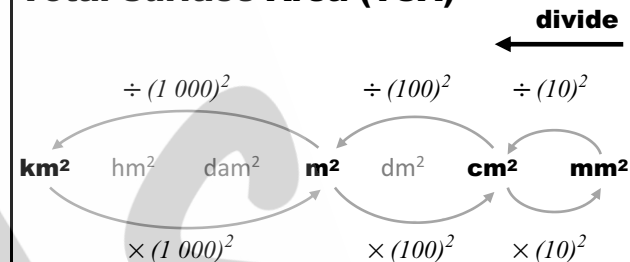


## SI Units & Conversions

Small unit → big unit: ÷

Big unit → small unit: ×

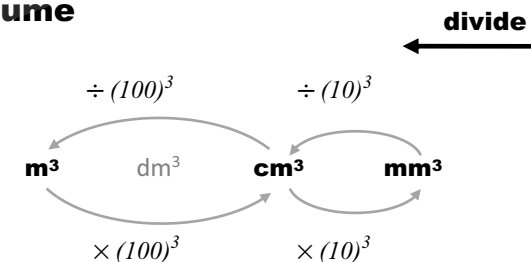
### Total Surface Area (TSA)



**multiply** →

Since the area is the product of **2D** lengths, we need to × or ÷ by the (conversion factor)<sup>2</sup>.

### Volume



**multiply** →

Since volume is the product of **3D** lengths, we need to × or ÷ by the (conversion factor)<sup>3</sup>.

# 4 EXPONENTS

## Exponential Expressions



### Exercise 4.1

Questions on p. 12

**Note:**

When multiplying and dividing: consider signs, numbers and then letters one at a time.



Apply the given law provided in each of the 6 cases to simplify all these expressions.

**Law 1:  $a^m \times a^n = a^{m+n}$**

1.1  $x^5 \times 2x^3 = 2x^8$

1.2  $3x^2 \times 2x^3$

$= 6x^5$

$3 \times 2 = 6$   
&  
 $x^2 \times x^3$   
 $= x^{2+3}$   
 $= x^5$

1.3  $2^{-4} \times 2^5 \times 2^0$

$= 2^{-4+5+0}$

$= 2^1$

$= 2$

**Law 2:  $a^m \div a^n = a^{m-n}$**

1.4  $2^5 \div 2^2 = 2^{5-2}$

$= 2^3$

**Note:**  $\frac{a^m}{a^n} = a^{m-n}$



&  $\frac{a^m}{a^n} = \frac{1}{a^{n-m}}$

1.5  $\frac{p^7}{p^3} = p^{7-3}$

$= p^4$

1.6  $\frac{a^2}{a^{-1}} = a^{2+1}$

$= a^3$

**Law 3:  $(a^m)^n = a^{mn}$**

1.7  $(2^3)^2 = 2^{3 \times 2}$   
 $= 2^6$   
 $= 64$

1.8  $(s^3)^5 = s^{3 \times 5}$   
 $= s^{15}$

1.9  $(3^x)^2 = 3^{x \times 2}$   
 $= 3^{2x}$

Make sure your letters and numbers are clearly different, e.g. a **5** and an **s** can easily get mixed up!



**Law 4:  $(ab)^m = a^m b^m$**

1.10  $(2 \times 5)^2 = 2^2 \times 5^2$   
 $= 4 \times 25$   
 $= 100$

Each factor in the bracket needs to be raised to the power, i.e. the sign, the number & each letter.

1.11  $(3x^3)^3 = 3^3 \cdot x^{3 \times 3}$   
 $= 27x^9$

**Note:**  $(ab^m)^n = a^n b^{mn}$   
&  $(abc\dots)^n = a^n b^n c^n \dots$



1.12  $(-4a^5b^3)^2$   
 $= 4^2 a^{5 \times 2} b^{3 \times 2}$   
 $= 16a^{10} b^6$

**Law 5:  $a^0 = 1$  ;  $a \neq 0$**

1.13  $5^0 = 1$

1.14  $7a^0 = 7(1)$   
 $= 7$

1.15  $3p^0 + (3p)^0$   
 $= 3(1) + (1)$   
 $= 3 + 1$   
 $= 4$

**Law 6:  $a^{-n} = \frac{1}{a^n}$  ;  $a \neq 0$**

1.16  $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$

1.17  $x^{-4} = \frac{1}{x^4}$

1.18  $a^3 \cdot b^{-4} = \frac{a^3}{b^4}$



Questions including Laws 1 & 2



2.1  $3a \cdot 2ab \cdot 3abc$   
 $= 18a^3b^2c$

2.2  $2x^3 \times -3x^5$   
 $= -6x^{3+5}$   
 $= -6x^8$

2.3  $2a^4 \times 4a^2$   
 $= 8a^{4+2}$   
 $= 8a^6$

2.4  $a^{-4} \times a^7$   
 $= a^{-4+7}$   
 $= a^3$

2.5  $x^3(x^2)^3x^0x^{-2}$   
 $= x^3x^6x^0x^{-2}$   
 $= x^{3+6+0-2}$   
 $= x^7$

(OR  $x^3(x^2)^3x^0x^{-2}$   
 $= x^3x^6(1)x^{-2}$  ... using law 5  
 $= x^{3+6-2}$   
 $= x^7$ )

2.6  $6a^5 \times a \div 3a^2$   
 $= \frac{6a^6}{3a^2}$   
 $= 2a^{6-2}$   
 $= 2a^4$

2.7  $5x^3y^5 \times 3x^{-2}y^2$   
 $= 15x^{3-2}y^{5+2}$   
 $= 15xy^7$

2.8  $8y^3 \times \frac{1}{4}y^2$   
 $= \frac{2y^3}{1} \times \frac{y^2}{4}$   
 $= 2y^{3+2}$   
 $= 2y^5$

2.9  $\frac{a^3b^2c}{ab^2c^3}$   
 $= \frac{a^2}{c^2}$  ...

$\frac{a^{3-1}}{c^{3-1}}$   
&  
 $\frac{b^2}{b^2} = 1$



$$2.10 \quad \frac{5^2 \cdot 5^3 \cdot x^2 \cdot y^4}{5^6 \cdot x^3 \cdot y^2}$$

$$= \frac{5^5 \cdot x^2 \cdot y^4}{5^6 \cdot x^3 \cdot y^2} \quad \dots \text{ Law 1 only}$$

$$= \frac{y^4 \cdot 2}{5^{6-5} \cdot x^{3-2}} \quad \dots \text{ Law 2 only}$$

$$= \frac{y^2}{5x}$$

OR

$$\frac{5^2 \cdot 5^3 \cdot x^2 \cdot y^4}{5^6 \cdot x^3 \cdot y^2}$$

$$= \frac{y^4 \cdot 2}{5^{6-2-3} \cdot x^{3-2}}$$

$$= \frac{y^2}{5x}$$

Law 1 & 2 together

$$2.11 \quad \frac{{}_1^3 ab^4 c^0}{{}_6^{18} a^2 bc}$$

$$= \frac{b^4 - 1}{6a^{2-1}c}$$

$$= \frac{b^3}{6ac}$$



**Note:**  
Gather exponents where their sum will be positive.

$$2.12 \quad \frac{{}_1^4 x^3 y^7}{{}_2^8 x^2 y^9}$$

$$= \frac{x^{3-2}}{2y^{9-7}}$$

$$= \frac{x}{2y^2}$$

$$2.13 \quad \frac{{}_2^8 b^{-3} c^5}{{}_3^{12} b^2 c^{-6}}$$

$$= \frac{2c^{5+6}}{3b^{2+3}}$$

$$= \frac{2c^{11}}{3b^5}$$

$$2.14 \quad \frac{{}_{11}^{121} x^7 y^{13}}{{}_1^{11} x^{-4} y^2}$$

$$= 11x^{7+4} y^{13-2}$$

$$= 11x^{11} y^{11}$$

$$2.15 \quad \frac{{}_5^{55} x^3 y^{12}}{{}_{11}^{11} x^{-2} y^{17}}$$

$$= \frac{5x^{3+2}}{y^{17-12}}$$

$$= \frac{5x^5}{y^5}$$



$$2.16 \quad \frac{15xy^{-2}}{3x^{-3}y^4}$$

$$= \frac{{}_5^5 15x^{1+3}}{{}_1^3 y^{4+2}}$$

$$= \frac{5x^4}{y^6}$$

$$2.17 \quad \frac{{}_3^{18} a^{-2} b^3 c}{{}_2^{12} a^3 b^5 c^{-4}}$$

$$= \frac{3c^{1+4}}{2a^{3+2} b^{5-3}}$$

$$= \frac{3c^5}{2a^5 b^2}$$

$$2.18 \quad \frac{{}_{13}^{-39} x^{17} y^{10}}{{}_1^{-3} x^4 y^{-3}}$$

$$= 13x^{17-4} y^{10+3}$$

$$= 13x^{13} y^{13}$$



Questions extending to Laws 3 & 4

$$3.1 \quad (-3x^2)^2 = 9x^4$$

$$3.3 \quad (2^3 x^2 y)^3 = 2^9 x^6 y^3 = 512x^6 y^3$$

$$3.5 \quad 4(-a)^2 - (-2a)^2 = 4a^2 - 4a^2 = 0$$

$$3.2 \quad (-3x^4)(2x^3) = -6x^7$$

$$3.4 \quad (3p^5 q^3)^2 = 9p^{10} q^6$$

$$3.6 \quad x^{-7}(x^{13} + x^{11}) = x^6 + x^4 \dots$$

These are not like terms.  
∴ You cannot add them.



$$3.7 \quad \frac{ab^2 \times a^3 b}{(a^2 b^2)^2}$$

$$= \frac{a^4 b^3}{a^4 b^4} \dots$$

$$= \frac{1}{b^{4-3}}$$

$$= \frac{1}{b}$$

$$\frac{a^4}{a^4} = \frac{1}{1} = 1$$

$$\& \frac{b^3}{b^4} = \frac{1}{b^{4-3}} = \frac{1}{b}$$



$$3.8 \quad \frac{ab(-2a^2 b^3)^3}{-56b^3}$$

$$= \frac{ab(-8a^6 b^9)}{-56b^3} \dots$$

$$= \frac{1-8a^7 b^{10}}{7-56b^3}$$

$$= \frac{a^7 b^{10-3}}{7}$$

$$= \frac{a^7 b^7}{7}$$



$$3.9 \quad \frac{(3m^3 n)(-2mn^4)}{(-2mn)^4}$$

$$= \frac{-3m^{3+1} n^{1+4}}{8m^4 n^4}$$

$$= \frac{-3m^4 n^5}{8m^4 n^4}$$

$$= -\frac{3n^{5-4}}{8}$$

$$= -\frac{3n}{8}$$



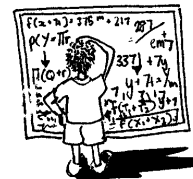
... A reminder ...  
When multiplying, start with **signs**, then **numbers**, then **letters**; one at a time.

$$3.10 \quad \frac{(x^2 y) \times (x^3 y^3)}{(x^2 y)^3}$$


$$= \frac{x^5 y^4}{x^6 y^3}$$

$$= \frac{y^{4-3}}{x^{6-5}}$$

$$= \frac{y}{x}$$



Questions extending to Laws 5 & 6


4.1  $\frac{x^{-3}}{y^{-4}} = \frac{y^4}{x^3} \dots x^{-3} = \frac{1}{x^3} \ \& \ \frac{1}{y^{-4}} = y^4$  

4.2  $\frac{m^5 \cdot n^{-2} \cdot p^0}{m \cdot n^3}$   
 $= \frac{m^{5-1} \cdot (1)}{n^{3+2}}$   
 $= \frac{m^4}{n^5}$


4.3  $\frac{x \cdot y^{-2}}{x^0 \cdot y}$   
 $= \frac{x}{(1)y^{1+2}}$   
 $= \frac{x}{y^3}$

4.4  $\frac{x^0}{0,25}$   
 $= \frac{1}{\frac{1}{4}}$

4.5  $\frac{(5m)^0}{5m^0}$   
 $= \frac{1}{5(1)} = \frac{1}{5}$

$= 1 \times \frac{4}{1} \dots 1 \div \frac{1}{4} = 1 \times \frac{4}{1}$  

4.6  $(a^{-2})(-3a^0) \dots$  *the 0 only applies to the a*  
 $= (a^{-2})(-3)$   
 $= \left(\frac{1}{a^2}\right)\left(\frac{-3}{1}\right)$   
 $= -\frac{3}{a^2}$

4.7  $(-2a^4b^3)^2(-3a^2b^0) = 4a^8b^6(-3a^2) \dots b^0 = 1$  

4.8  $\frac{(2^2q^{-3})^0(p^{-3}q)^{-2}}{(-2pq^{-1})^2}$   
 $= \frac{(1)(p^6q^{-2})}{4p^2q^{-2}}$   
 $= \frac{p^6q^{-2}}{4p^2q^{-2}}$   
 $= \frac{p^4}{4}$

**Remember...**


Each factor in the bracket needs to be raised to the exponent, i.e. the **sign**, the **number** & each **letter**.



4.9  $\frac{(4x^3)^2(-3x)^0}{-6x^2} = \frac{(16x^6)(1)}{-6x^2}$   
 $= -\frac{8 \cdot 16x^6}{3 \cdot 6x^2}$   
 $= -\frac{8x^4}{3}$

Mixed Questions

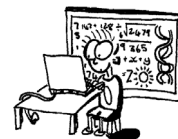
5.1  $3^{-2} + 3^0 + 3^2$   
 $= \frac{1}{3^2} + 1 + 9$   
 $= \frac{1}{9} + \frac{1}{1} + \frac{9}{1}$   
 $= \frac{1+9+81}{9}$   
 $= \frac{91}{9}$

5.2  $-5^2 \div \sqrt[3]{-1}$   
 $= -25 \div (-1)$   
 $= 25$  

5.3  $\sqrt{3^2 + 3^1 + 3^0 + 3^{-1} + 3^{-2}}$   
 $= \sqrt{9 + 3 + 1 + \frac{1}{3} + \frac{1}{9}}$   
 $= \sqrt{\frac{13}{1} + \frac{1}{3} + \frac{1}{9}}$   
 $= \sqrt{\frac{117+3+1}{9}}$   
 $= \sqrt{\frac{121}{9}}$   
 $= \frac{11}{3}$

Check that you know your exponent laws:

- 1  $a^m \times a^n = a^{m+n}$
- 2  $a^m \div a^n = a^{m-n}$
- 3  $(a^m)^n = a^{mn}$
- 4  $(ab)^n = a^n b^n$
- 5  $a^0 = 1$
- 6  $a^{-n} = \frac{1}{a^n} = \left(\frac{1}{a}\right)^n$



5.4  $\sqrt[3]{27} - \sqrt{49} + 4$   
 $= 3 - 7 + 4$   
 $= 0$

5.5  $\sqrt{9+16}$   
 $= \sqrt{25}$   
 $= 5$

5.6  $\sqrt{\frac{18}{2}} - (-2)^3$   
 $= \sqrt{9} - (-8)$   
 $= 3 + 8$   
 $= 11$

5.7  $\sqrt[3]{\frac{27}{8}} + \sqrt{2\frac{1}{4}}$   
 $= \frac{3}{2} + \sqrt{\frac{9}{4}}$   
 $= \frac{3}{2} + \frac{3}{2}$   
 $= \frac{6}{2}$   
 $= 3$

5.8  $\sqrt{36x^{36} + 64x^{36}} \dots$  *Add first (like terms)!*  
 $= \sqrt{100x^{36}}$   
 $= 10x^{18}$

$10 \times 10 = 100$   
 $\therefore \sqrt{100} = 10$   
 $\& \ x^{18} \times x^{18} = x^{18+18} = x^{36}$   
 $\therefore \sqrt{x^{36}} = x^{18}$

5.9  $\sqrt{9x^{10} + 16x^{10}}$   
 $= \sqrt{25x^{10}}$   
 $= 5x^5$

5.10  $\sqrt{144a^6b^{10}} \cdot (-2ab^2)$   
 $= (12a^3b^5)(-2ab^2)$   
 $= -24a^4b^7$

5.11  $\sqrt{9(a+2b)^2x^4}$   
 $= 3(a+2b)x^2$   
 $= 3x^2(a+2b)$   
 $= 3ax^2 + 6bx^2$

*Just as  $(\sqrt{9}) = \sqrt{3^2} = 3$   
so  $\sqrt{(a+2b)^2} = a+2b$*

5.12  $\sqrt{169x^{14}y^{22}}$   
 $= 13x^7y^{11}$

$13 \times 13 = 169$  ;  
 $x^7 \times x^7 = x^{14}$  ;  
 $y^{11} \times y^{11} = y^{22}$



**PAPER D1***Answers on p. M10**Approved scientific calculators (non-programmable and non-graphical) may be used.***QUESTION 1**

Four options are given for each of the following questions. Only one answer is correct. Write the correct letter next to the question number, e.g. 1.7 A.

1.1 Simplify:  $2x - x(x + y) = \dots$ 

- A  $x^2 + xy$                       B  $2x - x^2 - xy$   
 C  $x^2 - xy$                       D  $2x - x^2 + xy$                       (1)

1.2 The number 1 is NOT a(n) ...

- A rational number              B whole number  
 C irrational number              D integer                      (1)

1.3 All the fractions can be written as a(n) ...

- A percentage                      B decimal  
 C ratio                              D option A; B and C                      (1)

1.4  $\sqrt{\frac{16x^4}{y^{16}}} = \dots$ 

- A  $\frac{4x^2}{y^4}$                               B  $\frac{8x^2}{y^8}$   
 C  $\frac{8x^2}{y^4}$                               D  $\frac{4x^2}{y^8}$                               (2)

1.5 What is the missing number in the sequence?

2 ; 5 ; 10 ; ... ; 26

- A 15                              B 25  
 C 17                              D 20                              (2)

1.6 The ratio  $\frac{2}{5} : \frac{4}{6} : \frac{7}{15}$  simplifies to:

- A 2:4:7                              B  $\frac{12}{30} : \frac{20}{30} : \frac{14}{30}$   
 C 12:20:14                              D 6:10:7                              (2) [9]

**QUESTION 2**

2.1 Determine the following products:

- 2.1.1  $3x(y + 4z)$                               (1)  
 2.1.2  $(2x + 1)^2$                               (2)  
 2.1.3  $(2x - 1)(3x + 2)$                               (2)  
 2.1.4  $(2x - 5)(x - 3) + (x + 2)^0 - (x - 2)^2$                               (6)

2.2 Simplify the following:

- 2.2.1  $10x^3 \div \frac{1}{2}x^2$                               (2)  
 2.2.2  $(2x^2 \times \frac{1}{4}xy \times 8x^0) \div (3x \times 4y)$                               (3)  
 2.2.3  $\frac{6x^2 - 24}{3x^2 + 6x}$                               (4)  
 2.2.4  $\frac{2x^2 \times 4y}{3y^2 \times 4x} + \frac{2x^2 \times 3}{2x \times 9y}$                               (4)



2.3 An isosceles triangle is constructed by connecting three lines, two of which are equal to  $2x^2 + 2x$ . Determine the length of the remaining side in terms of  $x$  if the perimeter of the triangle is equal to  $7x^2 + 10x$ .                              (4) [28]

**QUESTION 3**

Factorise the following expressions:

- 3.1  $2x^2 + 14x$                               3.2  $x^4 - 16$                               (2)(3)  
 3.3  $4x^2 - 36$                               3.4  $x^2 + 7x + 12$                               (3)(2)  
 3.5  $(2x - 3y)x^2 + (3y - 2x)$                               (4) [14]

**QUESTION 4**

4.1 Solve the following equations:

- 4.1.1  $10 - 3x = 1$                               (2)  
 4.1.2  $3(x + 2) = 2 - 1(x + 4)$                               (4)  
 4.1.3  $3x(x + 2) = 2x^2 + 12 + x^2$                               (3)  
 4.1.4  $\frac{2x}{4} + \frac{1}{3} = \frac{-4}{6}$                               (4)  
 4.1.5  $\frac{2x^2 + x}{x} = \frac{4x^2 + 3}{2x}$                               (4)

- 4.2 Mrs Foster is very fussy about which colour jellybeans she eats. Her favourite colour is orange, and her least favourite is yellow. There are  $x + 9$  orange jellybeans in a packet and  $2x - 3$  yellow jellybeans in the same packet, in which there is a total of  $2(x + 8)$  yellow and orange jellybeans.
- 4.2.1 Set up an equation that represents the above paragraph.                              (2)
- 4.2.2 How many of her favourite jellybeans did Mrs Foster get in the packet of sweets?                              (3)

- 4.3 Mrs Louw loves her girls hockey team and after looking at their season she decided that they had an excellent season as the ratio of games won to games lost was 5 : 1. In terms of  $x$ , they only lost  $x + 4$  games.
- 4.3.1 Determine, in terms of  $x$ , how many games the girls won.                              (2)
- 4.3.2 Using your answer in Question 4.3.1 determine how many matches the girls played if in terms of  $x$  they played a total of  $10x$  games.                              (4) [28]





**PAPER D1**

Questions on p. E10

1.1 B ...  $2x - x(x + y)$   
 $= 2x - x^2 - xy$



1.2 C ... an irrational number is a number with non-recurring decimals

1.3 D ... these are 3 different ways of expressing a fraction

1.4 D ...  $\sqrt{16} = 4$ ;  $\sqrt{x^4} = x^2$ ;  $\sqrt{y^{16}} = y^8$   
 $\therefore 4 \times 4 = 16$ ;  $x^2 \times x^2 = x^4$ ;  $y^8 \times y^8 = y^{16}$

1.5 C ...  $\frac{2}{3} \times \frac{5}{5} \times \frac{10}{7} \times \frac{17}{9} \times \frac{26}{9}$

1.6 D ...  $\frac{2}{5} : \frac{4}{6} : \frac{7}{15}$   
 $\times 30$   $12 : 20 : 14 \rightarrow$  to make whole no's  
 $\div 2$   $6 : 10 : 7 \rightarrow$  to simplify

2.1.1  $3x(y + 4z)$   
 $= 3xy + 12xz$

**Note**

These two terms will always be the same when multiplying out a squared bracket. Noticing these kinds of patterns will make you more efficient and more confident.

2.1.2  $(2x + 1)^2$   
 $= (2x + 1)(2x + 1)$   
 $= 4x^2 + 2x + 2x + 1$   
 $= 4x^2 + 4x + 1$



2.1.3  $(2x - 1)(3x + 2)$   
 $= 6x^2 + 4x - 3x - 2$   
 $= 6x^2 + x - 2$

2.1.4  $(2x - 5)(x - 3) + (x + 2)^0 - (x - 2)^2$   
 $= 2x^2 - 6x - 5x + 15 + 1 - (x - 2)(x - 2)$   
 $= 2x^2 - 11x + 16 - (x^2 - 2x - 2x + 4)$   
 $= 2x^2 - 11x + 16 - (x^2 - 4x + 4) \dots$   
 $= 2x^2 - 11x + 16 - x^2 + 4x - 4$   
 $= x^2 - 7x + 12$

Brackets are essential here for signs to be correct.

There is no rush: take the pressure off by doing one thing at a time and focus on accuracy!



2.2.1  $10x^3 \div \frac{1}{2}x^2$   
 $= 10x^3 \div \frac{x^2}{2}$  ...  $\frac{1}{2}x^2 = \frac{1}{2} \times \frac{x^2}{1} = \frac{x^2}{2}$   
 $= \frac{10 \cancel{x}^3}{1} \times \frac{2}{\cancel{x}^2}$  ... **Tip** When multiplying a whole number by a fraction, put the whole number over 1.  
 $= 20x$

2.2.2  $(2x^2 \times \frac{1}{4}xy \times 8x^0) \div (3x \times 4y)$   
 $= \left( \frac{2x^2}{1} \times \frac{xy}{1} \times \frac{8(1)}{1} \right) \div (12xy)$   
 $= \frac{1}{4} \times \frac{8}{3} \times \frac{1}{1} \times \frac{1}{1} \times \frac{1}{1}$   
 $= \frac{x^2}{3}$

$\frac{1}{4}xy$   
 $= \frac{1}{4} \times \frac{x}{1} \times \frac{y}{1}$   
 $= \frac{xy}{4}$  &

$8x^0 = 8(1) = 8$

**NB:** Different to  $(8x)^0 = 1$ .

2.2.3  $\frac{6x^2 - 24}{3x^2 + 6x} \dots$   
 $= \frac{6(x^2 - 4)}{3x(x + 2)}$   
 $= \frac{2 \cancel{6} (x + 2)(x - 2)}{1 \cancel{3} x(x + 2)}$   
 $= \frac{2(x - 2)}{x}$

You can only cancel factors. So, factorise fully first!



2.2.4  $\frac{2x^2 \times 4y}{3y^2 \times 4x} + \frac{2x^2 \times 3}{2x \times 9y}$   
 $= \frac{2 \cancel{8} \cancel{x}^2 y}{3 \cancel{12} \cancel{x} y^2} + \frac{1 \cancel{6} \cancel{x}^2 3}{3 \cancel{18} \cancel{x} y}$  ...  
 $= \frac{2x}{3y} + \frac{x}{3y}$  ...  
 $= \frac{3x}{3y}$  ...  
 $= \frac{x}{y}$  ...

Cancel carefully when there is so much going on.

Same denominators! So you can add numerators which happen to be like terms.

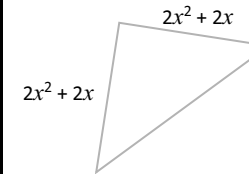
Always check to see if you can simplify

2.3

P = Sum of all 3 sides

$\therefore$  The remaining side

$= P - (\text{sum of the 2 given sides})$   
 $= 7x^2 + 10x - (2x^2 + 2x + 2x^2 + 2x)$   
 $= 7x^2 + 10x - (4x^2 + 4x)$   
 $= 7x^2 + 10x - 4x^2 - 4x$   
 $= 3x^2 + 6x$





**LINES**

|  |  |
|--|--|
| The adjacent angles on a straight line are supplementary.                                      | $\angle^s$ on a str line                             |
| If the adjacent angles are supplementary, the outer arms of these angles form a straight line. | adj $\angle^s$ supp                                  |
| The adjacent angles in a revolution add up to $360^\circ$ .                                    | $\angle^s$ around a pt <b>OR</b> $\angle^s$ in a rev |
| Vertically opposite angles are equal.  | vert opp $\angle^s$                                  |
| If $AB \parallel CD$ , then the alternate angles are equal.                                    | alt $\angle^s$ ; $AB \parallel CD$                   |
| If $AB \parallel CD$ , then the corresponding angles are equal.                                | corresp $\angle^s$ ; $AB \parallel CD$               |
| If $AB \parallel CD$ , then the co-interior angles are supplementary.                          | co-int $\angle^s$ ; $AB \parallel CD$                |
| If the alternate angles between two lines are equal, then the lines are parallel.              | alt $\angle^s =$                                     |
| If the corresponding angles between two lines are equal, then the lines are parallel.          | corresp $\angle^s =$                                 |
| If the co-interior angles between two lines are supplementary, then the lines are parallel.    | co-int $\angle^s$ supp                               |

**TRIANGLES**

|  |  |
|--|--|
| The interior angles of a triangle are supplementary.   | $\angle$ sum in $\Delta$ <b>OR</b> sum of $\angle^s$ in $\Delta$<br><b>OR</b> int $\angle^s$ in $\Delta$ |
| The exterior angle of a triangle is equal to the sum of the interior opposite angles.  | ext $\angle$ of $\Delta$   |
| The angles opposite the equal sides in an isosceles triangle are equal.  | $\angle^s$ opp equal sides   |
| The sides opposite the equal angles in an isosceles triangle are equal.  | sides opp equal $\angle^s$   |
| In a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.  | Pythagoras <b>OR</b><br>Theorem of Pythagoras  |
| If the square of the longest side in a triangle is equal to the sum of the squares of the other two sides then the triangle is right-angled.                   | <b>Converse</b> Pythagoras<br><b>OR Converse</b> Theorem of Pythagoras                                   |
| If three sides of one triangle are respectively equal to three sides of another triangle, the triangles are congruent.   | SSS  |
| If two sides and an included angle of one triangle are respectively equal to two sides and an included angle of another triangle, the triangles are congruent. | SAS <b>OR</b> S $\angle$ S   |

|  |                                 |
|--|---------------------------------|
| If two angles and one side of one triangle are respectively equal to two angles and the corresponding side in another triangle, the triangles are congruent.                   | AAS <b>OR</b> $\angle \angle S$ |
| If in two right angled triangles, the hypotenuse and one side of one triangle are respectively equal to the hypotenuse and one side of the other, the triangles are congruent. | RHS <b>OR</b> $90^\circ$ HS     |

**QUADRILATERALS**

|   |   |
|---|---|
| The interior angles of a quadrilateral add up to $360^\circ$ .  | sum of $\angle^s$ in quad   |
| The opposite sides of a parallelogram are parallel.   | opp sides of $\parallel m$  |
| If the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.                   | opp sides of quad are $\parallel$ <b>OR</b><br><b>converse</b> opp sides of $\parallel m$ |
| The opposite sides of a parallelogram are equal in length.  | opp sides of $\parallel m$  |
| If the opposite sides of a quadrilateral are equal, then the quadrilateral is a parallelogram.                      | opp sides of quad are = <b>OR</b><br><b>converse</b> opp sides of a parm                  |
| The opposite angles of a parallelogram are equal.   | opp $\angle^s$ of $\parallel m$   |
| If the opposite angles of a quadrilateral are equal then the quadrilateral is a parallelogram.                      | opp $\angle^s$ of quad are = <b>OR</b><br><b>converse</b> opp angles of a parm            |
| The diagonals of a parallelogram bisect each other.   | diag of $\parallel m$   |
| If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.                   | diags of quad bisect each other<br><b>OR</b> <b>converse</b> diags of a parm              |
| If one pair of opposite sides of a quadrilateral are equal and parallel, then the quadrilateral is a parallelogram. | pair of opp sides = and $\parallel$   |
| The diagonals of a parallelogram bisect its area.   | diag bisect area of $\parallel m$   |
| The diagonals of a rhombus bisect at right angles.  | diags of rhombus  |
| The diagonals of a rhombus bisect the interior angles.  | diags of rhombus  |
| All four sides of a rhombus are equal in length.  | sides of rhombus  |
| All four sides of a square are equal in length.   | sides of square   |
| The diagonals of a rectangle are equal in length.   | diags of rect   |
| The diagonals of a kite intersect at right-angles.  | diags of kite   |
| A diagonal of a kite bisects the other diagonal.  | diag of kite  |
| A diagonal of a kite bisects the opposite angles.   | diag of kite  |