The NBT Mathematics (MAT) test: Exemplar questions

Questions in the section *Functions and their Graphs* will determine whether a test writer understands the properties of parabolas. Questions 1 and 2 are examples.

1. The function f defined by $y = f(x) = -x^2 + 6x - 5$ has
   (A) A minimum y value and a negative y-intercept.
   (B) A maximum y value and a positive y-intercept.
   (C) A minimum y value and a positive y-intercept.
   (D) A maximum y value and a negative y-intercept.

Under the heading Algebraic Processes one of the topics listed is *Algebraic Manipulation*. Question 2 is an example of a question where the answer cannot be deduced by substituting into the given options to rule out those that are correct.

2. The sum of the roots of the equation $-x^2 + 6x - 5 = 0$ is

   (A) -5 (B) -4 (C) 3 (D) 6

Another topic listed under the heading Algebraic Processes is *Number Sense.* The following question depends on *Number Sense* (the bigger a number, the bigger its square root) as well as the concepts tested in the first two questions above.

3. The expression $\sqrt{-x^2 + 6x - 5}$ has a

   (A) maximum value of 2 (B) minimum value of 2

   (C) maximum value of 3 (D) minimum value of 3

Question 4 is an example of the category *Transformations* and related concepts.

4. If the graph of $y = -x^2 + 6x - 5$ is reflected in the x-axis and the resulting graph is then reflected in the y-axis, the new equation is

   (A) $y = -(x-3)^2 + 4$ (B) $y = -x^2 - 6x - 5$

   (C) $y = (x+3)^2 + 4$ (D) $y = x^2 + 6x + 5$

One of the categories listed is *Competent use of logical skills in making deductions and determining the validity of given assertions.* Question 5 (which is also an example *Number Sense)* illustrates what this means. Writers need to assess the various options and make deductions about their validity.

5. For any real number x, which one of the following statements is **always** true?

(A) $-x < 0$ (B) $\frac{1}{x}$ is rational

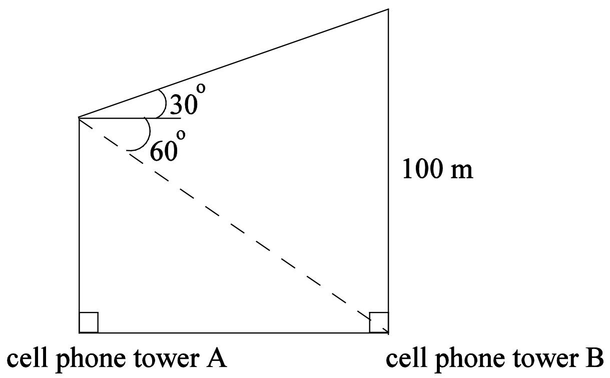
(C) $\frac{x}{x+1} < 1$ (D) $\frac{1}{x} > 1$ if $0 < x < 1$

The next two questions are in the *Trigonometry* category. Question 6 depends on an understanding of *compound angles*, and question 7 involves an application of *trigonometric ratios* in a two-dimensional situation.

6. $\sin 43^\circ \cos 23^\circ - \cos 43^\circ \sin 23^\circ$ is equal to

(A) $\cos 66^\circ$ (B) $\cos 20^\circ$
(C) $\sin 66^\circ$ (D) $\sin 20^\circ$

7.

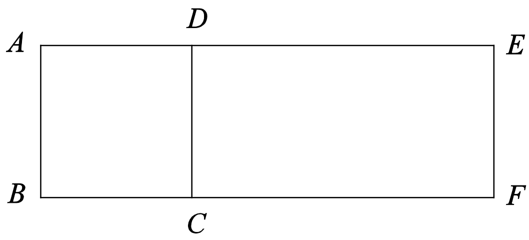


The angle of elevation of the top of cell phone tower B from the top of cell phone tower A is 30°. The angle of depression of the foot of cell phone tower B from the top of cell phone tower A is 60°. The height of cell phone tower B is 100m. The foot of cell phone tower A and the foot of cell phone tower B are in the same horizontal plane. The height of cell phone tower A is

(A) 60 m (B) 65 m (C) 70 m (D) 75 m

Question 8 combines an understanding of *Algebraic Manipulation* (in this case quadratics) and *Spatial Awareness* (rectangles) and Question 9 tests understanding of the *Properties of two- and three-dimensional objects,* as well as *surface area*.

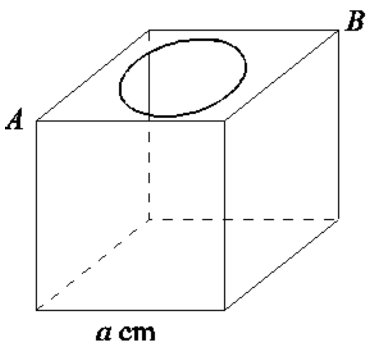
8.



Suppose ABCD is a square with side length $(x-1)$ cm. If the area of rectangle ABFE is (x^2+x-2) cm², then the length of FC, in cm, is

(A) 2 (B) 3 (C) 4 (D) 5

Question 9 is also an example of *Spatial Awareness*.



Use this QR code to access the solutions

9. The figure represents an empty cube with a circular opening at the top. The diameter of the opening is half the length of the diagonal AB. The outer surface area of the cube (in square centimetres) is:

(A) $6a^2 - \frac{\pi a^2}{4}$ (B) $6a^2 - 2\pi a^2$

(C) $6a^2 - \frac{\pi a^2}{8}$ (D) $6a^2 - \frac{\pi a^2}{2}$

Question 10 (from the *Algebraic Processes* subcategory *Financial Calculations*) shows what we mean when we say that calculators are unnecessary.

10. An amount of R1 000 is invested at an annual interest rate of 6%. Interest is compounded **every three months** (quarterly). After 5 years the investment, in rands, will be worth

(A) $1\,000\,(1{,}015)^{20}$ (B) $1\,000\,(1{,}02)^{15}$

(C) $1\,000\,(1{,}03)^{20}$ (D) $1\,000\,(1{,}06)^{5}$

The options given above show that we are interested in the *expression* you would use to carry out the calculation, and not in the final answer.

NBT MAT Exemplar Solutions

(These are unofficial solutions provided by The Answer Series)

Ignore the multiple choice options and solve the problem first, if possible.

1. (D)

The function f defined by $y = f(x) = -x^2 + 6x - 5$ has:

- $a = -1 < 0$ so, f has a maximum value
- $c = -5 < 0$ so, f has a negative y-intercept

2. (D)

Note that substitution is not an option because the roots are not given.

The sum of the roots requires the learner to understand how to solve the equation and find both roots. This question can be solved by knowing the formula for the sum of the roots, but this is not the intention of the question.

$$x^2 - 6x + 5 = 0$$

$$\therefore (x-1)(x-5) = 0$$

$$\therefore x = 1 \text{ or } x = 5$$

The sum of the roots is 6.

3. (A)

Number sense (extracts from "Preparing your learners for the MAT NBTs")

Manipulation/simple calculations involving integers, rational and irrational numbers (Page 12). In this question, number sense is required for a learner to understand "the bigger the number, the bigger its square root".

Completing the square

$$\sqrt{-x^2 + 6x - 5}$$

$$= \sqrt{-(x^2 - 6x + 5)}$$

$$= \sqrt{-\left[(x-3)^2 - 9 + 5\right]}$$

$$= \sqrt{-(x-3)^2 + 4}$$

When $x = 3$, the maximum value of $y = \sqrt{4} = 2$

Finding the TP first …

without using calculus | **using calculus**

$$f(x) = -x^2 + 6x - 5$$

Without using calculus:

$$x = -\frac{b}{2a} = -\frac{6}{2(-1)} = 3$$

Using calculus:

$$f'(x) = -2x + 6 = 0$$

$$\therefore x = 3$$

$$y = f(3) = -(3)^2 + 6(3) - 5 = 4$$

$$\therefore \text{ the max value of } y = \sqrt{4} = 2$$

4. (D)

Reflections of graphs require theoretical knowledge and/or intuitive understanding:

- reflections in the x-axis require the sign of y to change
- reflections in the y-axis require the sign of x to change
- reflections in the line $y = x$ create inverse graphs, with x and y swapping places

The reflection of $y = -x^2 + 6x - 5$ in the x-axis gives:

$$-y = -x^2 + 6x - 5$$

$$\therefore\ y = x^2 - 6x + 5$$

The subsequent reflection of $y = x^2 - 6x + 5$ in the y-axis gives:

$$y = (-x)^2 - 6(-x) + 5 = x^2 + 6x + 5$$

5. (D)

Number sense is needed in this question too.

To show that something is not "**always**" true requires just one counter example.

Option A: Is $-x < 0$ always true?

If $x = -3$, then $-(-3) = 3 > 0$, so this is not always true.

Option B: Is $\frac{1}{x}$ always rational?

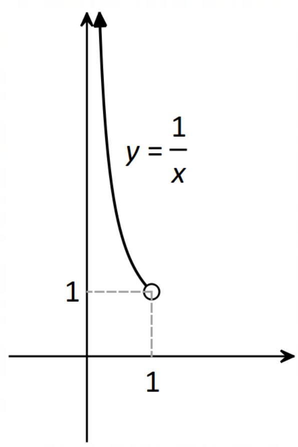
If $x = \pi$, then $\frac{1}{\pi}$ is not rational, so this is not always true.

Option C: Is $\frac{x}{x+1} < 1$ always true?

If $x = -2$, then $\frac{-2}{-2+1} = \frac{-2}{-1} = 2$ and this is not less than 1.

$\therefore \frac{x}{x+1} < 1$ is not always true.

Option D: Is $\frac{1}{x} > 1$ if $0 < x < 1$?



This is always true, but … it is not that easy to explain.

A sketch of a hyperbola, for the given

domain, shows that the statement is **always** true.

6. (D)

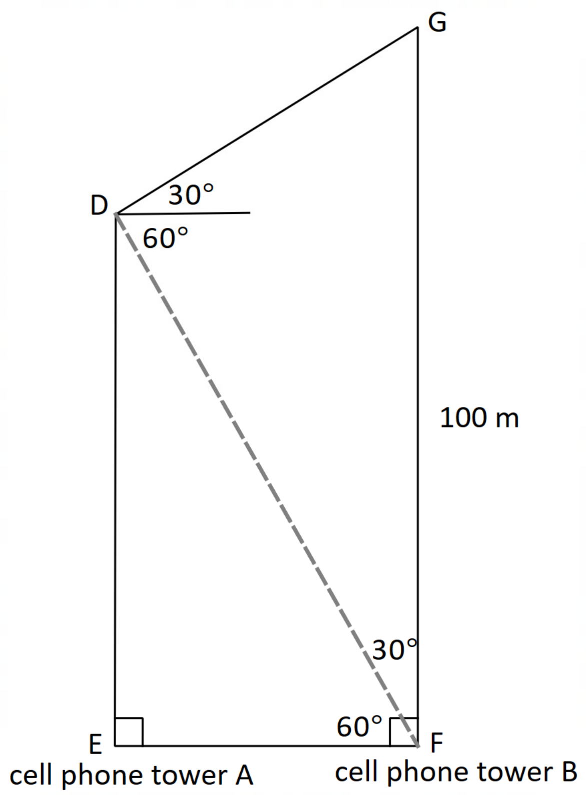
$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$\therefore\ \sin 43^\circ \cos 23^\circ - \cos 43^\circ \sin 23^\circ$

$= \sin(43^\circ - 23^\circ)$

$= \sin 20^\circ$

7. (D)



$D\hat{F}E = 60^\circ$ (alt $\angle$s, $\|$ lines)

$D\hat{F}G = 30^\circ$ (adj compl $\angle$s)

$\frac{DF}{100} = \cos 30^\circ = \frac{\sqrt{3}}{2}$

$\therefore\ DF = \frac{100\sqrt{3}}{2} = 50\sqrt{3}$

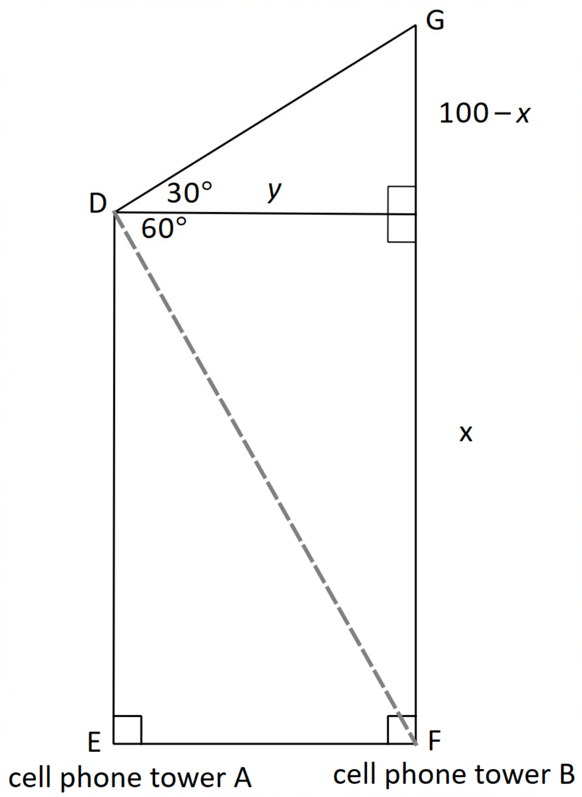
$\frac{DE}{50\sqrt{3}} = \sin 60^\circ = \frac{\sqrt{3}}{2}$

$\therefore\ DE = \frac{(50\sqrt{3})(\sqrt{3})}{2}$

$= 25 \times 3$

$= 75$ m

or



$\tan 30^\circ = \frac{100-x}{y} = \frac{1}{\sqrt{3}}$

$\therefore\ y = \sqrt{3}(100-x)$

$\tan 60^\circ = \frac{x}{\sqrt{3}(100-x)} = \frac{\sqrt{3}}{1}$

$\therefore\ x = 3(100-x)$

$\therefore\ x = 300 - 3x$

$\therefore\ 4x = 300$

$\therefore\ x = 75$ m

8. (B)

Area of rectangle ABFE

$= x^2 + x - 2$

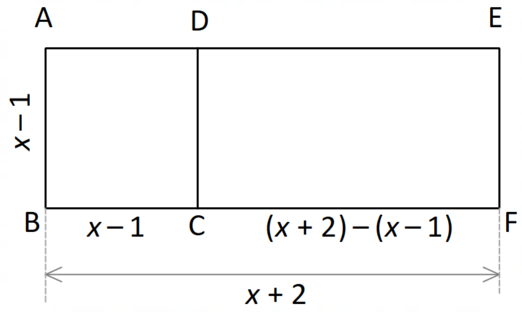
$= (x-1)(x+2)$

$\therefore$ BF $= (x+2)$cm

$\therefore$ CF $= (x+2) - (x-1)$

$= x + 2 - x + 1$

$= 3$ cm



or

BF $= x - 1 + y$

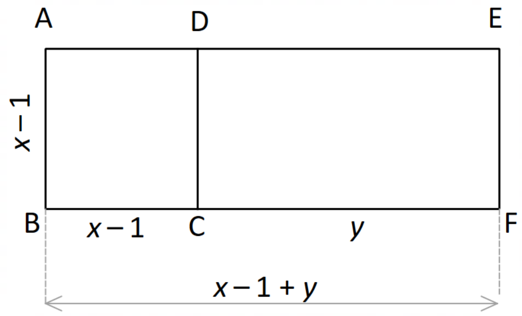
$\therefore (x-1)(x-1+y) = x^2 + x - 2$

$\therefore (x-1)(x-1+y) = (x-1)(x+2)$

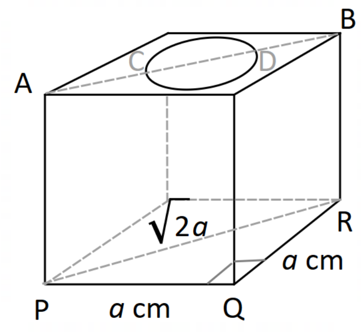
$\therefore x - 1 + y = x + 2 \qquad (x \neq 1)$

$\therefore y = 3$

$\therefore$ CF $= 3$cm



9. (C)



$$AB = PR = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a$$

$$CD = \frac{1}{2}AB = \frac{1}{2}\left(\sqrt{2}a\right) = \frac{\sqrt{2}a}{2}$$

$$\therefore\ r = \frac{1}{2}\left(\frac{\sqrt{2}a}{2}\right) = \frac{\sqrt{2}a}{4}$$

Area circle with diameter CD

$$= \pi r^2$$

$$= \pi\left(\frac{\sqrt{2}a}{4}\right)^2$$

$$= \pi\left(\frac{2a^2}{16}\right)$$

$$= \frac{\pi a^2}{8}$$

TSA of the cube

$$= 6a^2 - \frac{\pi a^2}{8}$$

10. (A)

4 quarters per year gives 20 quarters in 5 years.

6% p.a. compounded quarterly gives $\frac{6}{400} = \frac{3}{200} = \frac{1,5}{100} = 0,015$ per quarter.

$$1\,000(1 + 0,015)^{20} = 1000(1,015)^{20}$$