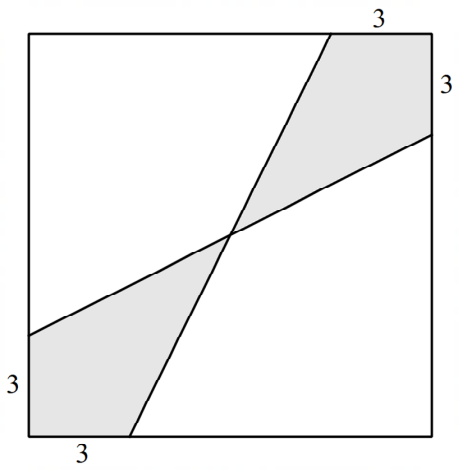
Level 4 Questions in the FET Phase

1. Without the use of a calculator, determine the value of $\dfrac{999999^2}{222222\times 666666}$

2. A square of side length 12 cm has straight lines drawn as shown below.

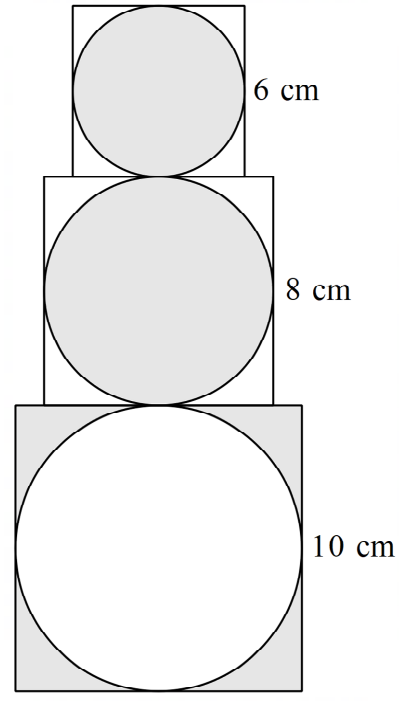


Determine the shaded area.

3. Determine the units digit of $(5+1)\times(5^2+1)\times(5^3+1)\times\ldots\times(5^{2024}+1)$

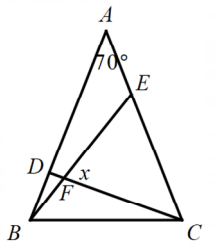
4. If $(3p-5)^2+(2q-p)^2\leq 1$ where p and q are integers, determine the value of $p-q$.

5. Determine the shaded area.



6. Determine the number of digits in $8^4 \times 35^3 \times 5^9$

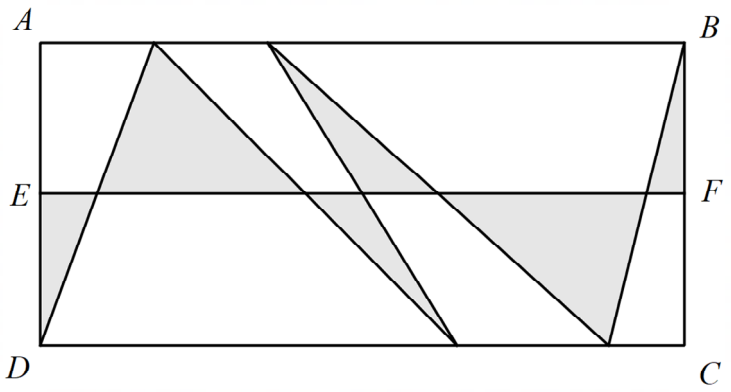
7. In the diagram, $AB = AC$, $A\hat{B}E = B\hat{C}D$ and $\hat{A} = 70°$.



Determine the size of x.

8. A piece of rope 130 cm long, is cut into four pieces so that each piece is one and a half times the length of the previous piece. Determine the length of the longest piece.

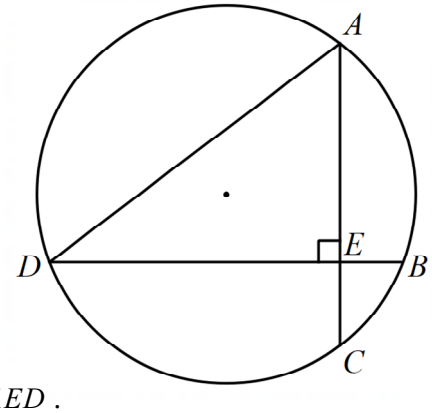
9. ABCD is a rectangle with E and F the midpoints of AD and BC respectively.



Determine the ratio of the shaded area to the unshaded area.

10. Determine the highest common factor of $2^{2024} + 2^{2025}$ and $3^{2024} + 3^{2025}$.

11. The diagram, which is not drawn to scale, has $AE = 7x - 5$, $DE = 5x - 7$, $BE = 2x - 13$ and $CE = x - 5$.

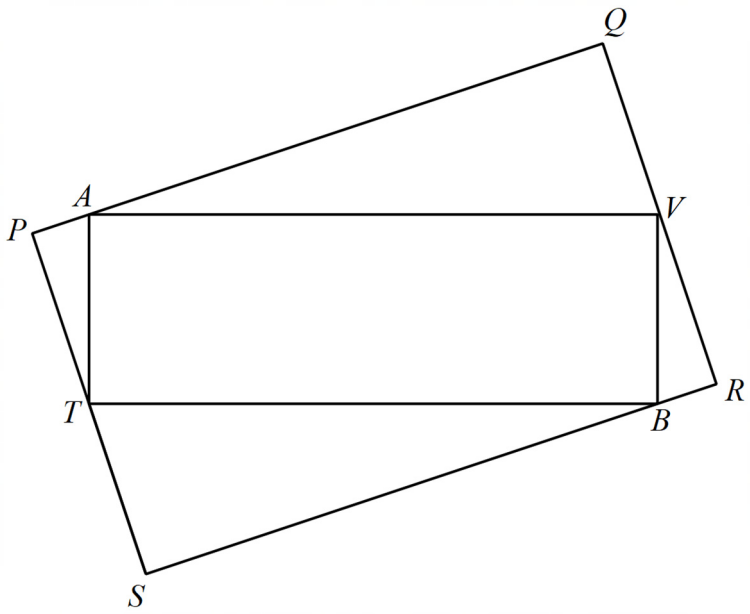


Determine the area of ΔAED.

12. Without the use of a calculator, determine the value of x if $6! \times 7! = x!$

13. Without the use of a calculator, determine the value of $2^{0^{2^4}} + 4^{2^{0^2}} + 2^{4^{2^0}} + 0^{2^{4^2}}$

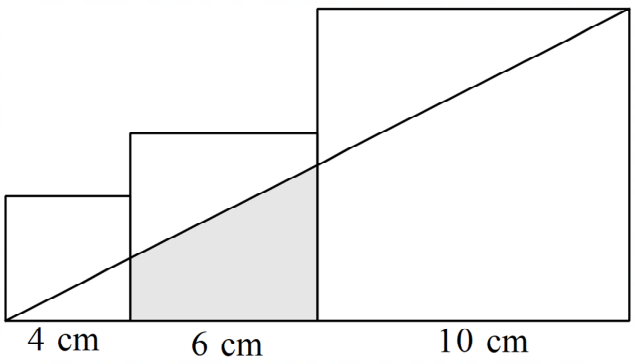
14. PQRS is a rectangle with T the midpoint of PS and V the midpoint of QR. A and B are drawn on PQ and SR respectively so that AVBT forms a rectangle.



Determine the area of PQRS in terms of x, if the area of AVBT is x units^2.

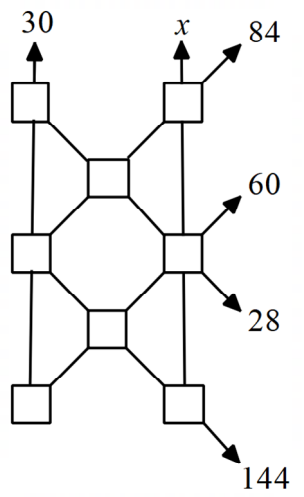
15. If today is Monday, what day of the week will it be in 2024 days time?

16. Three squares are drawn with sides 4 cm, 6 cm and 10 cm.



Determine the area of the shaded trapezium.

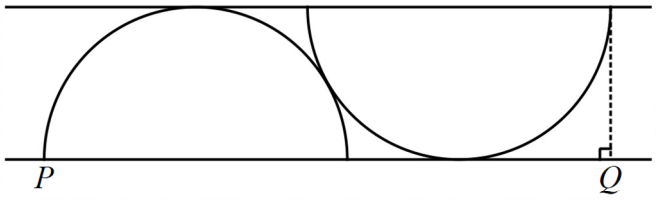
17. The numbers 1 to 8 are to be inserted in the boxes so that the product of the three numbers along each line gives the answer at the end.



Determine the value of x.

18. Determine the length of the shortest side of a right angled triangle if the perimeter is 168 units and the area is 756 units^2.

19. Two semi-circles of radius 1 unit are drawn between two parallel lines.



Determine the length of PQ.

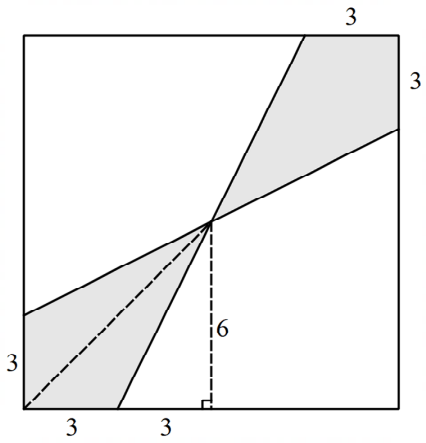
20. A right angle is to be divided using only three lines in such a way so that you have angles of $10°$; $20°$; $30°$; $40°$; $50°$; $60°$; $70°$ and $80°$. Draw a sketch to show how this is possible.

Note: you cannot add non-adjacent angles to form one of the required angles.

Level 4 Questions in the FET Phase Solutions

1. $$\frac{999999^2}{222222 \times 666666}$$
$$= \frac{\cancel{999999}^{\,9} \times 999999^{\,3}}{\cancel{222222}^{\,2} \times \cancel{666666}^{\,6}}$$
$$= \frac{27}{4}$$

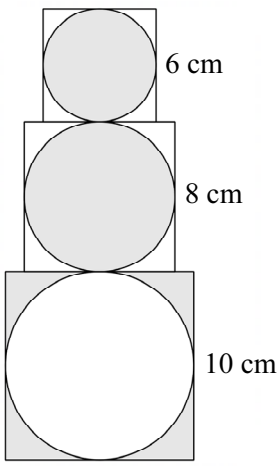
2. $A = 4\left[\frac{1}{2} \times 3 \times 6\right]$
$\therefore A = 36 \text{ cm}^2$

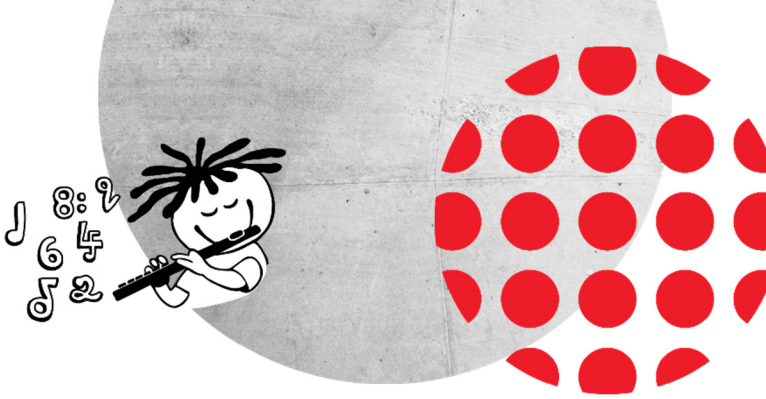


3. $(5+1) \times (5^2+1) \times (5^3+1) \times \ldots \times (5^{2024}+1)$
$= 6 \times 26 \times 126 \times \ldots \times (\ldots 6)$
$\therefore$ the units digit is 6

4. $-1 \leq 3p - 5 \leq 1$
$\therefore \frac{4}{3} \leq p \leq 2$
$\therefore p = 2$ ... p has to be an integer
$\therefore (3(2) - 5)^2 + (2q - 2)^2 \leq 1$
$\therefore (2q - 2)^2 \leq 0$
$\therefore 2q - 2 = 0$
$\therefore q = 1$
$\therefore p - q = 2 - 1 = 1$

5. $A = \left(10^2 - \pi.5^2\right) + \pi.4^2 + \pi.3^2$
$\therefore A = 100 - 25\pi + 16\pi + 9\pi$
$\therefore A = 100 \text{ cm}^2$





6. $8^4 \times 35^3 \times 5^9$

$= (2^3)^4 \times (5 \times 7)^3 \times 5^9$

$= 2^{12} \times 5^3 \times 7^3 \times 5^9$

$= 2^{12} \times 5^{12} \times 7^3$

$= 10^{12} \times 343$

$= 343\ 000\ 000\ 000\ 000$

$\therefore$ there are 15 digits.

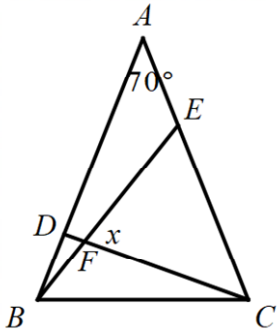
7. $A\hat{B}C + A\hat{C}B = 110°$ ($\angle$ sum of ΔABC)

$\therefore A\hat{B}C = A\hat{C}B = 55°$ ($\angle$s opp = sides)

Let $A\hat{B}E = B\hat{C}D = y$

$\therefore F\hat{B}C = 55° - y$

$\therefore x = 55° - y + y = 55°$ (ext $\angle$ of ΔFBC)



8. $x + \frac{3}{2}x + \frac{9}{4}x + \frac{27}{8}x = 130$

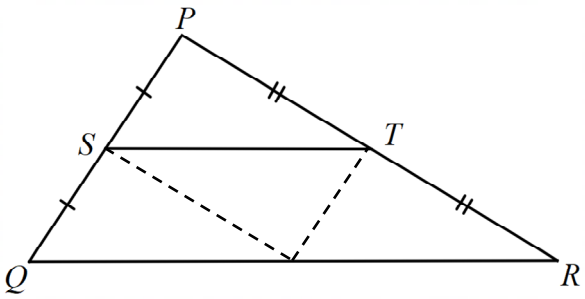
$\therefore 8x + 12x + 18x + 27x = 1\ 040$

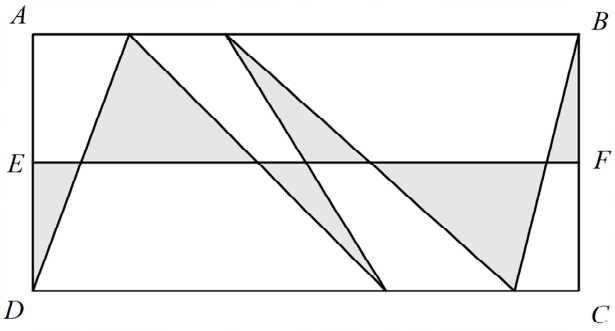
$\therefore 65x = 1\ 040$

$\therefore x = 16$

$\therefore$ the longest piece is 54 cm.

9.





Looking at ΔPQR above:

From the midpoint theorem, $ST = \frac{1}{2}QR$

$\therefore$ Area $\Delta PST = \frac{1}{4}$ Area ΔPQR . . . there are four congruent triangles

Looking at rectangle ABCD:

There are six triangles with the above situation.

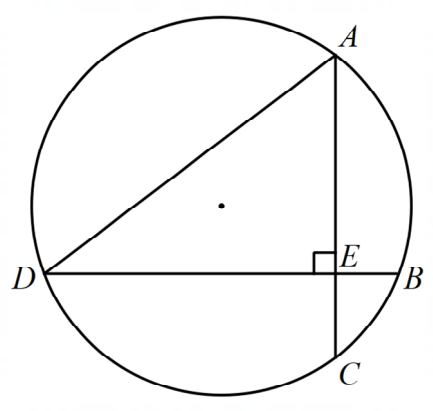
$\therefore$ the ratio of the shaded area to the unshaded area in ABCD = 1:3.

10. $2^{2024} + 2^{2025} = 2^{2024}(1+2) = 3.2^{2024}$

$3^{2024} + 3^{2025} = 3^{2024}(1+3) = 4.3^{2024} = 2^2.3^{2024}$

$\therefore$ the highest common factor is $2^2.3 = 12$

11. $\Delta AED \,|||\, \Delta BEC$ (AAA)



$$\therefore \frac{AE}{BE} = \frac{ED}{EC}$$

$$\therefore \frac{7x-5}{2x-13} = \frac{5x-7}{x-5}$$

$$\therefore 7x^2-40x+25 = 10x^2-79x+91$$

$$\therefore 3x^2-39x+66 = 0$$

$$\therefore x^2-13x+22 = 0$$

$$\therefore (x-11)(x-2) = 0$$

$$\therefore x = 11 \text{ or } x \neq 2$$

$$\therefore \text{Area } \Delta AED = \frac{1}{2} \times DE \times EA$$

$$\therefore \text{Area} = \frac{1}{2}(5(11)-7)(7(11)-5)$$

$$\therefore \text{Area} = 1\,728 \text{ units}^2$$

12. $6! \times 7!$

$= 6\times5\times4\times3\times2\times1\times7!$

$= 6\times5\times3\times8\times7!$

$= 2\times3\times5\times3\times8\times7!$

$= 2\times5\times9\times8\times7!$

$= 10\times9\times8\times7!$

$= 10!$

$\therefore x = 10$

13. $2^{0^{2^4}} + 4^{2^{0^2}} + 2^{4^{2^0}} + 0^{2^{4^2}}$

$= 2^{0^{16}} + 4^{2^0} + 2^{4^1} + 0^{2^{16}}$

$= 2^0 + 4^1 + 2^4 + 0^{\text{big}}$

$= 1+4+16+0$

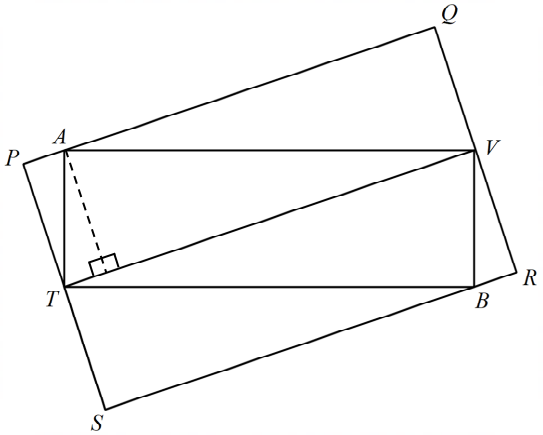
$= 21$

14. ΔATV and rectangle PQVT have the same base, TV, and the same perpendicular height.

$$\therefore \text{Area } \Delta ATV = \frac{1}{2} \text{Area } PQVT$$

Similarly Area $\Delta BTV = \frac{1}{2}$ Area $SRVT$

$\therefore$ Area $PQRS = 2x$ units^2



15. $\frac{2024}{7} = 289$ rem 1

$\therefore$ in 2024 days time it will be a Tuesday.

16. In ΔADE and ΔAFG, AD = DF and DE || FG.

$\therefore DE = \frac{1}{2}FG = 5$ (conv. midpt thm)

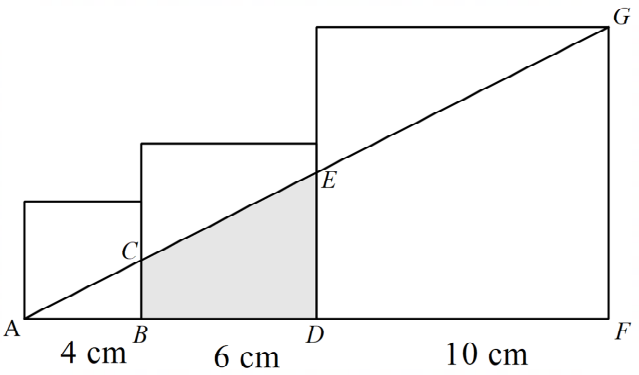
$\Delta ABC \,|||\, \Delta ADE$ (AAA)

$\therefore \frac{BC}{DE} = \frac{AB}{AD}$

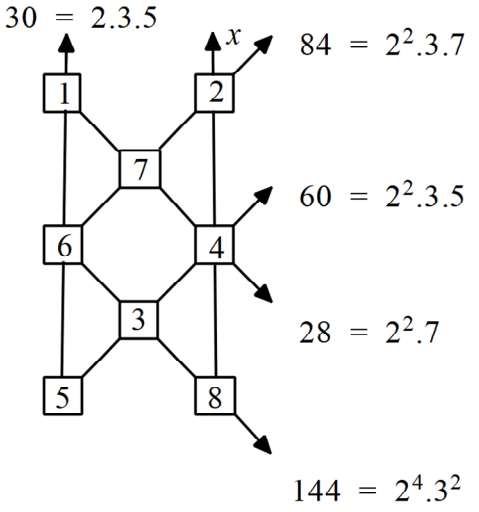
$\therefore \frac{BC}{5} = \frac{4}{10}$

$\therefore BC = 2$

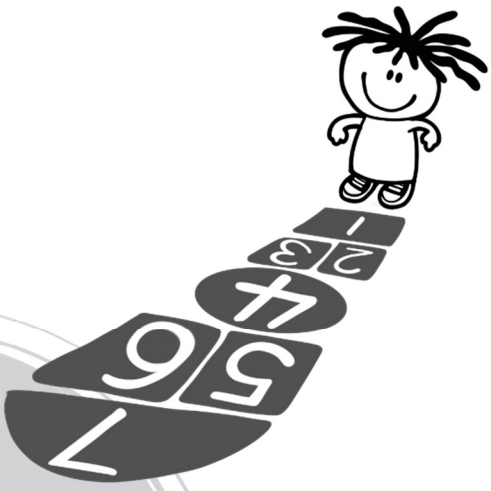
$\therefore$ Area $BCED = \frac{1}{2}(2+5)\times 6 = 21 \text{ cm}^2$



17.

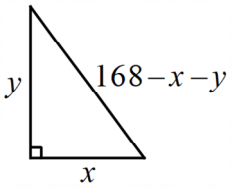


$\therefore x = 8\times 4\times 2 = 64$



18. Area: $\frac{1}{2}xy = 756$

$\therefore xy = 1\,512$



$x^2 + y^2 = (168 - x - y)^2$ (Pythag)

$\therefore x^2 + y^2 = 28\,224 + x^2 + y^2 - 336x - 336y + 2xy$

$\therefore 336(x + y) = 28\,224 + 2(1\,512)$

$\therefore 336(x + y) = 31\,248$

$\therefore x + y = 93$

$\therefore y = 93 - x$

$\therefore x(93 - x) = 1\,512$

$\therefore 93x - x^2 = 1\,512$

$\therefore x^2 - 93x + 1\,512 = 0$

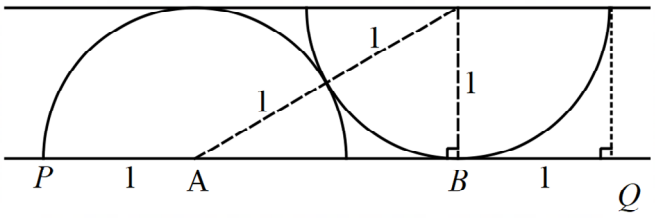
$\therefore (x - 72)(x - 21) = 0$

$\therefore x = 72$ or $x = 21$

$\therefore$ the shortest side is 21 units.

19. $AB = \sqrt{2^2 - 1^2} = \sqrt{3}$

$\therefore PQ = 2 + \sqrt{3}$ units.



20.