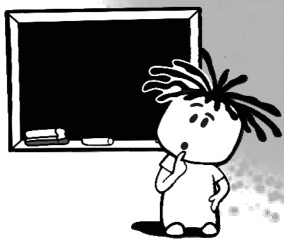
Problem Solving with a focus on Grade 7 (and 8 & 9)

Problem solving sums need you to do more than simply add, subtract, multiply or divide.
You may not know where to begin!

Lateral thinking is needed to find the solution.



Helpful Steps

1. Read the question carefully and think about the following:
   - What do you know?
   - What options do you have?
2. Is there anything you CAN do just to get started?

Below are five helpful strategies to get you started

The more you practise, the easier it will become to know which of the strategies is most suited to solving a particular problem. Some problems can be solved using two or more of the strategies shared here.

❶ **Trial and Improvement**

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If the sum has large numbers, it may help to work it out with simpler numbers first.
If you need to work out a result for a large number of values, start with just a few.
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❺ **Given a part, work out the whole**

You can work back to 1% or 10% or … any other percentage that makes sense in the context of the question.

Worksheet

1. There are hippos and ducks swimming in a pond.
   They have a combined total of 28 heads and 80 legs.
   How many hippos are there in the pond?

2. Two numbers have a product of 90 and a sum of 21.
   What are the numbers?

3. Joe travels from at 100 km/h and leaves home at 7:00. Mandla travels at 120 km/h and leaves home an hour later. They travel the same route as each other and don't stop along the way.

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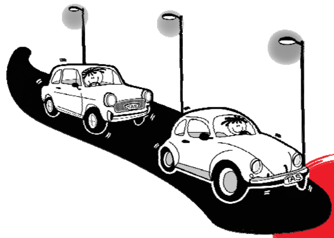

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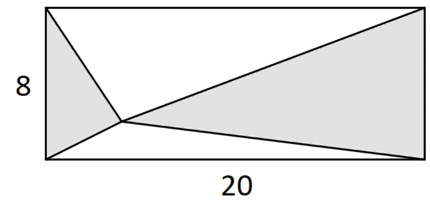

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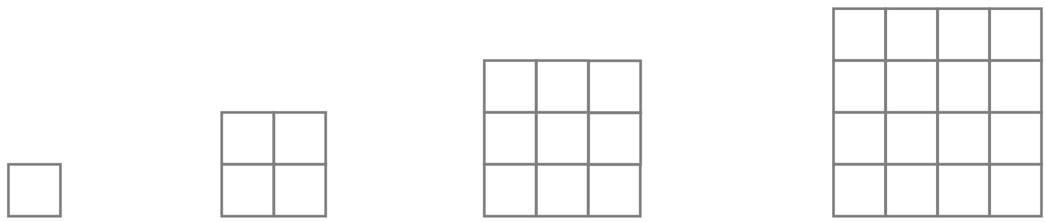

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   6.1 Work out how many squares, of any size, there are in a 4×4 square.

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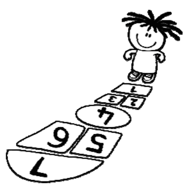

8. The sum of two numbers is 100.
   What is the largest possible product of these numbers?

9. A water tank is 60% full. After 4 155 litres of water have been used, it is only 45% full.
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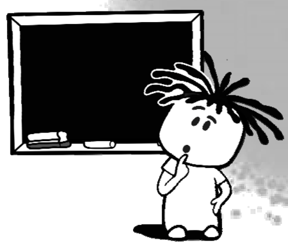

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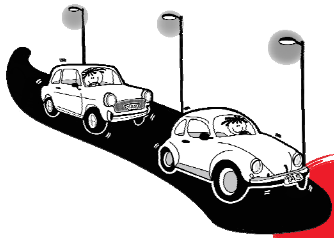

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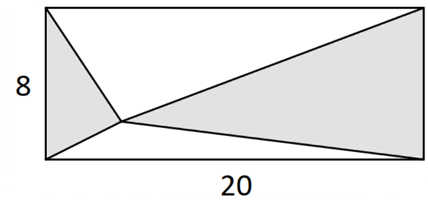

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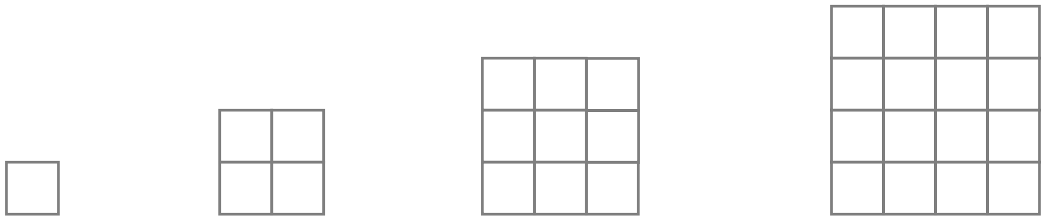

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   6.1 Work out how many squares, of any size, there are in a 4×4 square.

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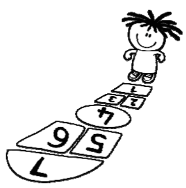
7. Find the sum of the first 100 EVEN numbers.



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Worksheet Solutions

1. There are hippos and ducks swimming in a pond.
They have a combined total of 28 heads and 80 legs.
How many hippos are there in the pond?

Hint
After each trial, stop and check.
Use the answer to improve your next trial.

Answer

1st Trial: 14 hippos and 14 ducks
$4 \times 14 + 2 \times 14 = 56 + 28 = 84$

The are 4 more legs than there should be, so we need less hippos and more ducks.

2nd Trial: 13 hippos and 15 ducks
$4 \times 13 + 2 \times 15 = 52 + 30 = 82$

Replace one more hippo with a duck.

3rd Trial: 12 hippos and 16 ducks
$4 \times 12 + 2 \times 16 = 48 + 32 = 80$

$\therefore$ there are 12 hippos in the pond

Algebraic solution

Let the number of hippos be x.
$\therefore$ the number of ducks is $28 - x$.

The total number of legs (in terms of x)

$= 4x + 2(28 - x)$

$= 4x + 56 - 2x$

$= 2x + 56$

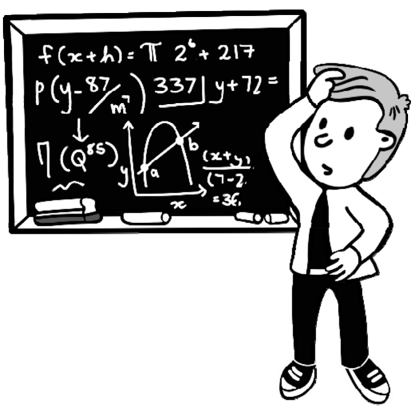
$\therefore 2x + 56 = 80$

$\therefore 2x = 24$

$\therefore x = 12$

$\therefore$ there are 12 hippos (and 16 ducks).

Check: $12(4) + 16(2) = 48 + 32 = 80$ legs



2. Two numbers have a product of 90 and a sum of 21.

What are the numbers?

Answer

Trial 1: $10+11=21$ and $10\times11=110$ ✗

Trial 2: $5+16=21$ and $5\times16=80$ ✗

Trial 3: $15+6=21$ and $15\times6=90$ ✓

$\therefore$ the numbers are 15 and 6

Hint

One of the numbers must end in 0 or 5 because 90 ends in 0.

Algebraic solution

Let one of the numbers be *x*.

$\therefore$ the other number is $21-x$.

$$\therefore\ x(21-x)=90$$

$$\therefore\ 21x-x^2=90$$

$$\therefore\ x^2-21x+90=0$$

$$(x-6)(x-15)=0$$

$$\therefore\ x=6 \text{ or } x=15$$

$\therefore$ the numbers are 6 and 15.

Check: $6+15=21$ and $6\times15=90$

3. Joe travels from at 100 km/h and leaves home at 7:00. Mandla travels at 120 km/h and leaves home an hour later. They travel the same route as each other and don't stop along the way.

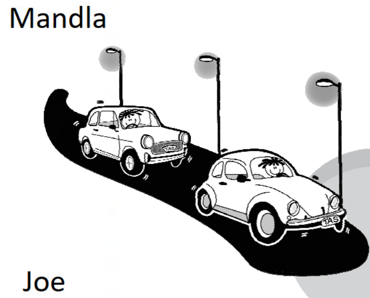
3.1 At what time does Mandla catch up with Joe?

3.2 How far had Joe travelled when Mandla caught up with him?

Answer

3.1

8:00	9:00	10:00	11:00	12:00	13:00
0	120	240	360	480	600



7:00	8:00	9:00	10:00	11:00	12:00	13:00
0	100	200	300	400	500	600

Mandla caught up with Joe at 13:00.

Take Note

Once the problem makes sense, you can consider more efficient methods.

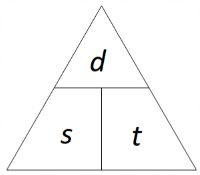
For example, the LCM of 100 and 120 is 600.

3.2 Joe had travelled 600 km when Mandla caught up with him.

Algebraic solution

3.1

	distance (in km)	speed (km/h)	time (in hours)
Joe	$100t$	100	t
Mandla	$120(t-1)$	120	$t-1$



$$120(t-1) = 100t$$

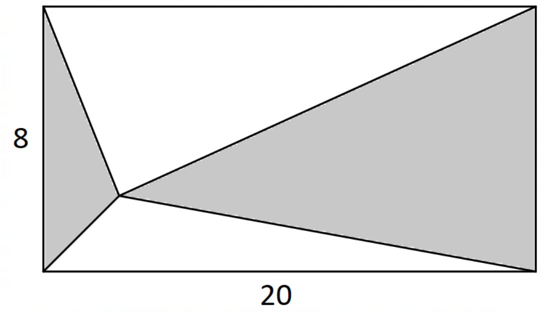
$$\therefore\ 120t - 120 = 100t$$

$$\therefore\ 20t = 120$$

$$\therefore\ t = 6$$

Mandla left at 8:00 and travelled for 5 hours, so he caught up with Joe at 13:00.

4.



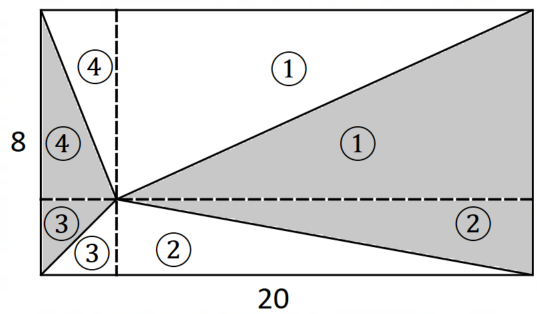
Take Note

This question can be asked without any dimensions being given if it is changed to a ratio question.

For example, what is the ratio of the shaded region to the unshaded region.

A rectangle has length 20 cm and width 8 cm. Find the area of the shaded region.

Answer



The area of the whole rectangle is 20 cm $\times$ 8 cm = 160 cm^2

Divide the rectangle into 4 smaller rectangles.

In each small rectangle, the diagonal halves the area.

The shaded area is 80 cm^2 (It is exactly half of the total area).

What is the ratio of the area of the shaded region to the area of the unshaded region.?

Divide the rectangle into 4 smaller rectangles. (refer to the sketch above)

In each small rectangle, the diagonal halves the area.

The area of the shaded region : the area of the unshaded region $= 1:1$

What is the ratio of the area of the shaded region to the area of the whole region?

The area of the shaded region : the area of the whole region $= 1:2$

5. Consider the letters A, B, C, D and E.

5.1 In how many different ways can the letters A, B and C be arranged?

5.2 In how many ways can the letters A, B, C, D and E be arranged?

Answer

Draw a diagram to help you understand the problem:

①	②	③	④	⑤	⑥
A B C	A C B	B A C	B C A	C A B	C B A

If you start with A, there are only two letters left, so arrange them both ways.

If you start with B, there are also two letters left, arrange them both ways.

Starting with C, the same logic applies.

5.1 A, B and C can be arranged in 6 different ways.

Take Note

When a question is broken up into parts

1) Are the questions linked or do they appear to be similar?
2) If they are linked or similar
   - can we use the same logic for both questions?
   - can we find a quicker way to solve the problem?

In 5.1 you are asked to work with the letters A, B and C.

In 5.2 you are asked to work with the letters A, B, C, D and E.

Another look at 5.1 shows us that we could have used the logic below.

- 3 ways to choose the 1st letter
- 2 ways to choose the 2nd letter
- 1 way to choose the 3rd letter
- $3 \times 2 \times 1 = 6$

Use this logic to arrange the letters A, B, C and D.

There are 4 ways to choose the 1st letter, 3 ways to choose the 2nd letter, 2 ways to choose the 3rd letter and only 1 way to choose the last letter.

Now answer question 5.2.

5.2 $5 \times 4 \times 3 \times 2 \times 1 = 120$

There are 120 different arrangements of the numbers A, B, C, D and E.

6.

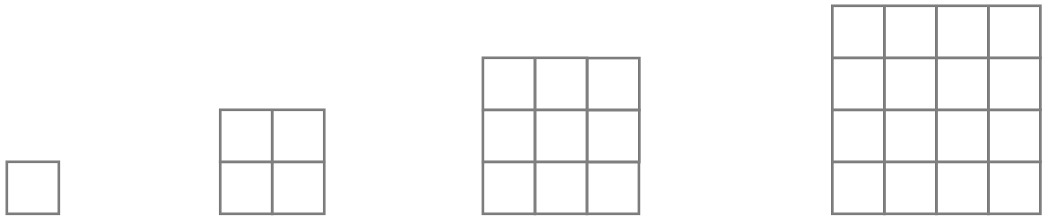


Figure 1 Figure 2 Figure 3 Figure 4

6.1 Work out how many squares, of any size, there are in a 4×4 square.

6.2 Without drawing a sketch, work out the number of squares, of any size, in a 5×5 square.

Hint

You need to be very systematic in questions like this.

Watch out for a pattern that may emerge from your working.

Patterns help you predict what will happen next.

They can save a lot of time and effort!

Answer

Draw a table to record the number of squares of each size:

Number of Squares

Size of square	1×1	2×2	3×3	4×4	Total number of squares
Figure 1	1				1
Figure 2	4	1			5
Figure 3	9	4	1		14
Figure 4	16	9	4	1	30

6.1 There are 30 squares in the fourth figure.

6.2 In a 5×5 square the total number of squares
$= 1 + 4 + 9 + 16 + 25$
$= 55$ squares in total.

Hint

1, 4, 9 and 16 are square numbers

You have found a pattern.

$1 = 1 \times 1 = 1^2$

$4 = 2 \times 2 = 2^2$

$9 = 3 \times 3 = 3^2$

$16 = 4 \times 4 = 4^2$

Continue the pattern

$5 \times 5 = 25$

There is a formula for this pattern, but … it is something to look forward to in Further Studies Maths.

The sum of consecutive square numbers $= \frac{n^3}{3} + \frac{n^2}{2} + \frac{n}{6}$, where n is the number of perfect squares.

7. Find the sum of the first 100 EVEN numbers.

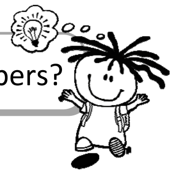
Answer

		Notice the Pattern
The 1^{st} EVEN number is	2	1×2
The sum of the 1^{st} TWO EVEN Numbers	$2 + 4 = 6$	2×3
The sum of the 1^{st} THREE EVEN Numbers	$2 + 4 + 6 = 12$	3×4
The sum of the 1^{st} FOUR EVEN Numbers	$2 + 4 + 6 + 8 = 20$	4×5

The sum of the 1^{st} 100 EVEN Numbers = 100×101 = 10 100

Food for Thought

Can you find the sum of the first 100 ODD numbers?



Answer		Notice the Pattern
The 1^{st} ODD number is	1	$1 = 1^2$
The sum of the 1^{st} TWO ODD Numbers	$1 + 3 = 4$	$4 = 2^2$
The sum of the 1^{st} THREE ODD Numbers	$1 + 3 + 5 = 9$	$9 = 3^2$
The sum of the 1^{st} FOUR ODD Numbers	$1 + 3 + 5 + 7 = 16$	$16 = 4^2$

The sum of the 1^{st} 100 ODD Numbers = 100×100 = 10 000

More Food for Thought

What is the sum of the first 200 numbers?

Answer

The first 100 ODD numbers + the first 100 EVEN numbers

$= 100 \times 101 + 100^2$

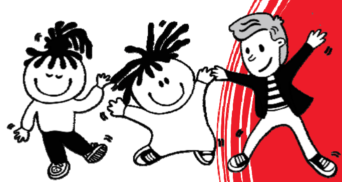
$= 10\ 100 + 10\ 000$

$= 20\ 100$

Can we find a formula to make these calculations easier?

$$\begin{array}{l} \quad 1 + \quad 2 + \quad 3 + \ldots + 198 + 199 + 200 \\ \underline{200 + 199 + 198 + \ldots + \quad 3 + \quad 2 + \quad 1} \\ \underline{201 + 201 + 201 + \ldots + 201 + 201 + 201} \text{ for 200 terms} \end{array}$$

$$\frac{200 \times 201}{2} = 100 \times 201 = 20\ 100$$



Yes we can 😊 😊

The sum of the first n numbers is $\frac{n(n+1)}{2}$

8. The sum of two numbers is 100.

What is the largest possible product of these numbers?

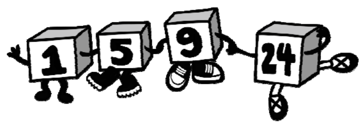
Try to understand the problem using 6.

Use a small number like 6 first so you can try all possible combinations.

Answer

If the sum of two numbers is 6, the options are:

Sum	$1 + 5 = 6$	gives product	$1 \times 5 = 5$
Sum	$2 + 4 = 6$	gives product	$2 \times 4 = 8$
Sum	$3 + 3 = 6$	gives product	$\mathbf{3 \times 3 = 9}$
Sum	$4 + 2 = 6$	gives product	$4 \times 2 = 8$
Sum	$5 + 1 = 6$	gives product	$5 \times 1 = 5$

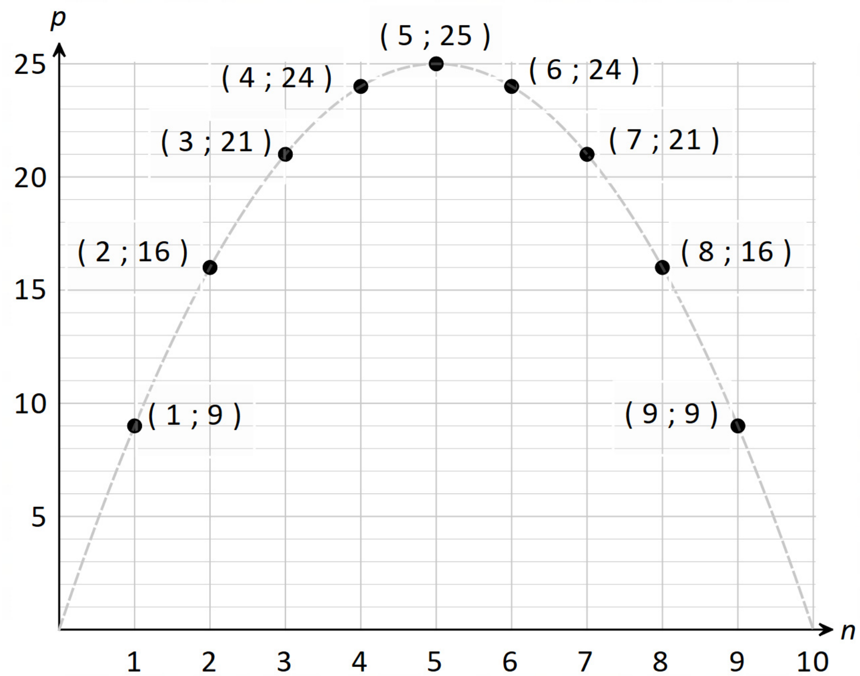


Maximum product is 9 (found when numbers are equal).

If the sum of the numbers is 100, the maximum product is $50 \times 50 = 2\ 500$.

Below is a graph showing the relationship two numbers that have a sum of 10, and the product of these two numbers. The points that are plotted show one of the two numbers, and their product.

The point (3; 21) shows that one of the numbers is 3, which means the other number must be 7. The 7 is not shown, but the product of 3 and 7, which is 21, is shown.



9. A water tank is 60% full. After 4 155 litres of water have been used, it is only 45% full.

How much water does the tank hold when it is full?

Answer

Working back to 1%

15% of the water in the tank equates to 4 155 litres of water.

1% of the water in the tank equates to $\dfrac{4\ 155 \div 5}{15 \div 5} = \dfrac{831}{3} = 277$ litres of water

$\therefore$ 100% of the water in the tank equates to 27 700 litres of water

Working back to 5%

15% of the water in the tank equates to 4 155 litres of water.

5% of the water in the tank equates to $\dfrac{4\ 155}{3} = 1\ 385$ litres of water

$\therefore$ 100% of the water in the tank equates to $1\ 385 \times 20 = 27\ 700$ litres of water

Algebraic solution

Let the tank hold x litres of water when it is full.

15% of the tank holds 4 155 litres of water.

$\therefore\ 0{,}15x = 4\ 155$

$$\therefore\ x = \frac{4\ 155}{0{,}15} = \frac{415\ 500}{15} = \frac{83\ 100}{3} = 27\ 700 \text{ litres of water}$$

10 All sporting equipment is marked down by 20% and a particular item now costs R1 800.

What was the original price of the item?

Answer

Working back to 1%

80% of the original price is R1 800.

1% or the original price is $\dfrac{1\ 800}{80} = \text{R}22{,}50$

100% of the original price is $100 \times 22{,}5 = \text{R}2\ 250$

Working back to 20%

80% of the original price is R1 800.

20% or the original price is $\dfrac{1\ 800}{4} = \text{R}450$

100% of the original price is $\text{R}1\ 800 + 450 = \text{R}2\ 250$

or

100% of the original price is $5 \times 450 = \text{R}2\ 250$

Algebraic solution

Let the original price be x rand.

$\therefore\ 0{,}8x = 1\ 800$

$\therefore\ x = \dfrac{1\ 800}{0{,}8} = \dfrac{18\ 000}{8} = 2\ 250$

The original price of the item was R2 250.